Approach to a Holistic Modelling of Cycling Dynamics

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***Abstract*—Model-based approaches for the development of complex systems are an established method for component or function development. With an increased interest in cycling due to the political promotion of cycling and the spread of technological innovations, such as e-bikes, model-based approaches can support the corresponding research and development activities, e.g., for a simulation-based assessment of cycling infrastructure. For this purpose, a holistic modelling of bicycle riding should be used, which is able to represent the physical aspects of cycling including a representation of the cyclist, bicycle as well as the environment and dynamics independently. Crucial for such a modelling approach is a comprehensive description of the vehicle's longitudinal, lateral and vertical dynamics as well as its dynamic limitations, which is presented in this paper. Based on a propulsion force given by the overall model, the trajectory and all relevant environmental properties, the resulting model calculates the dynamics relevant for the process of cycling and the cyclist. The necessary parameters are derived from the characteristics of the cyclist and the bicycle as well as the characteristics of the route to be travelled.**

Keywords—Cycling; Simulation; Bicycles; Bicycle Dynamics.

I. INTRODUCTION

The consideration and technologisation of cycling and bicycles is an emerging field of interest in research and development applications. On the one hand, an understanding of bicycle usage as well as improvements of the corresponding infrastructure is required to further improve cycling, which is promoted by governments as part of new mobility strategies, e.g., Germany's national cycling plan [1]. On the other hand, new technologies emerge that make the use of bicycles more accessible and convenient, which gets particularly visible in the rapid spread of e-bikes over the last few years [2][3].

To address current research challenges, model-based methods to simulate the whole process of cycling may provide interesting approaches. The utilisation of model-based methods is well established throughout the entire development process of complex systems and software [4], especially in the field of automotive engineering [5]. This makes it possible to test functions or components in the context of an entire vehicle and its environment at early stages of development. For a realistic representation, simulations include a model of the vehicle environment and the driving dynamics according to the physical relationships. Furthermore, there are model-based approaches using a physical-technical description for calculations, e.g., a prediction of electrical energy consumption [6]. However, this extensive use of model-based methods is still fairly uncommon for bicycles.

Possible applications of a physical overall cycling simulation could be the development of functions and components or an energy demand prediction for e-bikes [7][8]. In addition, a holistic modelling of cycling can be used to consider the quality of cycling infrastructure based on variables such as a cyclist's power-/energy expenditure or travel time [9]. In this way, objective aspects of cycling, which can be expressed in physical terms, are made observable from a rider's perspective. For this, a stand-alone simulation must model the rider, the bicycle as well as the environment and the dynamics. The cycling dynamics result from the route and environmental influences as well as the propulsion generated by the cyclist as pedalling torque and bicycle model as propulsion force on the rear wheel, which itself results in relation to the cycling dynamics. Accordingly, this paper presents an approach of a model to simulate the dynamics relevant to the process of cycling and to the cyclist.

Section II provides an overview of the state of the art of cycling model approaches. In Section III, the model of cycling dynamics is derived by physical description and equations. Section IV presents the implementation of the model regarding the resulting model and the required parameters for simulation. Thus, in Section V, the model can be simulated and the corresponding results verified. To conclude, Section VI summarises this paper and provides a preview to future work.

II. State of the Art

The description of the dynamics of cycling primarily needs to consider that a bicycle is a single-track vehicle. Thus, corresponding models exist for the resulting stability problem, as well as associated concepts for the stability control of a bicycle [10], e.g., on the basis of a given steering angle. Furthermore, there are models for describing the resulting vertical dynamics according to the surface profile, for example to represent the comfort of cycling on different road surfaces [11][12]. These modelling approaches can be used, for example, in human-in-the-loop bicycle simulators [13]. This involves generating the feedback of the cycling dynamics according to the human handling and realising it with corresponding actuators. However, modelling approaches that depict a holistic description and representation of a vehicle's longitudinal, lateral and vertical dynamics as well as dynamic limitations are, to the best of our knowledge, not available. Thus, this work proposes an approach for a holistic model of cycling dynamics to be used in a stand-alone model, e.g., to achieve a simulation-based assessment of cycling infrastructure [9]. To realise this, the corresponding physical descriptions found in the literature of bicycle or motorbike technology are used [14][15]. Nevertheless, the corresponding relationships and equations must first be applied to our specific model concept. In addition, the parameters required for the description of the cycling dynamics must be derived from the characteristics of the cyclist and the bicycle.

III. Modelling Cycling Dynamics

To achieve a holistic description for simulating the dynamics of cycling, the model environment provides a simulated propulsion force. The trajectory of the cycled path is given as a-priori knowledge that is derived by the course of the route. Furthermore, the frame of the bicycle is considered inflexible and the cyclists position on the bicycle does not change over the course of a journey. In addition, all further relevant information, e.g., slope, weather/wind, underground, for calculating the dynamics is specified as data given over distance of the route.

Based on these information, the given trajectory and the propulsion force, the resulting model must describe the movement of a cyclist and the bicycle as well as specifying the dynamic limitations of cycling. Accordingly, the necessary description of longitudinal, lateral and vertical dynamics are derived.

A. Longitudinal Dynamics

To simulate the process of cycling the description of longitudinal dynamics is a crucial aspect to describe the locomotion of a bicycle and a cyclist. For this the resistance forces of cycling are considered. Thus, the acceleration is calculated on basis of given propulsion $F_{prop.}$ and braking forces F_{brake} on the wheels as well as the resistance forces (1), while the velocity of the bicycle results from the integration of acceleration (2).

$$m\ a = F_{prop.} - F_{brake} - F_{roll} - F_{slope} - F_{drag} \qquad (1)$$

$$v\,(t) = \int a\,(t)\ dt \qquad (2)$$

Accordingly, Figure 1 shows the forces acting on the bicycle and the cyclist as well as the direction of speed and wind.

1) Slope Resistance: For cycling uphill a positive slope value generates the slope resistance force F_{slope}. This refers to the component of gravity F_G depending on the weight of bicycle and cyclist m that occurs in the direction of movement (3). In contrast, this force acts as propulsion when cycling downhill, caused by a negative slope angle α.

$$F_{slope} = F_G\ sin(\alpha)\ = m\ g\ sin(\alpha) \qquad (3)$$

2) Drag Resistance: The drag force F_{drag} (4) considers the resistance from the air displacement caused by the movement of bicycle and rider. The front area A depends on the type of the bicycle and the corresponding resulting seating position of cyclists. This also applies to the drag coefficient c_W. In addition, the density of the air ρ_{air} varies according to the altitude, temperature and humidity. Furthermore, the wind can exert a significant force in dependence of the actual wind speed v_{wind}. Thus, the drag force can support or hinder the propulsion depending on the intensity and direction of the wind.

$$F_{drag} = \frac{1}{2}\ \rho_{air}\ c_W\ A\ (v_{bike} - v_{wind})^2 \qquad (4)$$

3) Rolling Resistance: Rolling resistance occurs at both wheels of a bicycle and results from flexing, roll off and road resistance. The flexing resistance results from the deformation of the tyres, specifically a flattening around the contact point. The intensity of the deformation and resistance depends on the type of tyre used, in particular its width and profile, as well as the actual tyre pressure. Depending on these properties, the flexing resistance is represented by a factor i_F. Furthermore, the deformation of the tyre causes the contact point of the tyre and the tyre axis to be displaced in relation to each other, so that the normal force F_N generates a torque. This roll off resistance is also represented by a factor c_R, which is calculated according to the distance between the axle and the contact point e (Figure 1) and the wheel radius r_{whl}. The road resistance, which depicts the rolling over the unevenness of a road surface as well as any sinking into the subsoil, is also mapped via a factor i_R. Accordingly, the rolling resistance force F_{roll} (5) is calculated on the basis of the above-mentioned factors as a function of the normal force.

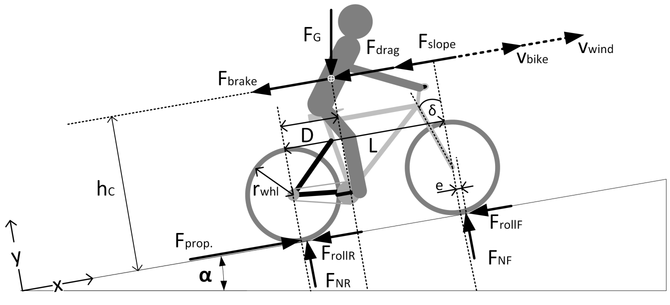

Figure 1. Representation of the dynamics and resistance forces for cycling.

For this purpose, the product of the individual factors can be combined to form the rolling resistance coefficient f_R.

$$F_{roll} = F_N \ i_F \ i_R \ c_R = F_N \ f_R \quad (5)$$

The normal force equals the gravity force in the vertical direction and is thus dependent on the slope angle (6).

$$F_N = F_G \ cos(\alpha) \ = m \ g \ cos(\alpha) \quad (6)$$

4) Transient Resistance: The transient resistance is the force required to overcome the inertia of a vehicle in a direction of motion and of the rotating parts of the power train. Since newtonian mechanics are used to derive the dynamics along the longitudinal axis (1), the mass inertia of the vehicle is already taken into account. In addition, the inertia of the rotating parts of the power train Θ_{PT}, the inertia of front Θ_{FW} and rear wheel Θ_{RW} must be taken into account (7).

$$F_{a,rot} = \frac{\Theta_{PT} \ a}{r_{whl}^2} + \frac{\Theta_{RW} \ a}{r_{whl}^2} + \frac{\Theta_{FW} \ a}{r_{whl}^2} \quad (7)$$

This rotatory acceleration resistance can be added with the translatory resistance ($F_{a,tran} = m \ a_{bike}$) to a total acceleration resistance (8).

$$F_{a,total} = a \ (m + \frac{\Theta_{PT}}{r_{whl}^2} + \frac{\Theta_{RW}}{r_{whl}^2} + \frac{\Theta_{FW}}{r_{whl}^2}) \quad (8)$$

In order to represent the rotational inertia in the given longitudinal dynamic equation (1), a factor λ (9) is used for representation.

$$\lambda = \frac{m + \frac{\Theta_{PT}}{r_{whl}^2} + \frac{\Theta_{RW}}{r_{whl}^2} + \frac{\Theta_{FW}}{r_{whl}^2}}{m} \quad (9)$$

This results in the following equation for the description of the longitudinal dynamics:

$$\lambda \ m \ a = F_{prop.} - F_{brake} - F_{roll} - F_{slope} - F_{drag} \quad (10)$$

B. Lateral Dynamics

Since the proposed cycling model does not deal with the balance or stability control of cycling, the consideration of lateral dynamics emphasis to the resulting movement that occurs in curve situations. In particular, the tilt angle and the steering angle are to be depicted.

1) Bicycle Balance: For the calculation of the tilt angle according to the representation in Figure 2 an ideal curve is assumed. The tilt to a side of the bicycle is caused by a deflection of the centre of gravity in relation to the point of contact. The tilting angle γ of the bicycle is determined by the ratio between the centrifugal force F_C (11) corresponding to the actual curve radius r_c and the normal force F_N.

$$F_C = \frac{m \ v^2}{r_c} \quad (11)$$

These act at a bicycle's centre of gravity and generate a torque around the point of contact corresponding to its height h_c and angular deflection. For stable cornering, these generated torques must be equal (12), so that the tilt angle results (13).

$$F_G \ h_c \ sin(\gamma) = \frac{m \ v^2}{r_c} \ h_c \ cos(\gamma) \quad (12)$$

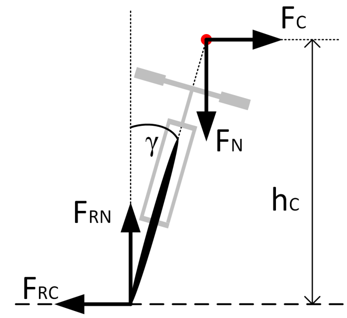

Figure 2. Representation of an idealised stationary cycling trough a curve.

$$tan(\gamma) = \frac{v^2}{g \ r_c} \quad (13)$$

2) Bicycle Steering: In addition to the tilt angle, the handlebars must be turned in for cornering. This steering angle results from the geometry of the vehicle's track model shown in Figure 3. The calculation is based on the yaw angle β (14) around the centre of gravity, the wheelbase L and the distance between the centre of gravity and the rear axle D.

$$sin(\beta) = \frac{D}{r_c} \quad (14)$$

However, this only represents the angle of the front wheel turning from its neutral position ψ (15).

$$tan(\psi) = \frac{L}{D} \ tan(\beta) \quad (15)$$

Based on this, the rotation of the handlebar θ is calculated (16) assuming a small head tube angle δ (Figure 1).

$$tan(\theta) = \frac{\psi}{sin(\delta)} \quad (16)$$

C. Braking Dynamics

For cycling, dynamic limitations occur primarily during the braking process. This is due to the propulsion force that does not usually cause the static friction to be exhausted and thus the wheels to spin. However, when braking intensively, the friction limit is quickly exceeded, so that the corresponding wheel locks up and slips. To enable the cyclist to prevent this, the maximum braking force $F_{B_{max}}$ that can be applied is calculated using the static friction force F_S (17) using the static friction coefficient μ_S as a function of the normal force.

$$F_{B_{max}} = F_S = \mu_S \ F_N = \mu_S \ m \ g \ cos(\alpha) \quad (17)$$

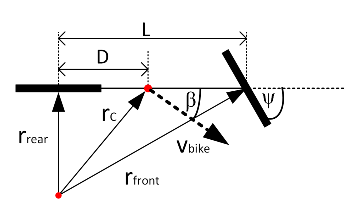

Figure 3. Representation of a bicycle's geometry model for curve cycling.

1) Normal Force Distribution: Furthermore, it must be taken into account that especially during braking there is a nodding movement, which is caused by the mass inertia/acceleration. This leads to an increase in the front wheel load and a reduction in the rear wheel load. The wheel loads at the front/rear wheel (19)/(18) can be calculated from the moment equilibria corresponding to the normal forces (6) and the inertia force resulting from the acceleration.

$$F_{N_{front}} = \frac{F_N \ D - m \ a \ h_S}{L} \tag{18}$$

$$F_{N_{rear}} = \frac{F_N \ (L - D) + m \ a \ h_S}{L} \tag{19}$$

Applied to (17), this results in different maximum braking forces for the front and rear wheel.

2) Braking in Curve Situations: Braking during cornering is subjected to further limitations, so that the maximum possible braking force is decreased accordingly. This is due to the existing static friction force being used for lateral traction (20). As lateral guidance force, the centrifugal force must be applied.

$$F_{B_{max}} = \sqrt{F_S^2 - F_C^2} \tag{20}$$

To calculate the maximum braking force on the front/rear wheel, the centrifugal forces must be calculated based on the weight distribution, the speed and the corresponding curve radius. The curve radius of the front and rear wheel result from the geometry of the bicycle (Figure 3). The equivalent masses for front and rear wheel result according to the position of the centre of gravity on the longitudinal axis. With the resulting centrifugal/lateral guidance forces (21)/(22), the maximum braking force for front/rear wheel can be calculated (20).

$$F_{C_{front}} = \frac{m \ D \ v^2}{L \ \sqrt{r_c^2 - D^2 + L^2}} \tag{21}$$

$$F_{C_{rear}} = \frac{m \ (L - D) \ v^2}{L \ \sqrt{r_c^2 - D^2}} \tag{22}$$

D. Vertical Dynamics

The vertical dynamics represent the effects of the road surface on the cyclist. For this purpose, the tyre and the suspension fork are modelled as a serially connected spring-damper unit for the front axle, while only the tyre is modelled for the rear axle. According to a given profile of the underground, the deflection passed on to the respective wheel can thus be calculated.

IV. Implementation

The physical equations given from literature and the description of the dynamics of cycling derived from those are modelled in *MATLAB/Simulink* and simulated with the numerical methods offered there. This results in our definition of an overall structure of the dynamics model. In addition, the parameters used for the modelling must be provided and determined.

A. Resulting Model

The resulting model is presented in Figure 4 and consists of five subsystems. According to the formulated description of the driving dynamics, all quantities are handled as scalars and apply in the direction defined in the corresponding figures (Figure 1, 2, 3). The longitudinal dynamics system calculates the acceleration and velocity of the bicycle according to the slope angle α_{slope}, the roll resistance factor f_R, the resulting wind speed v_{wind}, the density of the air ρ_{air} and a given propulsion force $F_{propulsion}$. The gravity subsystem calculates the normal forces for front and rear wheel $F_{N_{front}}/F_{N_{rear}}$ by considering the slope angle and the acceleration of the bicycle. The curve dynamics use the bicycles velocity and the curve radius r_{curve} (measured to the centre of gravity) and include the calculation of steering angles ϕ, θ, the tilt angle γ as well as the centrifugal forces at front and rear wheel $F_{C_{front}}/F_{C_{rear}}$. In combination with the normal forces these are considered by the braking subsystem to calculate the maximum braking forces for each wheel $F_{B_{max,front}}/F_{B_{max,rear}}$. The vertical dynamics use the calculated normal forces to determine the translation/deflection passed on to the front and rear wheel y_{front}/y_{rear} which can be given by a road surface profile.

B. Modelling Parameters

The required parameters are calculated from the characteristics of the rider, bicycle and the route. A distinction is made between parameters that are variable over the course of the route and those that are constant for a ride. Both the constant parameters and the parameters depicted over the route are calculated by appropriate pre-processing before the simulation is executed. In order to retrieve the distance-dependent parameters, the distance travelled so far on the given route is required. For this purpose, the model of the longitudinal dynamics is extended by the integration of the velocity over the simulation time.

1) Ride Constant Parameter: To calculate the ride constant parameters, it is assumed that the characteristics of the rider and bicycle are sufficiently known. Among other things, the total weight of rider and bicycle must be calculated. The parameters for the aerodynamic description, c_W-value and frontal area A, are determined depending on the type of bicycle used, e.g., mountain bike (MTB) or road bike and the physique of the rider. In the same way, the centre of gravity

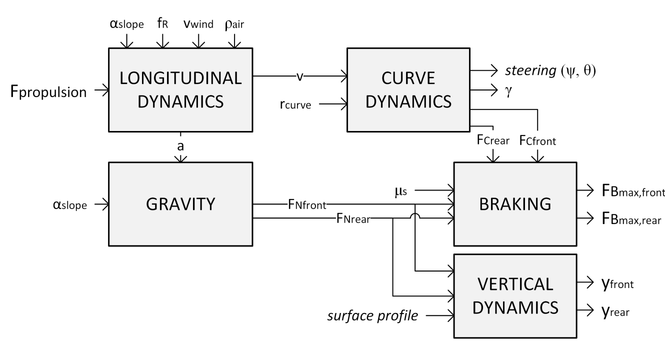

Figure 4. Resulting approach of a holistic dynamic model of cycling.

position h_S, D can be derived, while the head tube angle δ and wheelbase L are only determined on the basis of the bicycle used. The parameters describing the spring-damper unit should also be derived according to the bicycle type, while the vertical dynamic properties of the tyres are calculated depending on the type, e.g., MTB or road bike tyres, as well as the tyre pressure. For a general consideration, the derivation of the parameter values is based on defined rules, whereas for a concrete bicycle these data can be specified individually.

2) Distance Dependent Parameter: For synthetic routes, the user must define the required properties, whereas for routes based on real infrastructure, tools and methods for route data generation can be used. In our case, a self-developed route data generation algorithm and environment is used that has been applied to the application for bicycles [16].

This tool enables the calculation of the curve radius r_{curve} and the altitude profile of the route, from which the slope angle α_{slope} can be calculated. The air pressure ρ_{air} is calculated depending on the altitude profile and the generated air pressure, humidity and temperature. The resulting wind speed v_{wind} is calculated according to the general wind speed and direction as well as the trajectory. The rolling resistance f_R and the static friction coefficient μ_S depend on the surface, but also on the type of tyre and its pressure.

C. Determination of Parameters

For the modelling of the cycling dynamics of a bicycle it should be possible to depict specific bicycles as well as generic bicycle types.

1) Parameters for Specific Bicycles: The geometric data as well as the weight can be taken from the technical data for a known bicycle or measured directly. The required characteristics for modelling the specific spring-damper unit, are usually not publicly available. However, the specific values are only required if the vertical dynamics are to be considered in detail.

Furthermore, experiments can be realised to determine parameters of longitudinal dynamics, for example a coasting test to determine the drag coefficients ($c_W A$) and the rolling resistance coefficient (f_R). For this purpose, the coasting process on a flat road, in which only the air and rolling resistance act, is simulated using (1) and (2) with a given initial speed. The searched parameters are iterated and the mean square error between simulated and measured speed is calculated. Based on the parameter combination with the smallest error, the drag and the rolling resistance coefficient for the given surface are obtained, as shown in Figure 5.

2) Parameters for Generic Bicycles: The required database for deriving the dynamic parameters on the basis of the bicycle or tyre type is derived from literature and the results of experiments. Assumptions are also made according to qualitative data from literature, which in turn must be evaluated through experimental tests. To determine the geometric information, data sheets can be used from which average values can be obtained. For the description of the spring-damper unit, standard values are used, which result from components specific to the type of vehicle.

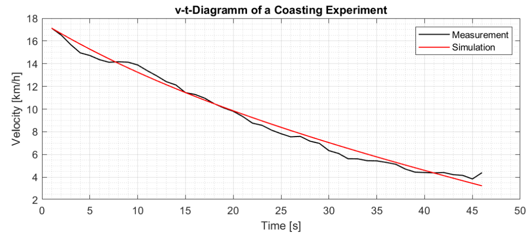

Figure 5. Coasting experiment to determine parameters of longitudinal dynamics (f_R= 0.006, $c_W A$ = 0.6 m²).

V. Simulation and Results

To verify whether the developed dynamics model is correctly formulated, it is integrated into the model environment in *Simulink*. This simulates the propulsion force applied to the rear wheel and provides the required route data, such as the parameters for calculating the longitudinal dynamics. Figure 6 shows the corresponding results for driving on a forest road. The first part of the route is uphill, which is reflected in the gradient resistance and plausibly results in a lower speed. As soon as the uphill and overall resistance decreases, the speed increases as expected. Furthermore, it shows correctly that the increased velocity neutralises the tailwind that acts on the cyclist in the course of the ride. To demonstrate the dynamics of cycling curves Figure 7 illustrates this process. Therefore a curve radius between 10 to 30 meters is given and the resulting tilt and steering angles are displayed. It can therefore be shown that, as expected, narrower curves or smaller curve radii result in increased steering and tilt angles.

Figure 8 shows forces involved in the breaking process on front and rear wheel. For this purpose, the brakes are applied from a speed of 24 km/h while driving through a curve with a radius of 100 metres and, for illustration purposes, a high static friction value of 0.98 is selected. It can be seen that plausibly an increased load is applied to the front wheel due to the deceleration and thus an increased maximum braking force occurs.

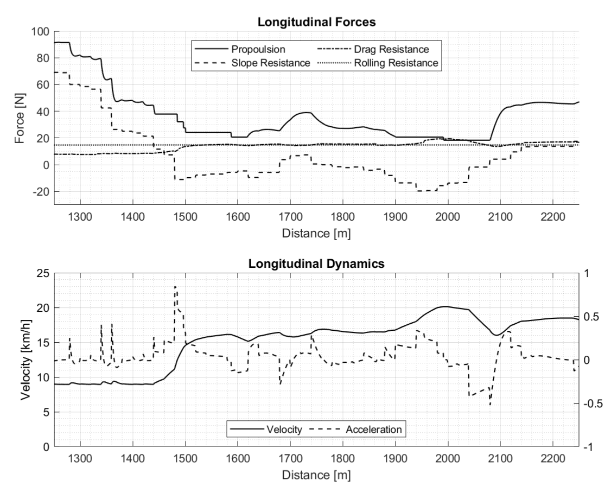

Figure 6. Results of the subsystem for calculating the longitudinal dynamics of cycling on a forest road.

 ISBN: 978-1-68558-102-2

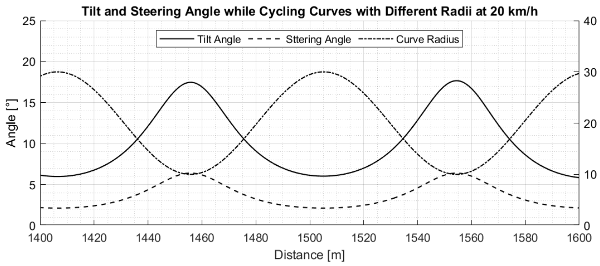

Figure 7. Results of the subsystem for calculating the dynamics of curve cycling.

As expected, an opposite effect appears at the rear wheel. Furthermore, the model correctly represents that the decreasing centrifugal force caused by the reduced speed on both wheels leads to an increase of the maximum braking force.

VI. Conclusion and Future Work

This paper presents an approach towards a holistic model of cycling dynamics to be used for simulating the whole process of cycling, e.g., to be used for the assessment of cycling infrastructure. Therefore the proposed model calculates the longitudinal, lateral and vertical dynamics as well as dynamic limitations as a function of a given propulsion force and trajectory. Furthermore, all relevant parameter describing the route, the cyclist or the bicycle are available as a-priori knowledge. Accordingly, physical descriptions of the cycling process provided by literature are applied to achieve a suitable model for given use-cases, e.g., a model-based assessment of cycling infrastructure. Additionally a brief overview of the determination of these parameters for specific and generic bicycles is provided. The resulting model is implemented as a *Simulink* model while the required parameters are calculated by pre-processing functions. Thus, a verification shows that the developed model is capable of plausibly representing the required representation of cycling dynamics.

Further development of the model will mainly focus on the qualitative evaluation of the model. Literature research and experiments will be used to develop functions for determining the required parameters. Once the parameters have been determined precisely, an extended evaluation of the dynamics model can be carried out especially in comparison to real cycling situations. Furthermore, optimisations of individual subsystems are possible as well as a continuous development of the model according to given requirements from the model environment or given use-cases.

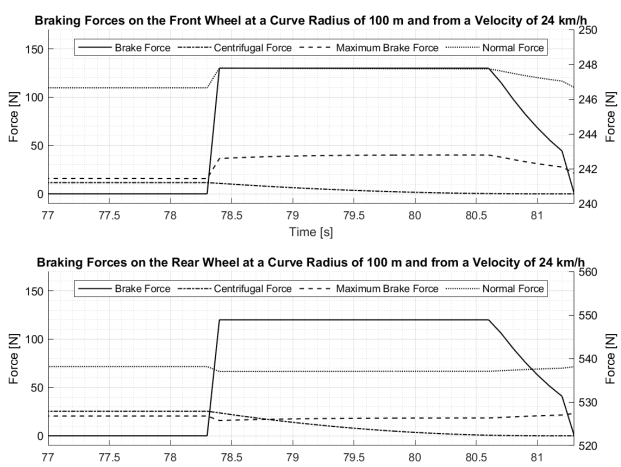

Figure 8. Results of the forces involved in braking dynamics.

Acknowledgment

This work was carried out as part of MODELO-Rad project [9], sponsored by the Ministry of Digitalisation and Traffic of the Federal Republic of Germany.

References

[1] Federal Ministry for Digital and Transport, "National Cycling Plan 3.0," 2021.

[2] Zweirad-Industrie-Verband e.V., "Market data bicycles and e-bikes 2022 - Marktdaten Fahrräder und E-Bikes 2022," 2023.

[3] Royal RAI Association, "Mobility in Figures Two-Wheelers," 2023, [Online]. Available from: https://www.raivereniging.nl/file/upload/doc/clickable-pdf-mic-mobiliteit-in-cijfers-tweewielers-2022-1.pdf 2023.08.04.

[4] M. Glöckler, "Simulation als Teil moderner Entwicklungsprozesse," in *Simulation mechatronischer Systeme*, ser. Lehrbuch, M. Glöckler, Ed. Springer Vieweg, 2018, pp. 251–261.

[5] J. Schäuffele and T. Zurawka, *Automotive Software Engineering*. Wiesbaden: Springer Fachmedien Wiesbaden, 2013.

[6] K. Kruppok, C. Gutenkunst, R. Kriesten, and E. Sax, "Prediction of energy consumption for an automatic ancillary unit regulation," in *17. Internationales Stuttgarter Symposium*, ser. Proceedings, M. Bargende, H.-C. Reuss, and J. Wiedemann, Eds. Springer Fachmedien Wiesbaden, 2017, pp. 41–56.

[7] E. Burani, G. Cabri, and M. Leoncini, "An Algorithm to Predict E-Bike Power Consumption Based on Planned Routes," *Electronics*, vol. 11, no. 7, p. 1105, 2022.

[8] Y. Rauch and F. May, "Online Energy and Range Prediction for E-Bikes," in *Reports on Energy Efficient Mobility – Volume 3*, D. Feßler, M. Kettner, R. Kriesten, P. Nenninger, and P. Offermann, Eds. Zenodo, 2023, pp. 100–110.

[9] J. Eckart, R. Kriesten, and Y. Rauch. (2022) Modelo-Rad. [Online]. Available from: https://www.mobilitaetsforum.bund.de/DE/Themen/Wissenspool/Projekte/Projektbeispiele/Projekte/MODELO_Rad.html 2023.08.04.

[10] A. L. Schwab and J. P. Meijaard, "A review on bicycle dynamics and rider control," *Vehicle System Dynamics*, vol. 51, no. 7, pp. 1059–1090, 2013.

[11] W. Du, D. Zhang, and X. Zhao, "Dynamic modelling and simulation of electric bicycle ride comfort," in *2009 International Conference on Mechatronics and Automation*. IEEE, 2009, pp. 4339–4343.

[12] C.-P. Chou *et al.*, "Simulation of Bicycle-Riding Smoothness by Bicycle Motion Analysis Model," *Journal of Transportation Engineering*, vol. 141, no. 12, p. 04015031, 2015.

[13] D.-S. Kwon *et al.*, "KAIST interactive bicycle racing simulator: the 2nd version with advanced features," in *IEEE/RSJ International Conference on Intelligent Robots and System*. IEEE, 2002, pp. 2961–2966.

[14] M. Gressmann, *Bicycle Physics and Biomechanics - Fahrradphysik und Biomechanik: Technik - Formeln - Gesetze*, 12th ed., ser. Tour. Bielefeld: Delius Klasing Verlag, 2017.

[15] J. Stoffregen, *Motorradtechnik*. Wiesbaden: Vieweg+Teubner Verlag, 2012.

[16] T. Nguyen and Y. Rauch, "Real Route Generation for Simulation Based Developement," in *Reports on Energy Efficient Mobility – Volume 2*, D. Feßler, M. Kettner, R. Kriesten, P. Nenninger, and P. Offermann, Eds. Zenodo, 2022, pp. 58–64.