# Microscopic Simulation of Synchronized Flow in City Traffic: Effect of Driver's Speed Adaptation

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***Abstract*—Recently, the synchronized flow traffic pattern has been found in empirical data of oversaturated city traffic. Traffic simulation models based on classical theories cannot reproduce synchronized flow. We present simulation results based on Kerner's three-phase traffic theory. By using a version of the Kerner-Klenov stochastic microscopic traffic flow simulation model, we are able to reproduce all traffic patterns found in city traffic, including synchronized flow. The key role in understanding the emergence of synchronized flow within the model plays the driver's speed adaptation effect.**

***Keywords*–*Microscopic simulation; Three-phase traffic theory; City traffic.***

## I. Introduction

On an urban road with a traffic signal various congested traffic patterns can be observed. The understanding of these city traffic phenomena is important for the development of effective tools for traffic management and intelligent transportation systems.

There are two main types of city traffic on multilane roads at traffic signals: under- and over-saturated traffic. In accordance with the classical theories [1][2][3][4][5][6], when the flow rate increases above some signal capacity value, a transition from under- to over-saturated traffic occurs. In undersaturated traffic, all vehicles, which are waiting within a queue during the red phase, can pass the signal during the green phase. An opposite case occurs in oversaturated traffic: some of the vehicles in the queue cannot pass the signal location during the green phase resulting in the queue growth (see Figure 1 (a, b)). In accordance with the classical theories [1][2][3][4][5][6], well-developed oversaturated traffic consists of a sequence of moving queues with stopped vehicles separated by regions in which vehicles move from one moving queue to the adjacent downstream moving queue; the mean duration of the vehicle stop within a moving queue does not usually change while the moving queue propagates upstream of the signal (see Figure 1 (c)).

Based on simulations in the framework of the three-phase theory [7][8], Kerner et al. have recently predicted that in addition to classical sequences of moving queues (see Figure 1), in oversaturated traffic synchronized flow pattern(s) (SP) should also occur [9]. In comparison with [9], in this article we make the following new developments: i) we simulate and discuss single vehicle (microscopic) acceleration related to synchronized flow in oversaturated city traffic and ii) we make a briew comparison of simulated synchronized flow with empirical synchronized flow revealed recently in measured field data in oversaturated city traffic. For simulations of spatiotemporal features of SPs in oversaturated traffic we use the Kerner-Klenov stochastic microscopic traffic flow model [10][11][12] in the framework of Kerner's three-phase theory.

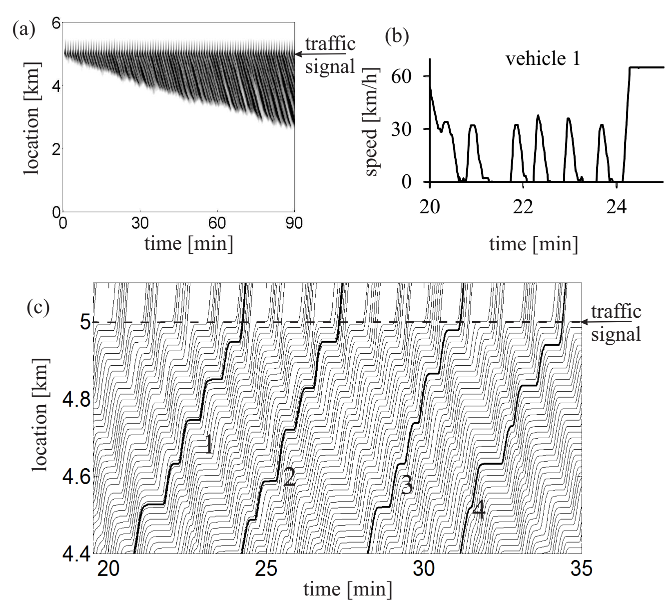


Figure 1. Spatiotemporal structure of oversaturated traffic at a traffic signal of the classical theory of city traffic. Taken from [9].

This article is organized as follows. In Section 1, a short introduction to the theory of synchronized traffic flow in a city is given, and the simulation model is described. In Section

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2, empirical findings of synchronized flow in city traffic are presented and compared with simulation results. In Section 3, a conclusion and an outlook to further work are given.

## II. KERNER-KLENOV SIMULATION MODEL

Urban traffic on a multilane road with a traffic signal at its end can either be oversaturad or undersaturated. In undersaturated city traffic all vehicles waiting in a queue in front of a traffic signal can pass the signal in the next green light phase. In oversaturated city traffic on the other hand, not all vehicles can pass and therefore the queue of waiting vehicles grows with each cycle of the signal. In classical theories the oversaturated city traffic consists of a series of moving queues, sequences of stopped vehicles interrupted by sequences of vehicles that move from one moving queue to the next queue downstream. In general the duration of the vehicle stop within a moving queue does not change in average while the moving queue propagates upstream of the signal (see Figure 1).

It is usually assumed [1][2][3][4][5][6] that the transition from under- to over-saturated traffic, i.e., traffic breakdown at the signal, occurs at a classical capacity of traffic signal $C_{\mathrm{cl}} = q_{\mathrm{sat}} T_{\mathrm{G}}^{(\mathrm{eff})}/\vartheta$, where $q_{\mathrm{sat}}$ is the saturation flow rate, i.e., the mean flow rate from a queue at the signal during the green phase when vehicles discharge from the moving queue to their maximum free speed $v_{\mathrm{free}}$. $\vartheta = T_{\mathrm{G}} + T_{\mathrm{Y}} + T_{\mathrm{R}}$ is the cycle time of the signal and assumed to be constant, $T_{\mathrm{G}}$, $T_{\mathrm{Y}}$, and $T_{\mathrm{R}}$ are the durations of the green, yellow, and red phases of the signal, respectively. $T_{\mathrm{G}}^{(\mathrm{eff})}$ is the effective green phase time that is the portion of the cycle time during which vehicles are assumed to pass the signal at constant rate $q_{\mathrm{sat}}$.

Within anonymized vehicle probe data from oversaturated city traffic provided by TomTom, rather than this stop-and-go pattern built by the moving queues, synchronized flow according to the three-phase traffic theory by Kerner has been found [13].

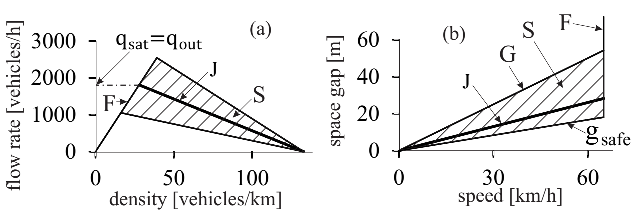


Figure 2. 2D-regions for steady states of synchronized flow in the flow–density (a) and the space-gap–speed planes (b) [10][11][12][18][19]. F – free flow, S – synchronized flow, J – line $J$ [7][8].

In contrast with two-phase traffic flow models with a fundamental diagram (e.g., [14][15][16][17]), in the stochastic model [10][11][12] used for all simulations, there is a 2D-region of synchronized flow associated with the fundamental hypothesis of three-phase theory (see Figure 2) [7]. When a driver approaches a slower moving preceding vehicle and she/he cannot overtake it, then the driver begins to decelerate and adapts its speed to the speed of the preceding vehicle, when the gap $g$ to the preceding vehicle becomes smaller than a synchronization gap $G$ (see Figure 2 (b)). This driver's speed adaptation occurs under condition $g_{\mathrm{safe}} \le g \le G$, where $g_{\mathrm{safe}}$ is a safe gap.

We use a discrete version of a stochastic three-phase microscopic model of Kerner and Klenov [10][11][12]. The physics of the model variables are explained in [7][8]. The parameteres of the model have been adapted for city traffic in [18][19], the vehicle speed $v_{n+1}$, the coordinate $x_{n+1}$, and the acceleration $A_{n+1}$ at time step $n+1$ are found from the following equations:

$$v_{n+1} = \max(0, \min(v_{\mathrm{free}}, \tilde{v}_{n+1} + \xi_n, v_n + a_{\max}\tau, v_{\mathrm{s},n})), \tag{1}$$

$$x_{n+1} = x_n + v_{n+1}\tau, \tag{2}$$

$$A_{n+1} = (v_{n+1} - v_n)/\tau, \tag{3}$$

$$\tilde{v}_{n+1} = \min(v_{\mathrm{free}}, v_{\mathrm{s},n}, v_{\mathrm{c},n}), \tag{4}$$

$$v_{\mathrm{c},n} = \begin{cases} v_{\mathrm{c},n}^{(1)} & \text{at } \Delta v_n + A_{\ell,n}\tau < \Delta v_{\mathrm{a}}, \\ v_{\mathrm{c},n}^{(2)} & \text{at } \Delta v_n + A_{\ell,n}\tau \ge \Delta v_{\mathrm{a}}, \end{cases} \tag{5}$$

$\Delta v_{\mathrm{a}}$ is constant.

$$v_{\mathrm{c},n}^{(1)} = \begin{cases} v_n + \Delta_n^{(1)} & \text{at } g_n \le G_n, \\ v_n + a_n\tau & \text{at } g_n > G_n, \end{cases} \tag{6}$$

$$\Delta_n^{(1)} = \max(-b_n\tau, \min(a_n\tau,\ v_{\ell,n} - v_n)), \tag{7}$$

$$v_{\mathrm{c},n}^{(2)} = v_n + \Delta_n^{(2)}, \tag{8}$$

$$\Delta_n^{(2)} = k_{\mathrm{a}} a_n \tau \max(0, \min(1,\ \gamma(g_n - v_n\tau))), \tag{9}$$

$$a_{\max} = \begin{cases} a & \text{at } \Delta v_n + A_{\ell,n}\tau < \Delta v_{\mathrm{a}}, \\ k_{\mathrm{a}} a & \text{at } \Delta v_n + A_{\ell,n}\tau \ge \Delta v_{\mathrm{a}}, \end{cases} \tag{10}$$

$$a_n = a\Theta(P_0 - r_1),\ b_n = a\Theta(P_1 - r_1), \tag{11}$$

$$P_0 = \begin{cases} p_0 & \text{if } S_n \ne 1, \\ 1 & \text{if } S_n = 1, \end{cases} \qquad P_1 = \begin{cases} p_1 & \text{if } S_n \ne -1, \\ p_2 & \text{if } S_n = -1, \end{cases} \tag{12}$$

$$S_{n+1} = \begin{cases} -1 & \text{if } \tilde{v}_{n+1} < v_n, \\ 1 & \text{if } \tilde{v}_{n+1} > v_n, \\ 0 & \text{if } \tilde{v}_{n+1} = v_n, \end{cases} \tag{13}$$

$r_1 = \mathrm{rand}(0,1)$, $\Theta(z) = 0$ at $z < 0$ and $\Theta(z) = 1$ at $z \ge 0$, $p_0 = p_0(v_n)$, $p_2 = p_2(v_n)$, $p_1$ is constant.

$$\xi_n = \begin{cases} \xi_{\mathrm{a}} & \text{if } S_{n+1} = 1, \\ -\xi_{\mathrm{b}} & \text{if } S_{n+1} = -1, \\ \xi^{(0)} & \text{if } S_{n+1} = 0, \end{cases} \tag{14}$$

$$\xi_{\mathrm{a}} = a^{(\mathrm{a})}\tau\Theta(p_{\mathrm{a}} - r),\ \xi_{\mathrm{b}} = a^{(\mathrm{b})}\tau\Theta(p_{\mathrm{b}} - r), \tag{15}$$

$$\xi^{(0)} = a^{(0)}\tau \begin{cases} -1 & \text{if } r \le p^{(0)}, \\ 1 & \text{if } p^{(0)} < r \le 2p^{(0)} \text{ and } v_n > 0, \\ 0 & \text{otherwise,} \end{cases} \tag{16}$$

$r = \mathrm{rand}(0,1)$; $a^{(\mathrm{b})} = a^{(\mathrm{b})}(v_n)$; $p_{\mathrm{a}}$, $p_{\mathrm{b}}$, $p^{(0)}$, $a^{(\mathrm{a})}$, $a^{(0)}$ are constants; synchronization gap $G_n$ and safe speed $v_{\mathrm{s},n}$ are

$$G_n = G(v_n, v_{\ell,n}), \tag{17}$$

$$G(u, w) = \max(0, \lfloor k\tau u + a^{-1}\phi_0 u(u - w)\rfloor), \tag{18}$$

$$v_{\mathrm{s},n} = \min\left(v_n^{(\mathrm{safe})}, g_n/\tau + v_\ell^{(\mathrm{a})}\right), \tag{19}$$

$$v_n^{(\mathrm{safe})} = \lfloor v^{(\mathrm{safe})}(g_n,\ v_{\ell,n})\rfloor, \tag{20}$$

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$v^{(\mathrm{safe})}\tau_{\mathrm{safe}} + X_{\mathrm{d}}(v^{(\mathrm{safe})}) = g_n + X_{\mathrm{d}}(v_{\ell,n})$, $X_{\mathrm{d}}(u) = b\tau^2\left(\alpha\beta + \frac{\alpha(\alpha-1)}{2}\right)$, $\alpha = \lfloor u/b\tau \rfloor$, $\beta = u/b\tau - \alpha$, $v_\ell^{(\mathrm{a})} = \max(0, \min(v_{\ell,n}^{(\mathrm{safe})}, v_{\ell,n}, g_{\ell,n}/\tau) - a\tau)$, $\tau_{\mathrm{safe}}$ is a safe time gap; $b$, $k > 1$, $a$, $k_{\mathrm{a}}$, and $\phi_0$ are constants; $\lfloor z \rfloor$ denotes the integer part of a real number $z$. In (1)–(20), $n = 0, 1, 2, \ldots$ is the number of time steps, $\tau = 1\,\mathrm{s}$ is a time step, $a_{\max}$ is a maximum acceleration, $v_{\mathrm{free}}$ is a maximum speed in free flow, $\tilde{v}_n$ is the vehicle speed without speed fluctuations $\xi_n$, $\ell$ marks the preceding vehicle, $g_n = x_{\ell,n} - x_n - d$ is the space gap between vehicles, $d$ is the vehicle length, $\Delta v_n = v_{\ell,n} - v_n$; $x_{\mathrm{n}}$ and $v_{\mathrm{n}}$ are measured in units $\delta x = 0.01\,\mathrm{m}$ and $\delta v = 0.01$ m/s, respectively.

In the model, if a vehicle reaches the upstream front of a moving queue at a signal it decelerates as it does at the upstream front of a wide moving jam propagating on a road without traffic signals [7][8]. During the green light phase, vehicles accelerate at the downstream front of the moving queue (queue discharge) with a random time delay as they do at the downstream jam front. During the yellow phase a vehicle passes the signal location, if the vehicle can do it until the end of the yellow phase; otherwise, the vehicle comes to a stop at the signal.

A key role in the simulation of synchronized flow plays the driver's speed adaptation for which we use a stochastic description through the probabilities $p_1$ and $p_2$ in (12). We write these probabilities as follows:

$$p_1 = \min(1, (1+\varepsilon)p_1^{(0)}), \quad p_2 = \min(1, (1+\varepsilon)p_2^{(0)}(v_n)), \tag{21}$$

where $p_1^{(0)} = 0.3$, $p_2^{(0)}(v_n) = 0.48 + 0.32\Theta(v_n - v_{21})$, $\varepsilon$ is the coefficient of speed adaptation. The larger $\varepsilon$, the stronger the speed adaptation and, therefore, the larger the mean space gap (the longer the mean time headway) between vehicles in synchronized flow. This can be explained as follows.

If on a multilane road a driver approaches a slower moving vehicle in front and if he cannot change lane, e.g., because of traffic on the other lane not permitting a sufficient safety gap for a lane change, than the driver has to decelerate. In the model he can start to decelerate as soon as he enters the synchronization gap to his leader. The driver will decelerate if his distance to the leader becomes smaller than the safety gap, and he will accelerate if the distance becomes greater than the synchronization gap (see Figure 2 (b)). But in contrast to models based on classical theories, in three-phase traffic flow models a vehicle doesn't try to abide to a specific distance to the preceeding car, usually the safety gap. Instead, the vehicle tends to adapt its speed to the speed of the preceeding car, but it will do this while taking an arbitrary distance to it as long as it stays within the synchronization gap.

Therefore, the driver could decelerate gradually to the speed of the leader as soon as he enters the synchronization gap, which would lead to a short headway to the preceeding vehicle (see Figure 3 (a)). This situation, described with $\varepsilon = 0$, is called usual or weak speed adaptation and leads to the classical behaviour of stop-and-go traffic: the mean duration of the stops of the vehicles remain almost constant along the moving queues that build upstream of the signal [9]. Alternatively, the driver can decelerate more sharply when reaching the synchronization gap distance, which would lead to a large headway to the preceeding vehicle (see Figure 3 (b)).

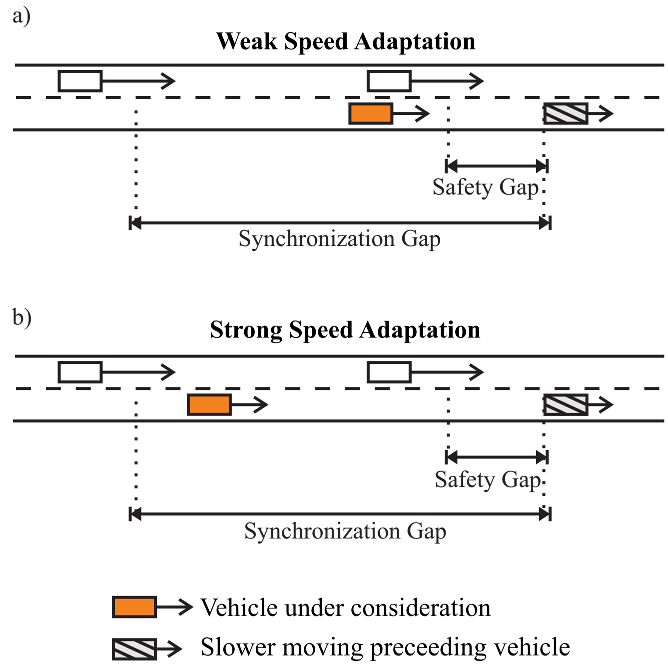


Figure 3. Explanation of speed adaptation effect: Weak speed adaptation (a) and strong speed adaptation (b).

This situation, described with $\varepsilon > 0$, is called strong speed adaptation and leads to the behaviour that cannot be described by classical theories: the mean duration of the stops of the vehicles decreases the further upstream of the traffic signal the moving queue is located [9]. The growth of the space gap leads to the dissolution of the jam upstream of the traffic signal. So in oversaturated city traffic adjacent to the traffic signal there is still the classical stop-and-go behaviour of the vehicles, but further away from the signal synchronized flow emerges due the dissolution of the jam.

## III. Synchronized flow in city traffic

For the simulation of synchronized flow in urban traffic we use a multilane road with a traffic signal at the end. We chose a road section of Völklinger Straße in the city of Düsseldorf (see Figure 4) as congested traffic is observed here regularly in the data of a stationary video detector [13]. The length of the section is 630 m, the speed limit is 60 km/h, and there are no junctions in between.

### A. Empirical findings

Figure 5 shows typical examples of anonymized empirical vehicle probe data provided by TomTom, the dotted (red) line denotes the point in time at which the vehicles pass the position of the traffic signal. Figure 5 (a) shows the velocity profile of a vehicle driving on Völklinger Straße in the morning rush hour, exhibiting the classical moving queue pattern in front of a traffic signal in oversaturated city traffic.

Figure 5 (b) shows an example of a velocity profile in the synchronized flow phase in the late morning rush hour: the vehicle at first drives with a speed between 20 km/h and 30 km/h without coming to a stop. Then as it moves further down the road it has to decelerate to a full stop about 100 m

 

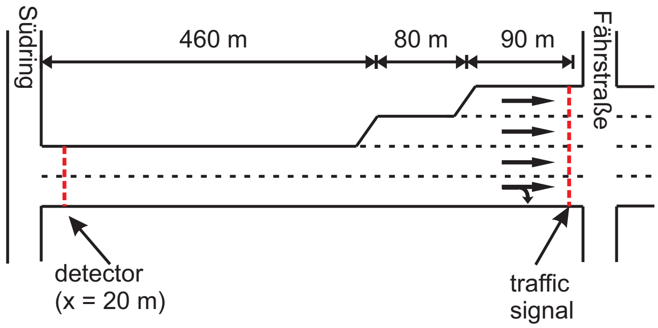


Figure 4. Layout of test track Völklinger Straße from Südring to Fährstraße. Video detector at $x = 20\,\text{m}$, traffic light at $x = 630\,\text{m}$ (cycle time $\vartheta = 70\,\text{s}$, red phase duration $T_\text{R} = 35\,\text{s}$, yellow phase duration $T_\text{Y} = 4\,\text{s}$), speed limit $v_\text{max} = 60\,\text{km/h}$.

in front of the traffic signal as it reaches the upstream front of the queue. In accordance with the microscopic criteria for traffic phases in congested traffic [7][8], this is a typical empirical example of synchronized flow in oversaturated city traffic. Following classical theories one would expect either a stop-and-go pattern along the road, or a higher speed at the beginning of the section followed by one or more stops in front of the traffic signal.

Finally, Figure 5 (c) shows an example of a vehicle driving through heavy congested traffic: the travel time for the 630 meters of Völklinger Straße is about 12 minutes.

### *B. Simulation of synchronized flow*

For our simulations we used infrastructure information from OpenStreetMap [20] that was improved by the use of infrastructure plans from the city of Düsseldorf that also provided the traffic signal cycle plans. The amount of vehicles emitted per minute into the simulation was matched to the data of the video detector at the beginning of the road. In all simulations we use the following parameters of model ((1)–(21)): $\tau_\text{safe} = \tau = 1$, $d = 7.5\,\text{m}$, $v_\text{free} = 18.0558\,\text{ms}^{-1}$ (65 km/h), $b = 1\,\text{ms}^{-2}$, $a = 0.5\,\text{ms}^{-2}$, $k = 3$, $\phi_0 = 1$, $\Delta v_a = 2\,\text{ms}^{-1}$, $k_a = 4$, $\gamma = 1$, $p_b = 0.1$, $p_a = 0.03$, $p^{(0)} = 0.005$, $p_0(v_n) = 0.667 + 0.083\min(1, v_n/v_{01})$, $v_{01} = 6\,\text{ms}^{-1}$, $v_{21} = 7\,\text{ms}^{-1}$, $a^{(a)} = a$, $a^{(0)} = 0.2a$, $a^{(b)}(v_n) = 0.2a + 0.8a\max(0, \min(1, (v_{22} - v_n)/\Delta v_{22})$, $v_{22} = 7\,\text{ms}^{-1}$, $\Delta v_{22} = 2\,\text{ms}^{-1}$, $\varepsilon = 0$. Open boundary conditions have been used in all simulations.

Figure 6 (a) shows the speed profile of a vehicle driving in free flow, and Figure 6 (b) shows the according acceleration profile: after entering the road section, the vehicle drives (nearly) at maximum speed at first. Then it decelerates and comes to a stop in front of the traffic signal. In the next green light phase it passes the traffic signal position and again accelerates to maximum speed. This is a typical simulation example for the situation of undersaturated city traffic. Since we describe free flow, the speed adaptation effect doesn't come into it.

Figures 7 (a) and (b) show the speed and acceleration profile of a vehicle driving in synchronized flow. The pattern is very similar to the one found in the empirical data (see Figure 5 (b)): the vehicle first drives with a speed between 10 km/h and 20 km/h. As it moves further down the road it first decelerates to a full stop at the end of the queue in front of the traffic signal and then a second time about 20 seconds

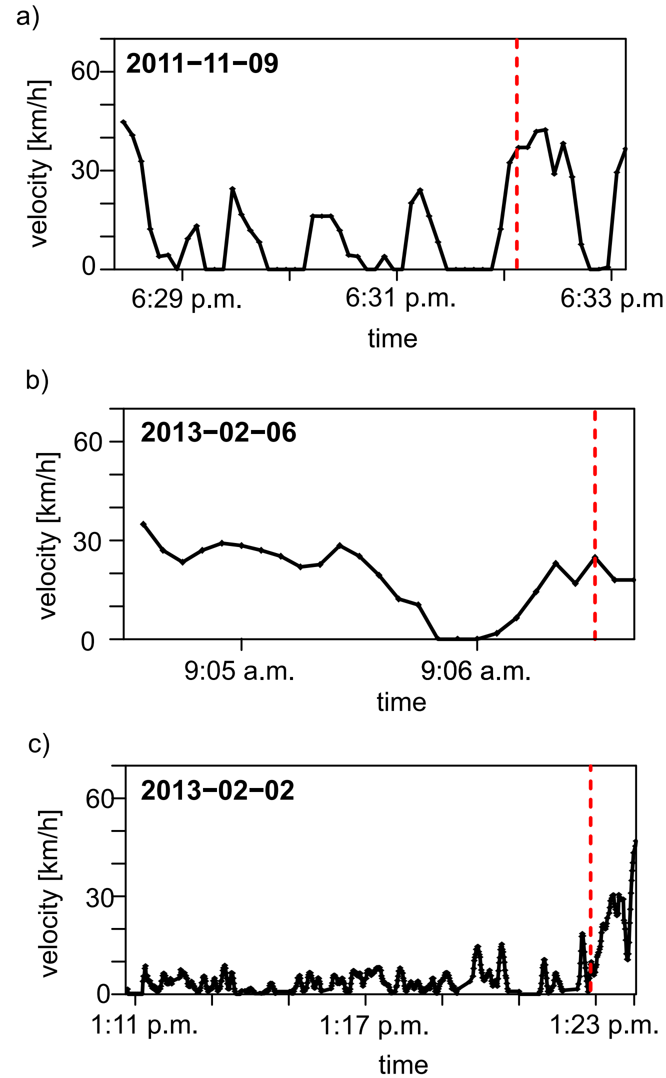


Figure 5. Empirical examples of speed profiles: classical stop-and-go pattern (moving queues) (a), synchronized flow (b) and highly congested traffic pattern (c). The dotted (red) line denotes the position of the traffic light.

later within the queue. This is an example of synchronized flow pattern reproduced in the simulation.

Finally, Figures 8 (a) and (b) show the speed and acceleration profile of a vehicle driving in heavily congested traffic. The vehicle is following a typical stop-and-go pattern along the entire strech of the road up to the position of the traffic signal.

All simulation results shown above are representatives of traffic patterns that were found in the empirical data. This demonstrates that we can reconstruct in our simulations the urban traffic patterns known from classical theories like stop-and-go traffic, as well as synchronized traffic that is unknown to classical theories but found in real data. By alternating the strength of the speed adaptation effect we can reproduce all traffic patterns found in the empirical data for synchronized flow.

As described in Section II, the stronger the speed adaptation effect, the larger the space gap (time headway) that a driver chooses on average to the preceeding vehicle. Since the drivers in this case have enough space in front and can decelerate gradually, they come to a stop less often and therefore the

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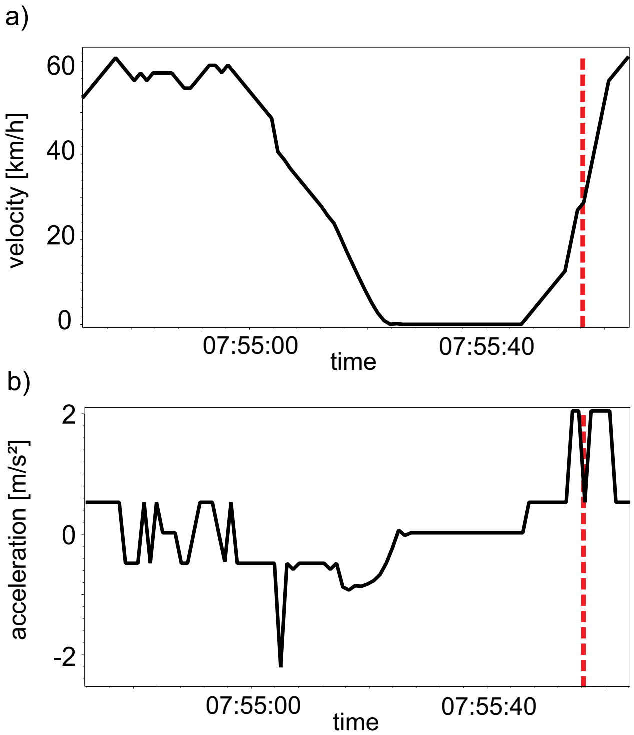


Figure 6. Typical example of simulation results for free flow: time-dependance of microscopic (single) vehicle speed (a), and the according acceleration profile (b) The dotted (red) line denotes the position of the traffic light. $\varepsilon = 0$.

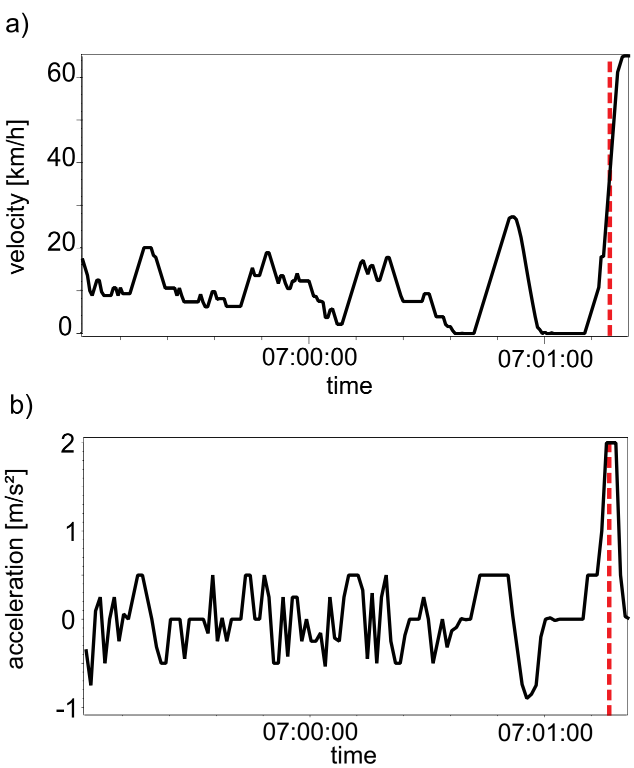


Figure 7. Typical example of simulation results for synchronization flow. The dotted (red) line denotes the position of the traffic light. Model parameters are the same as those in Figure 6 with the exception of the value of $\varepsilon$, that is taken as $\varepsilon = 1.333$.

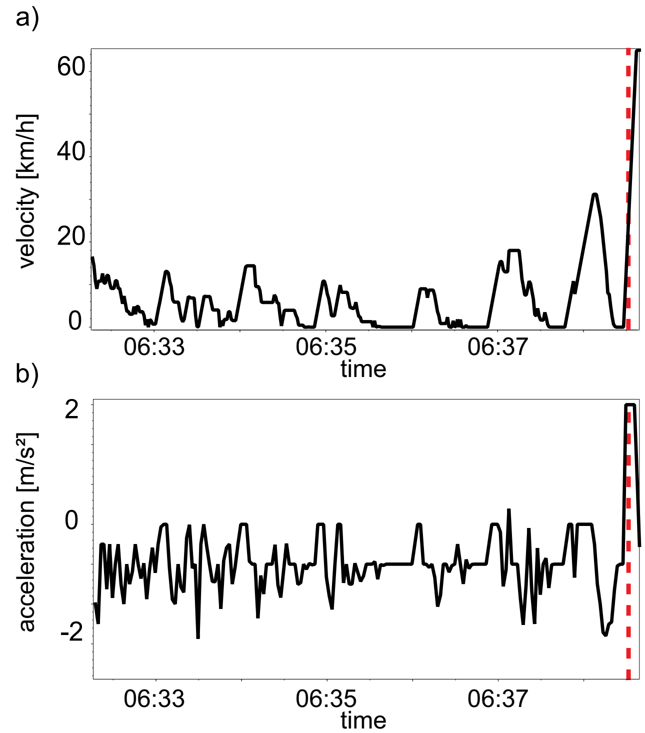


Figure 8. Typical example of simulation results for congested traffic. The dotted (red) line denotes the position of the traffic light. Model parameters are the same as those in Figure 6 with the exception of the value of $\varepsilon$, that is taken as $\varepsilon = 1.333$.

mean stop duration of the vehicle within the moving queue decreases. As a result, the absolute value of the upstream front velocity of a moving queue becomes smaller than that of the downstream front of the moving queue, resulting in moving queue dissolution. The moving queues dissolve into the synchronized flow phase. The strength of the speed adaptation effect influences the distance from the traffic signal, at which the moving queues dissolve. The greater $\varepsilon$, the shorter the distance from the traffic signals at which the moving queues dissolve [9].

## IV. Conclusion and Future Work

Empirical probe vehicle data measured by TomTom navigation devices show synchronized flow in oversaturated city traffic [13]. We have simulated oversaturated city traffic with the discrete stochastic microscopic Kerner-Klenov three-phase traffic flow model. The simulations show that under strong speed adaptation synchronized flow patterns which are very close to empirical data can be reproduced with this model. Strong speed adaptation is associated with an average increase of space gaps (time headways) which drivers choose moving in very dense city traffic. The stronger the speed adaptation effect, the shorter the distance at which moving queues dissolve into synchronized flow. Further work will include the evaluation of the strength of the speed adaptation effect as well as comparisons to more empirical examples.

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## REFERENCES

[1] F. V. Webster, “Traffic signal settings,” Road Research Technical Paper, vol. 39, 1958.

[2] G. F. Newell, “Approximation methods for queues with application to the fixed-cycle traffic light,” SIAM Review, vol. 7, no. 2, 1965, pp. 223–240.

[3] J. D. C. Little, “The synchronization of traffic signals by mixed-integer linear programming,” Operations Research, vol. 14, no. 4, 1966, pp. 568–594, ISSN: 0030-364X.

[4] D. I. Robertson, Transyt: A traffic network study tool. Transportation and Road Research Laboratory, Crowthorne, UK, 1969, Report no. LR 253, in Transportation and Road Research Laboratory Report, ISBN: 0968-4093.

[5] N. H. Gartner, and C. Stamatiadis, Traffic Networks, Optimization and Control of Urban. Springer, New York, 2009, pp. 9470–9500, in Meyers, R. A., Ed., Encyclopedia of Complexity and System Science, ISBN: 978-0-387-75888-6.

[6] F. Dion, H. Rakha, and Y.-S. Kang, “Comparison of delay estimates at under-saturated and over-saturated pre-timed signalized intersections,” Transportation Research Part B: Methodological, vol. 38, no. 2, 2004, pp. 99–1222, ISSN: 0191-2615.

[7] B. S. Kerner, The Physics of Traffic. Springer, Berlin, New York, 2004, ISBN: 978-3-540-40986-1.

[8] B. S. Kerner, Introduction to Modern Traffic Flow Theory and Control. Springer, Berlin, New York, 2009, ISBN: 978-3-642-02605-8.

[9] B. S. Kerner et al., “Synchronized flow in oversaturated city traffic,” Physical Review E: Statistical, Nonlinear, and Soft Matter Physics, vol. 88, no. 5, 2013, 054801.

[10] B. S. Kerner and S. L. Klenov, “A microscopic model for phase transitions in traffic flow,” Journal of Physics A: Mathematical and General, vol. 35, no. 3, 2002, L31-L43.

[11] B. S. Kerner and S. L. Klenov, “Microscopic theory of spatial-temporal congested traffic patterns at highway bottlenecks,” Physical Review E: Statistical, Nonlinear, and Soft Matter Physics, vol. 68, no. 3, 2003, 036130.

[12] B. S. Kerner and S. L. Klenov, “Phase transitions in traffic flow on multilane roads,” Physical Review E: Statistical, Nonlinear, and Soft Matter Physics, vol. 80, no. 5, 2009, 056101.

[13] P. Hemmerle et al., Increased Consumption in Oversaturated City Traffic Based on Empirical Vehicle Data. Springer International Publishing, 2014, pp. 71–79, in Fischer-Wolfarth, J. and Meyer, G., Eds., Advanced Microsystems for Automotive Applications 2014. Lecture Notes in Mobility, ISBN: 978-3-319-08086-4.

[14] P. G. Gipps, “A behavioural car-following model for computer simulation,” Transportation Research Part B: Methodological, vol. 15, no. 2, 1981, pp. 105–111, ISSN: 0191-2615.

[15] K. Nagel and M. Schreckenberg, “A cellular automaton model for freeway traffic,” Journal de Physique I, France 2, 1992, pp. 2221–2229.

[16] R. Barlović, L. Santen, A. Schadschneider, and M. Schreckenberg, “Metastable states in cellular automata for traffic flow,” The European Physical Journal B - Condensed Matter and Complex Systems, vol. 5, no. 3, 1998, pp. 793–800, ISSN: 1434-6028.

[17] S. Krauß, P. Wagner, and C. Gawron, “Metastable states in a microscopic model of traffic flow,” Physical Review E: Statistical, Nonlinear, and Soft Matter Physics, vol. 55, no. 5, 1997, pp. 5597–5602.

[18] B. S. Kerner, “Theory of self-organized traffic at light signal,” arXiv:1211.2535v1 [physics.soc-ph], retrieved: August, 2014.

[19] B. S. Kerner, “The physics of green-wave breakdown in a city,” Europhysics Letter, vol. 102, no. 2, 2013, pp. 28010.

[20] www.openstreetmap.org