The Matching Lego(R)-Like Bricks Problem: A Metaheuristic Approach

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Abstract—**We formulate and transform a real-world combinatorial problem into a constraint satisfaction problem: choose a restricted set of containers from a warehouse, such that the elements contained in the containers satisfy some restrictions and compatibility criteria. We set up a formal, mathematical model, describe the combinatorial problem and define a (nonlinear) system of equations, which describes the equivalent constraint satisfaction problem. Next, we use the framework provided by the Apache Commons Mathematics Library in order to implement a solution based on genetic algorithms. We carry out performance tests and show that a general approach, having business logic solely in the definition of the fitness function, can deliver satisfactory results for a real-world use case in the manufacturing industry. To conclude, we use the possibilities offered by the jMetal framework to extend the use case to multi-objective optimization and and compare different heuristic algorithms predefined in jMetal applied to our use case.**

Keywords–Constraint satisfaction problem; Combinatorial problem; Genetic algorithm; Crossover; Mutation; Multi-objective optimization; Apache Commons Math.; jMetal.

I. Introduction

We formulate a new real-world combinatorial problem, the motivation for our study [1]. Initially, we describe succinctly the real-world problem as it has been identified at a semiconductor company and present the general strategy to solve it. In addition to the Single-Objective Optimization [1] using Genetic Algorithms based on the Apache Framework, we present a Multi-Objective Optimization strategy using Genetic Algorithms based on the jMetal Framework and compare the results. In order to avoid the technical difficulties related to the industrial application, we present the equivalent problem based on LEGO® bricks.

A. Motivation

Some time ago we were facing a strategic problem at a big semiconductor company. The company produces Integrated Circuits (ICs), also termed *chips*, assembles them to modules on a circuit board according to guidelines and specifications, and ships the modules as the final product to the customer. The ICs are stored in bins before the last technological process (cleaning) is performed.

The difficulties arise due to technical limitations of the tool that assembles the ICs to modules. The tool can handle at most five bins at once. This means in particular, that the ICs required to fulfill an order from the customer have to be in not more than five bins. Once the bins have been identified, the modules are assembled and shipped to the customer. If it is not possible to identify five bins in connection with a customer order, then either cost-intensive methods (rearranging the content of some bins) or time-intensive methods (waiting some days till the production process delivers new ICs) have to be applied. Hence, identifying the bins necessary to fulfill an order is crucial for the economic success of the company.

B. Current State and Challenge

There has been a selection algorithm in place, based primarily on heuristics and inside knowledge regarding the patterns of the specifications of the modules. Although the existing selection algorithm delivered satisfactory results in most of the cases, it runs for days in some cases and is not flexible enough, in particular, it cannot handle slight deviations from the existing specification patterns.

To circumvent the above inconvenient, the main aim of our study is to determine alternative selection methods, which always deliver satisfactory results within an acceptable time frame, and which are easy adaptable to meet future requirements. Our main objective is to identify and formalize the industrial problem as a mathematical model and to transform the occurring Combinatorial Problem (CP) into a Constrained Satisfaction Problem (CSP). The exact method using MATLAB did not deliver results within a satisfactory time frame. A suitable heuristic method – including Simulated Annealing (SA), Ant Colony Optimization (ACO), Genetic Algorithms (GA), etc. – to solve the CSP within the requirements had to be identified and appropriate algorithms had to be developed, which satisfy both the accuracy and performance demands.

If the general task is to find optimal solution to a set of constraints, we speak about Constrained Optimization Problem (COP). The primarily purpose of the industrial problem is to find a satisfactory solution, since from the technical perspective undercutting the requirements of the specifications does not lead to better quality. However, a straightforward extensions of the CSP towards COP is mentioned later.

C. Problem Description

The following example is artificial, it does not occur in *real life* in this manner, although it is very close to it. It is used to best describe the problem without burden the reader with the technical details of a concrete "real life" example. Later on, we will present a "real life" example from the industry and specify the respective mappings between the two models.

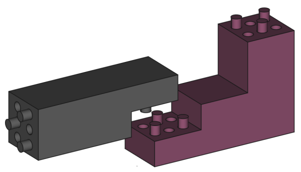


Figure 1: Illustration how two bricks, one of them a corner brick, can be pooled together.

We describe the problem succinctly by using an analogy of building structures out of LEGO®-like pieces (bricks). LEGO®-like pieces (also termed *blocks* or *bricks*) can be assembled to build sophisticated structures (in the following termed *objects*) like buildings, etc. Figure 1 shows how two bricks can be pooled together. The manufacturer of the bricks wants to facilitate and simplify the assembling of the bricks to the final objects as well as to cut manufacturing costs and establishes a two phases strategy when designing the layout plans of the final objects. The final object is parsed into components (termed *modules* or *assemblies*) in a straightforward way, such that these modules can also be reused to assemble other objects. This strategy of representing the final object as a composition of modules is very similar to the construction of buildings out of prefabricated structural components, i.e., modules. This way, by using a modular approach, the description and the design plans of quite sophisticated objects can be kept relatively simple and manageable and the complexity and the difficulty of building the final object is delegated to the assembly of the modules. Hence, the building specification of the final object is split into two guidelines, one regarding how to assemble the required modules, one regarding how to put together the modules to form the final object.

Each brick has numerical and non-numerical characteristics. A non-numerical attribute is, for example, a unique ID which characterizes the bricks like shape, approximate dimensions, number and the arrangement of the inner tubes, etc. Another non-numerical attribute is the color of the bricks, etc. There are very tight requirements in order to be able to assemble two or more bricks. In order to cut costs the technological process to manufacture the bricks is kept simple and cost-effective to the detriment of interchangeability. Thus, the pieces are measured after the production process and the measurement values are persisted in adequate storage systems.

In order to be able to assemble the bricks, they have to fit together, i.e., some measurement values (see Figure 2 for an example) have to fulfill some constraints. The respective measurement values must match in order that the bricks can be assembled. For example, putting four bricks together, side by side and on top of each other, strict restrictions concerning perpendicularity and planarity tolerance, have to be satisfied, such that for example, the overall maximum planarity error is 0.05 mm and the maximum perpendicularity error is 0.1 angular degree. Unfortunately, these restrictions can only be evaluated when all the measurement values of the bricks chosen to build the module are at the builder's disposal. Corresponding calculation prescription are available.

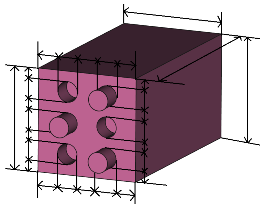


Figure 2: Exemplification of the measurements of a brick.

Once, the modules have been assembled, the object can be put together out of the pre-assembled modules with no limitations. Furthermore, all the modules are interchangeable with similar ones.

The manufacturing of the bricks is continuous, the bricks are packed into bins after the measuring process occurred and stowed in a warehouse. The ID, the non-numerical attributes and the numerical measurement values are stored in a database and associated to the bin ID. This way, the manufacturer knows exactly the content of each bin. In order to keep the manufacturing costs low, the bins are never repacked, after a bin is full and in the warehouse.

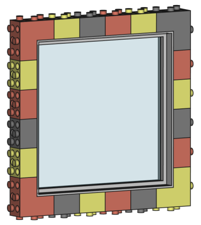


Figure 3: A frame with window as an example for a module.

The assembly plan for a particular structure (for example as in Figure 3) is not univocal, i.e., the number and the type of the bricks to build the envisaged structure is not unequivocally specified, the assembly plan contains more alternatives. Since the manufacturer provides detail information in digital form regarding each brick contained in the bins offered for sale, a computer program could easily verify that a house as given in Figure 4 could be built up from a particular set of bins. Unfortunately, identifying the set of bins necessary to build an object (for example the house as in Figure 4) turns out to be a very hard task to accomplish. In order to keep costs down, the number of the bins to be purchased, has to be limited to the necessary ones.

Let us suppose that the order can be assembled out of 5 bins, and the manufacturer offers 1000 bins for sale on his home page. Regrettably, the computer program can only verify if a particular set of five bins contains the bricks necessary to build the house. The brute force method to verify each set of 5 bins out of 1000 does not deliver a practical solution as elementary combinatorics show. Thus, other methods have to be applied.

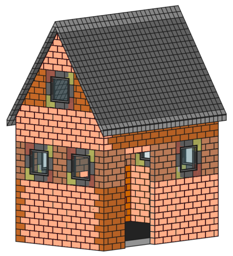


Figure 4: House as exemplification of an order composed of modules.

D. Outline

The remainder of the paper is structured as follows: Section II gives an overview about existing work related to the described problem. Section III introduces the mathematical model and describes how the combinatorial problem can be transformed into a constrained satisfaction problem. Section IV applies the proposed selection algorithm based on genetic algorithms to an industrial use case and shows the performance of an implemented solution which is based on genetic algorithms. A short investigation regarding multi objective optimization is considered in Section V, these investigations are extended by considering the porting and extension of the use case to the jMetal framework in Section VI. Additionally, the possibility offered by jMetal to compare different heuristic algorithms is taken advantage of, whereas Section VII concludes this paper and sketches the future work.

II. Related work

Generally speaking, combinatorial optimization problems are considered as being difficult [2] [3], which stimulated the development of effective approximate methods for their solutions. Combinatorial optimization problems appear in a multitude of real world applications, such as routing, assignment, scheduling, cutting and packing, network design, protein alignment, and in many fields of utmost economic, industrial, and scientific importance. The techniques for solving combinatorial optimization problems can be exact and heuristics. Exact algorithms guarantee optimal solutions, but the execution time often increases dramatically with the size of the underlying data, such that only small size of instances can be exactly solved. For all other cases, optimality is sacrificed for solvability in a limited amount of time [4].

The concept of a constraint satisfaction problem has also been formulated in the nineteen seventies by researchers in the artificial intelligence. Characteristic CSPs are the n queens problem, the zebra puzzle, the full adder circuit, the crossword puzzle, qualitative temporal reasoning, etc. Typical examples of constrained optimization problems are the knapsack problem and the coins problem [5]. Further examples of combinatorial optimization problems [6] are: bin packing, the traveling salesman problem, job scheduling, network routing, vehicle routing problem, multiprocessor scheduling, etc.

For the last decades, the development of theory and methods of computational intelligence regarding problems of combinatorial optimization was of interest of researchers. Nowadays, a class of evolutionary methods [7]–[10] is of particular interest, like simulated annealing, ant colony optimization, taboo search, particle swarm optimization, to which genetic algorithms belong [11]–[14]. Recent publications in this direction [15]–[21] prove the efficacy of applying genetic and other evolutionary algorithms in solving combinatorial optimization problems.

A genetic algorithm is an adaptive search technique based on the principles and mechanism of natural selection and of the survival of the fittest from the natural evolution. The genetic algorithms evolved from Holland's study [22] of adaptation in artificial and natural systems [6].

Typical examples of using evolutionary algorithms are the genetic algorithm approach to solve the hospital physician scheduling problem and an ant colony optimization based approach to solve the split delivery vehicle routing problem [23].

The report [24] offers an approach to use genetic algorithms to solve combinatorial optimization problems on a set of euclidean combinatorial configuration. The euclidean combinatorial configuration is a mapping of a finite abstract set into the euclidean space using the euclidean metric. The class of handled problems includes a problem of balancing masses of rotating parts, occurred in turbine construction, power plant engineering, etc.

III. The Formal Model

In the following, we will formalize the description of the combinatorial problem by introducing a mathematical model. This way, we use the advantages of the rigor of a formal approach over the inaccuracy and the incompleteness of natural languages. First, we introduce and tighten our notation, then we present the formal definition of the constraints which are considered in our formal model and which are the major components in the definition of the fitness function used to control and steer the genetic algorithm. Concluding, the combinatorial problem is defined as a constraint satisfaction problem.

A. Notation

In the following, we will formalize the description by introducing a mathematical model in order to use the advantages of the rigor of a formal approach over the inaccuracy and the incompleteness of natural languages.

Let $\mathfrak{V}$ be an arbitrary set. We notate by $\mathcal{P}(\mathfrak{V})$ the power set of $\mathfrak{V}$, i.e., the set of all subsets of $\mathfrak{V}$, including the empty set and $\mathfrak{V}$ itself. We notate by $\mathrm{card}(\mathfrak{V})$ the cardinality of $\mathfrak{V}$. We use a calligraphic font to denote index sets, such that the index set of $\mathfrak{V}$ is notated by $I^{\mathcal{V}}$.

The finite sets of bricks, bins, (non-numerical type of) attributes, (numerical type of) and measurements are denoted as follows:

$$\begin{aligned}
\mathfrak{S} &:= \{s_i \mid i \in I^{\mathcal{S}} \text{ and } s_i \text{ is a brick (stone)}\},\\
\mathfrak{B} &:= \{b_i \mid i \in I^{\mathcal{B}} \text{ and } b_i \text{ is a bin (carton)}\},\\
\mathfrak{A} &:= \{A^i \mid i \in I^{\mathcal{A}} \text{ and } A^i \text{ is an attribute}\},\\
\mathfrak{M} &:= \{M^i \mid i \in I^{\mathcal{M}} \text{ and } M^i \text{ is a measurement}\}.
\end{aligned}$$

Let $i \in I^{\mathcal{A}}$, $j \in I^{\mathcal{M}}$, and $k \in I^{\mathcal{S}}$. We denote by a_k^i the value of the attribute A^i of the brick s_k and by m_k^j the value of the measurement M^j at the brick s_k. We denote the list of assembly units (modules) by

$$\mathfrak{U} := \{U^i \mid i \in I^{\mathcal{U}} \text{ and } U^i \text{ is an assembly unit (module)}\}.$$

The construction (guideline) plan for a module $U \in \mathfrak{U}$ contains

a) the (three dimensional) design plan description, i.e., the position of each brick within the module,

b) the non-numerical attribute values for each brick and

c) prescriptions regarding the measurement values.

Analogously, we denote by

$$\mathfrak{O} := \{O^i \mid i \in I^O \text{ and } O^i \text{ is an object}\}$$

the list of objects for which there exists construction plans.

The non-numerical attributes values of the selected bricks have to match the corresponding values in the guideline plan.

We denote by

$$\mathfrak{R} := \{R^i \mid i \in I^{\mathcal{R}} \text{ and } R^i \text{ is a requirement (specification)}\}$$

the list of the requirements (specifications) of the objects.

B. Transformation of the CP into a CSP

Let $O \in \mathfrak{O}$ an order. Then, according to the specifications, there exists *(proxy) modules* $\hat{M}^{l_1}, \hat{M}^{l_2}, \ldots, \hat{M}^{l_k}$, such that O is an ordered list of modules, i.e., $O = (\hat{M}^{l_1}, \hat{M}^{l_2}, \ldots, \hat{M}^{l_k})$. The term *proxy* is used to denote an abstract entity according to the specifications. Analogously, each module $\hat{M}^i$ with $i \in \{l_1, l_2, \ldots, l_k\}$ is an ordered list of *proxy bricks* (stones), i.e., $\hat{M}^i = (\hat{s}_{i_1}, \hat{s}_{i_2}, \ldots, \hat{s}_{i_m})$. Hence, each module can be represented by an ordered list of proxy bricks, i.e., $O = (\hat{s}_{k_1}, \hat{s}_{k_2}, \ldots, \hat{s}_{k_n})$. This representation will be used later to define the individuals within the context of the genetic algorithms. Furthermore, we set as $\mathrm{card}(O)$ the number of proxy bricks associated to the order O, i.e., $\mathrm{card}(O) = n$.

Let $O = (\hat{s}_{k_1}, \hat{s}_{k_2}, \ldots, \hat{s}_{k_n})$ be an order. We say that the ordered list $(s_{k_1}, s_{k_2}, \ldots, s_{k_n})$ with $s_i \in \mathfrak{S}\ \forall i \in \{k_1, k_2, \ldots, k_n\}$ is an assignment (embodiment) of O. This means especially that the abstract unit of the specification is materialized within the production process. We set

$$\mathfrak{E} := \{E^i \mid i \in I^{\mathcal{E}} \text{ and } E^i \text{ is an assignment (embodiment)}\}.$$

Let $R \in \mathfrak{R}$ be a requirement (specification) of a specific module $U \in \mathfrak{U}$ and let $\{\hat{s}_1, \hat{s}_2, \ldots, \hat{s}_n\}$ be the proxy bricks of the specification. The design plan of the specification provides the three-dimensional assembly plan of the proxy bricks. Additionally, the specifications provide information regarding the restriction the bricks have to fulfill in order to be eligible. For each $j \in \{1, 2, \ldots, n\}$ the specifications contain the values $\{\hat{a}^i_j \mid i \in I^{\mathcal{A}}\}$ of the attributes $\mathfrak{A} := \{A^i \mid i \in I^{\mathcal{A}}\}$ at the proxy brick $\hat{s}_j$. We use the symbol $\hat{a}^i_j$ to denote the value of the attribute a^i of the proxy (placeholder) brick $\hat{s}_j$.

This means especially, that the brick s_j can substitute the proxy brick $\hat{s}_j$ if the values of the corresponding attributes coincide, i.e., $a^i_j = \hat{a}^i_j$ for all $i \in I^{\mathcal{A}}$.

More formally, the attributes must satisfy certain constraints:

$$C_A : \mathfrak{A} \times \mathfrak{S} \to \{yes, no\},$$
$$\{s^i_j \mid i \in I^{\mathcal{A}},\, j \in I^{S}\} \mapsto C_A(a^i_j).$$

$C_A(a^i_j) = yes$ if the attribute constraint is satisfied for the brick s_j i.e., $a^i_j = \hat{a}^i_j$, else $C_A(a^i_j) = no$.

On the other side, the (numerical) measurement values must also satisfy certain constraints (restrictions). For example, the standard deviation of the respective measurement values for some bricks of a specific module should not surpass some given limits. Formally, C_M can be represented as:

$$C_M : \mathfrak{M} \times \mathfrak{U} \to \{yes, no\},$$
$$\{m^i_j \mid i \in I^{\mathcal{M}},\, j \in I^{\mathcal{U}}\} \mapsto C_M(m^i_j).$$

$C_M(m^i_j) = yes$ if the constraint is satisfied for the bricks belonging to the module U^j, else $C_M(m^i_j) = no$.

In order to be able to reduce the constraints to brick level, i.e., to be able to decide whether the constraint is satisfied for a specific brick or not, we use the restriction C^S_M of C_M namely $C^S_M := C_M\big|_S$ such that $C^S_M(s^i_j) = yes$ if $s_j \in U^j$ and $C_M(m^i_j) = yes$; else $C^S_M(m^i_j) = no$. Let $U \in \mathfrak{U}$ be a module. The above means especially, that the measurement constraint on brick level is satisfied for $s \in U$ if the measurement constraint is satisfied (on module level) for U.

Since the measurement values do not really characterize the modules (they must only fulfill the requirements regarding the constraints), we introduce equivalence classes on the set of modules. Two modules belong to the same class if

a) they have both the same design plan,

b) the component bricks fulfill the same (non-numerical) attributes and

c) the prescriptions regarding the measurement values are satisfied for both modules.

Accordingly, two modules belonging to the same equivalence class are interchangeable.

Hence, all the bricks needed for a module must be selected in order to be able to finally decide if the constraints are satisfied or not.

As already mentioned, each object $O \in \mathfrak{O}$ should be assembled out of bricks contained in a reduced number of bins. We set $MaxB^O$ for the maximum number of bins as mentioned above.

Let $i \in I^{\mathcal{B}}$, $j \in I^O$, let $\{s_{i_1}, s_{i_2}, \ldots, s_{i_k}\}$ be the content of the bin b_i and let $\{s_{j_1}, s_{j_2}, \ldots, s_{j_l}\}$ be an assignment of $O^j \in \mathfrak{O}$. We set $\overline{b}^j_i := 1$ if $\{s_{i_1}, s_{i_2}, \ldots, s_{i_k}\} \cap \{s_{j_1}, s_{j_2}, \ldots, s_{j_l}\} \neq \emptyset$ else 0. This means especially, that $\overline{b}^j_i := 1$ if the bin contains bricks belonging to the respective assignment of O^j.

Analogously, the constraints regarding the bins (cartons) can be regarded formally as:

$$C_B : \mathcal{P}(\mathfrak{B}) \times \mathfrak{O} \to \{yes, no\},$$
$$\{\mathbb{B} \in \mathcal{P}(\mathfrak{B}),\, O^j \in \mathfrak{O}\} \mapsto C_B(\mathbb{B}, O^j).$$

Let $\mathbb{I}$ an index set, such that $\mathbb{B} = \{b_i | i \in \mathbb{I}\}$. Then $C_B(\mathbb{B}, O^j) = yes$ if

$$\sum_{i \in \mathbb{I}} \bar{b}_i^j \leq MaxB^{O^j}, \tag{1}$$

i.e., the bricks of the order O^j are contained in no more than $MaxB^{O^j}$ bins. Additionally, $C_B(\mathbb{B}, O^j) = no$ if the above condition is not satisfied.

Similar to the measurement constraint C_M, we reduce C_B to brick level. Let $\mathbb{S} := \{s_{i_1}, s_{i_2}, \ldots, s_{i_k}\}$ be an assignment of O^i. Let $\mathbb{B} = \{b_{l_1}, b_{l_2}, \ldots, b_{l_n}\}$ be a set of bins, such that each bin contains at least one $s \in \mathbb{S}$ and there is no brick $s \in \mathbb{S}$ which is not contained in one of the bins of the set $\mathbb{B}$. In this sense, $\mathbb{B}$ is minimal regarding the assignment $\mathbb{S}$. Then, for the restriction C_B^S on S of C_M we have $C_B^S(s) = yes$ if $C_B(\mathbb{B}, O^i) = yes$, i.e., all the bricks of the assignment $\mathbb{S}$ are stored in no more than $MaxB^{O^j}$ bins. Additionally, $C_B^{S,U}(s) = no$ if $C_B(\mathbb{B}, O^i)$ is not satisfied.

Until now, we considered the constraints related to the architecture of the object, i.e., related to the attributes of a particular brick, the measurement values of the bricks belonging to a module, and the restrictions regarding the bins which contain the bricks. We can condense the constraints mentioned above, such that they relate only to bricks. This means especially, that the measurement constraint are satisfied for a brick, if there is a group of bricks (module, or order), such that the given measurement constraint is satisfied as described above.

We set accordingly:

$$\mathfrak{C} := \{C^i \mid i \in I^C \text{ and } C^i \text{ is a distinct constraint}\}$$

the list of distinct constraints.

The constraints can be considered as a function. Please recall that I^S is the index set of $\mathfrak{S}$.

$$C : \mathfrak{S} \times I^C \to \{yes, no\},$$
$$\{s_k^i \mid i \in I^C,\, k \in I^S\} \mapsto C(s_k^i).$$

Please consider, that the above representation can be misinterpreted, such that the constraint is exclusively a property of the respected brick. This is not the case, for example the measurement constraints fulfilled or not for the bricks assigned to a module. Hence, if one brick is changed, then the constraints of all the bricks belonging to a module can be invalidated.

We define now formally the weights, (i.e., w is a weight if $w > 0$) which are necessary to be able to model the importance of the constraints within the genetic algorithm. We set

$$\mathfrak{W} := \{w_i \mid i \in I^C \text{ and } w_i \text{ is a weight}\}$$

the list of weights. This means especially, that each constraint has an associated weight.

The fitness function [25] characterizes the quality of an assignment of an order, such that a value closer to 1 means a better quality. It plays an important role in the decision, whether an assignment fulfills the specifications or not.

The purpose of the following function inv is purely technical, it is used to switch the values of the boolean values 1 and 0 to be used in the definition of an example of the fitness function, i.e., $inv : \{yes, no\} \to \{0, 1\}$ such that $inv(yes) = 0$ and $inv(no) = 1$.

Please find below an example for the fitness function. Let $I^{\mathbb{S}} \subset I^S$ and let $w_i \in \mathfrak{W}$ for all $i \in I^C$. Then:

$$F : \mathfrak{C} \times I^C \to (0, 1],$$
$$\{s_k^i \mid i \in I^C,\, k \in I^{\mathbb{S}}\} \mapsto \frac{1}{1 + \sum\limits_{i \in I^C, k \in I^{\mathbb{S}}} w_i \cdot inv(C(s_k^i))}. \tag{2}$$

Problem formulation (Combinatorial problem)

Let $O \in \mathfrak{O}$ a given object and let $n \in \mathbb{N}$.

Choose n bins from the warehouse, such that the object can be assembled out of the bricks contained in these bins according to the existing construction plans.

The construction plan for an object $O \in \mathfrak{O}$ specifies the lists of (non equivocally determined) modules, including the design plan, such that the object can be build out of these modules. Hence, it can be unambiguously decided, whether the n cartons contain the necessary bricks to assemble them to modules, which can be put together to form the required object. Let us suppose that $n \ll \operatorname{card}(\mathfrak{C})$, i.e., the number of bins in the warehouse exceeds the number of bins to be chosen by orders of magnitude. The difficulties of solving the problem in a straightforward way lie in the very large number of possibilities to combine n bins out of $\operatorname{card}(\mathfrak{B})$. Therefore, other strategies have to be used.

To summarize: the specification of an object (for example a house composed of bricks), contains very strict requirements regarding the components. The assembly plan specifies the strict order in which the bricks have to be assembled. Hence, the bricks must satisfy some attributes (like shape, type, color, etc., in order to satisfy the requirements of the construction plans. Moreover, some bricks have to fit together (for example the window frame) so they can be assembled in the order given by the construction plans. If the above requirements are satisfied for all the units (modules), the object can be assembled. Furthermore, the selected bricks have to be selected from a restricted number of bins (cartons). The latter makes the task so difficult.

From a formal point of view, the associated constraint satisfaction problem of the combinatorial problem can now be formulated:

Problem formulation (Constraint satisfaction problem)

Let $O \in \mathfrak{O}$ be an order with the representation $O = (\hat{s_1}, \hat{s_2}, \ldots, \hat{s_k})$. Set $w_i = 1$ for all $i \in I^C$.

Find an index set $\{l_1, l_2, \ldots, l_k\} \subset I^S$ such that $(s_{l_1}, s_{l_2}, \ldots, s_{l_k})$ is an assignment of O, having $F((s_{l_1}, s_{l_2}, \ldots, s_{l_k})) = 1$.

C. Formulation as a System of Equations

In the following, we will formulate the problem as a System of Equations. For readability reasons, we will refresh the some of the notations already introduced. Let $\mathbb{S} \subset \mathfrak{S}$ a subset of the bricks. We denote – as we have done before – by $I^{\mathbb{S}}$ the index set of $\mathbb{S}$. We recall that $I^{\mathcal{S}}$ is the index set of $\mathfrak{S}$, i.e., $I^{\mathbb{S}} \subset I^{\mathcal{S}}$ and I^C is the index set of the distinct constraints $\mathfrak{C}$. Moreover, we recall that C_B are the bin constraints and correspondingly, I^{C_B} is the index set of the bin constraints.

Let $O^j = (\hat{s}_{k_1}, \hat{s}_{k_2}, \ldots, \hat{s}_{k_n})$, $j \in I^O$ be an order. Then $\operatorname{card}(O^j)$ is equal to the number of proxy bricks, i.e., $\operatorname{card}(O^j) = n$. Let $i \in I^{\mathcal{B}}$. Please recall that $\overline{b}_i^j := 1$ if the bin b_i contains bricks belonging to the respective assignment of O^j.

Problem reformulation (Constraint satisfaction problem)
Find an assignment $\mathbb{S} \subset \mathfrak{S}$ of O^j such that all constraints are satisfied:

$$\sum_{i \in I^C,\ k \in I^{\mathbb{S}}} inv(C(s_k^i)) = 0. \tag{3}$$

Requiring that the sum of chosen bins is minimal, one can formulate the problem as an Optimization Problem (OP)

Problem formulation (Optimization problem)
Find an assignment $\mathbb{S} \subset \mathfrak{S}$ of O^j such that:

1) *all but the bin constraints are satisfied:*

$$\sum_{i \in (I^C \setminus I^{C_B}),\ k \in I^{\mathbb{S}}} inv(C(s_k^i)) = 0 \tag{4}$$

2) *and the number of bins is minimal:*

$$\textit{minimize} \sum_{i \in \mathbb{I}} \overline{b}_i^j \tag{5}$$

Please recall that according to the description of the assignment, the equality

$$\operatorname{card}(\mathbb{S}) = \operatorname{card}(O^j) \tag{6}$$

holds, i.e., the number of the bricks we seek is determined through the specification of the order O^j.

Equation (3) means especially that all constraints – including the bin constraint (i.e., relation (1)) – have to be satisfied. In contrast to the above, the optimization strategy does not require that the bin constraint is satisfied (Equation (4)), but the minimal number of bins (see condition (5)) is seeked.

There are either no solution to CSP / OP or one / multiple solutions. The multiple solutions are equivalent, i.e., two solutions which satisfy the constraint are regarded as of the same quality. Accordingly, only one assignment is seeked.

Let A be an integer matrix; b and c vectors, an Integer Linear Program (ILP) is expressed as [26]

$$\max_{x \in \mathbb{Z}^n} \{c^T x \mid Ax \leq b,\ x \geq 0\}. \tag{7}$$

Unfortunately, the measurement constraints do not apply on the brick level, moreover it has to be decided for a module (i.e., a bunch of bricks) if the measurements values are satisfied. This means especially, that the measurement constraints are satisfied for all of the bricks of the module or for none of them. This means that the reformulated CSP cannot be represented in a direct way as an ILP.

However, in order to linearize the relation (4) the OP can be formally transformed by considering modules instead of bricks. Accordingly, a module M satisfies a specific constraint if all the bricks composing the module M satisfy the constraint. Hence, the module is contained in a nonempty set of bins and not anymore in a single bin. Unfortunately, this strategy can be applied only on very small data set, since the measurement constraints must verified for all modules which can be build out of the existing bricks.

To summarize, it does not seem that the relation (4) can be meaningfully linearized – due to the constraints which act on a bunch of bricks – in such a way that the reformulated CSP can be transformed into a ILP, thus benefiting from the advantages of the research in this area.

D. Reformulation as a Nonlinear System of Equations

In the following, we will reformulate the problem as a nonlinear system of equations. Therefore, firstly we recall and strengthen the relating definitions and correlations between them in order to ease the reading of the following section.

- $I^{\mathcal{B}}$ is the index set of the set of bins (cartons); obviously $\operatorname{card}(I^{\mathcal{B}}) = \operatorname{card}(\mathfrak{B})$, where $\mathfrak{B}$ is the set of bins (cartons),
- $I^{(\mathcal{S}^i)}$ is the index set of the bricks (stones) contained in bin b_i for all $i \in I^{\mathcal{B}}$,
- $\mathfrak{S}^i := \{s_{i,k} \mid i \in I^{\mathcal{B}}, k \in I^{(\mathcal{S}^i)}$ and $s_{k,i}$ is a brick (stone) contained in bin $b_i\} \subseteq \mathfrak{S}$, i.e., $\mathfrak{S}^i$ is the set of bricks contained in bin b_i, for all $i \in I^{\mathcal{B}}$, It follows from the definition that $\operatorname{card}(I^{(\mathcal{S}^i)}) = \operatorname{card}(\mathfrak{S}^i)$
- Clearly, $\operatorname{card}(\mathfrak{S}) = \sum_{i \in I^{\mathcal{B}}} \operatorname{card}(I^{(\mathcal{S}^i)})$, the total number of bricks (stones) is obtained by summing up the bricks contained in the cartons.

As mentioned, we denote by $s_{i,k}$ the brick located at bin b_i having the local index k and introduce the following set of boolean variables:

$$\overline{s}_{i,k} := \begin{cases} 1 & \text{if brick } s_{i,k} \text{ has been selected,} \\ 0 & \text{otherwise} \end{cases}$$
$$\text{for all } i \in I^{\mathcal{B}}, k \in I^{(\mathcal{S}^i)}.$$

Please recall that $\mathfrak{U}$ is the set of all assembly units (modules), i.e., $\mathfrak{U} := \{U^i \mid i \in I^{\mathcal{U}}\}$; such that $I^{\mathcal{U}}$ is the index set of $\mathfrak{U}$. We denote:

$$\overline{s}_{i,k}^{U^j} := \begin{cases} 1 & \text{if } \overline{s}_{i,k} = 1 \text{ and brick } s_{i,k} \in U^j, \\ 0 & \text{otherwise} \end{cases}$$
$$\text{for all } i \in I^{\mathcal{B}}, k \in I^{(\mathcal{S}^i)}, j \in I^{\mathcal{U}}.$$

and

$$\overline{s}_i := \begin{cases} 1 & \text{if } \exists k \in I^{(\mathcal{S}^i)} \text{ such that } \overline{s}_{i,k} = 1, \\ 0 & \text{otherwise} \end{cases}$$
$$\text{for all } i \in I^{\mathcal{B}}.$$

As already defined, $I^{\mathfrak{A}}$ be the index set of the set of the non-numerical attributes $\mathfrak{A}$. The matching of the non-numerical attributes is modeled by introducing a new set of boolean variables:

$$\overline{d}_{i,k}^{g} := \begin{cases} 1 & \text{if } f_{i,k}^{g} = \hat{f}_{i,k}^{g}, \\ 0 & \text{otherwise} \end{cases}$$
$$\text{for all } g \in I^{\mathfrak{A}}, i \in I^{\mathcal{B}}, k \in I^{(\mathcal{S}^i)}.$$

Unfortunately, the measurement constraints are assembly unit (module) related, i.e., the measurement values of the bricks belonging to a specific assembly unit as a whole satisfy or not the constraints requirements. For example, the standard deviation of a specific measurement may not surpass a given threshold value for the bricks belonging considered assembly unit. Accordingly, all bricks belonging to a module have to be selected before measurement satisfaction decisions can be validated.

Let $U \in \mathfrak{U}$ a particular assembly unit (module). The fulfilling of the constraints is modeled by introducing a new set of boolean variables:

$$\overline{e}_{i,k}^{h,U} := \begin{cases} 1 & \text{if } \forall s_{i,k} \in U : s_{i,k} \text{ fulfills constraint } E_h, \\ 0 & \text{otherwise} \end{cases}$$
$$\text{for all } h \in I^{\mathcal{M}}, i \in I^{\mathcal{B}}, k \in I^{(\mathcal{S}^i)}.$$

This means in particular, that opposite to the non-numerical attributes, the measurement constraints do depend on the specifications of each module.

Let $O \in \mathfrak{O}$ an order. Then, according to the definition of the order, there exists (proxy) modules $\{\hat{U}^i \mid i \in I^{\mathfrak{U}_O}\}$, such that O is a ordered list of those modules. Let $I^{\mathfrak{U}_O}$ the index set of the set of modules and let $MaxB^O$ the maximal number of bins where the bricks may be located. For the sake of simplifying the notations, we set $p := card(O)$ as the number of the bricks we seek to determine. By defining a vector $t \in \{0,1\}^{\mathrm{card}(I^{\mathcal{B}}) \times \mathrm{card}(I^{(\mathcal{S}^i)})}$ with elements $t_{i,k} := \overline{s}_{i,k}$ $\forall i \in I^{\mathcal{B}}, k \in I^{(\mathcal{S}^i)}$, the problem can be reformulated as a system of equations:

Problem reformulation (Non-linear system of equations)
Given all quantities listed above except for $\overline{s}_{i,k}$, $i \in I^{\mathcal{B}}, k \in I^{(\mathcal{S}^i)}$, find $t \in \{0,1\}^{\mathrm{card}(I^{\mathcal{B}}) \times \mathrm{card}(I^{(\mathcal{S}^i)})}$ such that

1) *p bricks from all cartons are chosen:*

$$\sum_{i \in I^{\mathcal{B}}} \sum_{k \in I^{(\mathcal{S}^i)}} \overline{s}_{i,k} = p, \tag{8}$$

2) *maximal $MaxB^O$ bins are chosen:*

$$\sum_{i \in I^{\mathcal{B}}} \overline{s}_i \leq MaxB^O, \tag{9}$$

3) *the non-numerical features are matched:*

$$\sum_{i \in I^{\mathcal{B}}} \sum_{k \in I^{(\mathcal{S}^i)}} d_{i,k}^{g} \cdot \overline{s}_{i,k} \geq p \quad \textit{for all } g \in I^{\mathfrak{A}}, \tag{10}$$

4) *the measurement constraints are fulfilled:*

$$\sum_{i \in I^{\mathcal{B}}} \sum_{k \in I^{(\mathcal{S}^i)}} e_{i,k}^{h} \cdot \overline{s}_{i,k}^{U^j} \geq p \quad \textit{for all } h \in I^{\mathcal{M}}, j \in I^{\mathfrak{U}}, \tag{11}$$

The inequations (10) result as follows: The attributes have to match for at least p bricks, thus, the product $d_{i,k}^{g} \cdot \overline{s}_{i,k}$ of at least p terms has to be one. Each of the remaining terms will be either one or zero due to the boolean variables. Therefore, the total sum will be also greater than or equal to p. The inequations (11) have been determined analogously. In total, there are one equation from (8), one inequation from (9), $\mathrm{card}(I^{\mathfrak{A}})$ inequations from (10), and $\mathrm{card}(I^{\mathcal{M}}) \cdot \mathrm{card}(I^{\mathfrak{U}})$ inequations from (11) compared to $\sum_{i \in I^{\mathcal{B}}} card(I^{(\mathcal{S}^i)}) := card(I^{\mathcal{S}})$ unknown variables.

All equations are linear. Please note that the variables $s_{i,k}$ are boolean ones and thus, have to be integer. We only have to find one solution of the system. However, if we want to maximize the number of objects which can be assembled from the given set of cartons, then we can formulate an optimization problem based on the linear system defined above. It has to be examined if this optimization problem leads to an integer linear program (ILP). In case this is possible, then rewriting the ILP in standard form will be the basis for solving the problem.

There exists a lot of methods for solving ILPs. Branch-and-bound and branch-and-cut algorithms are two of them. They work in a heuristic way.

IV. Use case: An excerpt

In the following, we present a real-life use case [27] we came across at an international semiconductor company. We describe the problem by using the specific terminology in the semiconductor industry, utilizing them with care and only when it is inevitable and undeniable necessary. We describe the fundamentals of the genetic algorithms and show the way it is used to solve our problem. Finally, we conclude by presenting some performance tests.

A. Problem Description

The company manufactures integrated circuits (ICs, also termed chips), which are subsequently assembled on circuits boards to salable entities, termed modules. In order to keep production cost low, the specification of the ICs do not impose very tight constraints on the attributes of the ICs, such that the same IC can be used for different types of modules. On the contrary, the specification regarding the modules are very stringent, in order that the module should be fully functional at the customer side. As soon as the ICs are manufactured, a good dozen of electrical properties are measured and persisted in a data repository. Figure 5 shows a symbolic representation of an IC.

Usually, four to six ICs are assembled on the module. The specification of the modules contains the design (i.e., number and positioning) of the ICs on the integrated circuit board, the type of the IC (article, number of pins, etc.), and several constrains regarding the interaction of the ICs of the module. Figure 6 shows a symbolic representation of a module with

5 ICs. In order for the module to be fully functional, the corresponding measurement values of the ICs have to be in a narrow range. For example, for a specific measurement, the values of the voltage of the ICs have to be between 2.1 volt and 2.5 volt in order that the IC is not scrapped and can be used for further processing. Unfortunately, not all the ICs having the corresponding measurement value in the range as described above, can be assembled to a module. The values differ too much from each other, and the module will not work properly at the customer side. To circumvent this impediment further constraints are needed. These constraints apply on all ICs of the module or just on a subset of it. For example, an often used constraint is limiting the standard deviation of the voltage to 0.1 within one module.

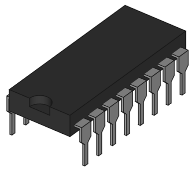


Figure 5: Symbolic representation of an IC.

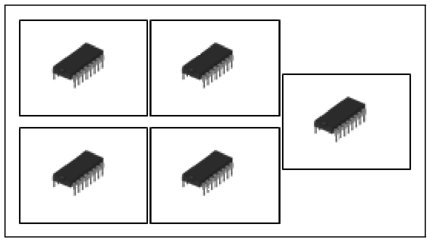


Figure 6: Symbolic representation of a module with 5 ICs.

The ordering unit (termed *work order*) contains the description of the modules, the customer expects to be shipped together at once. There are no additional constraints on the ICs regarding the work order. As soon as the manufacturing process of the ICs has been finished, the ICs are packed in boxes. That way the cleaning of the ICs can be performed and the ICs can be assembled to modules. The boxes are then transferred to the warehouse. Figure 7 shows a symbolic representation of the bins in the warehouse.

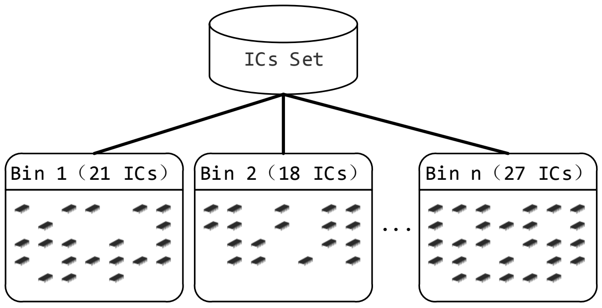


Figure 7: Symbolic representation of the Warehouse containing the bins and the bins containing the ICs.

The difficulties of the semiconductor company to honor each the order in general are also due to technological restrictions, since the tool that assembles the ICs can handle at most five boxes, i.e., all the ICs necessary to fulfill a work order should be located in five boxes. Due to the fact that the work orders always contain the same number of ICs, independent of the specification of the modules, the minimal number of boxes which are needed to meet the requirement of the work order is four with 9 percent surplus of ICs.

If we rephrase the above in a more concise form, the challenge is: Find five boxes in the warehouse, such that it contains the ICs needed to fulfill the requirements for a work order.

B. Used Methods

Simple combinatorics show that the brute force method, i.e., go through all the possibilities and check if the selected boxes fulfill the requirements, is not implementable for practical systems. Fortunately, there is an implementation in place for the selection strategy, based on heuristics, local optimum, and inside knowledge of the pattern of the modules. This way, we have a very good way to compare the results of the genetic algorithm with alternative solutions. Regretfully, our attempt to deliver exact solutions on the problem using MATLAB were not crowned by success due to the large amount of data and to the restricted computing power of the machines we used. The disadvantages of the already existing solution for the selection strategy were partly also the issues that made it possible to set up such a solution:

a) the unpredictability that the selection strategy delivers a solution within the expected time frame;

b) the inflexibility to even minor changes in the design and specifications of the modules, thus, the unpredictability that the software can be used in the future;

c) heavy maintenance efforts due to the sophisticated and architecture and implementation;

d) lack of the proprietary knowledge and documentation of the implementation on the low level side;

e) impossibility to reuse the existing code with reasonable efforts for further development and enhancements.

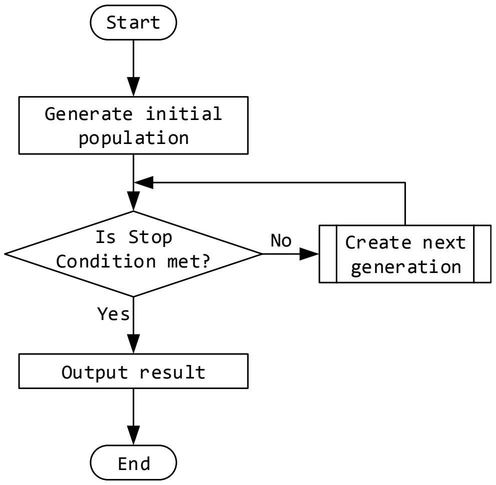


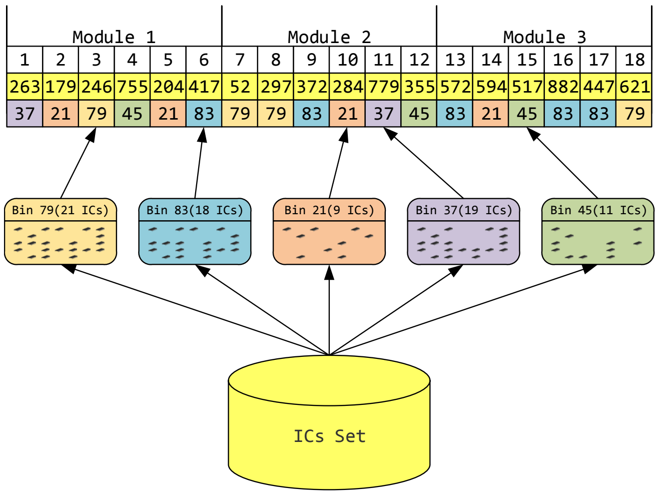
Figure 8: Overview of the genetic algorithm.

The concepts of the genetic methods are straightforward and easy to understand. The main idea is that we start with an initial population of individuals, and as time goes by, the genes of the descendants are improved, such that at least one individual satisfies the expectations. The individual incorporates the requirements of the problem. The expectation in the end is that these requirements are finally satisfied. Each individual owns genes, part of it is inherited by his descendants.

The principle of the genetic algorithm is straightforward, see Figure 8 for a basic representation. First, the initial population is randomly generated, then subsequent generations are created as long as the stopping condition is satisfied. The individuals with the highest quality characteristics constitute the main part of the output.

We define the individuals as an abstraction of the work order, such that each gene of the individual is the abstraction for an IC of the warehouse. Accordingly, the individual satisfies the requirements if the ICs can be assembled to modules, such that the corresponding work order is fulfilled.

Figure 9 is a symbolic representation of an individual (also termed chromosome) containing three modules. According to the specifications, each module contains six ICs. The slots of the modules – i.e., the plug-in location where the ICs are inserted – are numbered consecutively, from 1 to 18 (upper row of the table). Generally speaking, the specific slots – each of them corresponds to the specification of the components of the respective module – represent formally the requirements regarding the ICs composing the modules, hence the structure of the consecutive modules (Module 1 till Module 3) also determines the layout of the individual. Accordingly, the ICs assigned to the slots are the genes of the individual. The middle row of the table in Figure 9 shows an assignment of the slots with ICs, for example, the IC with the unique identifier (Id) 263 has been assigned to slot No. 1, etc. The bottom row illustrates the storage of the ICs in the bins, for example the IC with the Id 263 is stored in the bin No. 6. This latter allocation cannot be changed during the selection process, i.e., it is not possible to rearrange the content of the bins before or during the selection process.



Module 1						Module 2						Module 3					
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
263	179	246	755	204	417	52	297	372	284	779	355	572	594	517	882	447	621
37	21	79	45	21	83	79	79	83	21	37	45	83	21	45	83	83	79

Figure 9: Symbolic representation of the module based structure of an individual (chromosome) containing the respective ICs (genes).

The initial population is randomly generated out of the ICs in the warehouse. The criterion, which determines to what degree the individual fulfills the requirement of the associated work order, is the fitness function. The fitness function takes values between 0 and 1, a greater value means that the individual is more close to fulfill the specification of the work order. To achieve a value of 1 is the ultimate goal. It means that the corresponding individual satisfies the requirements to fulfill the associated work order. Hence, the definition of the fitness function is one of the most sensible parts of the genetic algorithms and the setup of this function should be considered very carefully.

Actually, the strategy of the genetic algorithm resembles very much to the evolution of the mankind. People marry, have children by passing their genes to them, divorce and remarry again, have children, and so on and so forth. The expectation is that the descendants have more “advantageous genes”, regardless of how the term “advantageous genes” is defined.

Establishing the fitness function is one of the most important strategical decision to be taken when setting up the genetic algorithm. In our case, there are a few constraints (more than one) which affect the quality of the individuals. Implementations which try to find a Pareto optimal state [28], [29] (i.e., a state from which it is impossible to make an individual better, without making at least one individual worse) use strategies as tournament selection [30] or the improved Pareto domination tournament [29].

As already mentioned, the starting population is selected aleatorically. Once, the first generation is constituted, the preparations to generate the next generation are met. Unfortunately, the Apache Commons Mathematics Library does not support multi-objective optimization problems, hence our algorithms cannot use the strategy of the *Niched Pareto Genetic Algorithm* (NPGA) [29].

Instead, for each individual, the fitness function is calculated such that the suitability to fulfill the expectations, is evaluated for each individual. The higher the computed value is, the better fitted are the individuals. Let us suppose, that the initial population is composed of 500 individuals. We use some of the concepts provided by Apache Commons Mathematics Library in our implementation, among others the *elitism rate,* which specifies the percentage of the individuals with the highest fitness value to be taken over / cloned to the new generation. We use an elitism rate of 10 percent, i.e., the 50 best individuals will be taken without any changes of the genes to the new generation.

The population of each generation remains constant in time. In order to choose the remaining 450 individuals (parents) to generate the next generation, we use the *tournament selection* [30] including the implementation of the Apache Commons Mathematics Library. The tournament strategy can be configured by the *arity* of the tournament strategy, which specifies the number of individuals who take part in the tournament. For our purpose, five individuals in the tournament proved to be efficient. Accordingly, five individuals are selected randomly out of the total population of 500 individuals to take part in the tournament. Out of the individuals taking part in the tournament, the fittest individual is selected as a parent for the new generation. This way, 500 parents are selected out of a population of 500 individuals. These parents are paired aleatorically and they always have two descendants. This way, the next generation is created. Accordingly, the size of each generation remains constant.

We use two major strategies in order to improve the quality of the genes of the descendants, the *crossover strategy* described in [31] and the *mutation strategy*. Generally speaking, during the crossover phase, the two descendants receive the partly interchanged genes of their parents. Additionally, some particular genes can suffer mutations, i.e., the values of those particular genes are modified. The mutation policy can be configured, such that it allows or rejects the inclusion of new bins to the descendants. This way, the bin constraints can be reinforced, or the flexibility to mutate to any gene ensured. The general strategy to generate the descendants is based with some restrictions on random decisions.

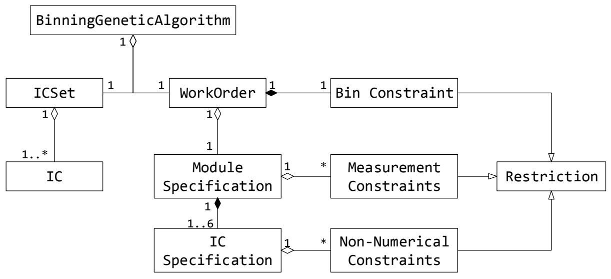


Figure 10: UML class diagram of algorithm and its parameters.

As shown in Figure 10, the genetic algorithm relies on the work order and the set of ICs as its principal input parameter. The ICs set holds the primary production result and the work order identifies the specification of the final product – i.e. set of modules –, which are delivered as the final product to the customer. The specification of the work order include the constraints of the IC, module and work order level. The constraints on the IC level were termed non-numerical constraints and relate to each particular IC, the constraints on module level were termed measurement constraints and they relate to the restriction the ICs that form the module have to satisfy as a group. Similar considerations apply for the bin constraints.

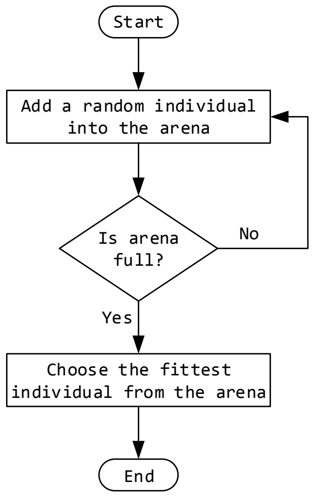

Start
Add a random individual into the arena
Is arena full?
No
Yes
Choose the fittest individual from the arena
End

Figure 11: Tournament selection.

As mentioned above, according to the percentage given by the elitism rate, a subset of individuals of the current generation having promising genes are taken over unmodified to the new generation. In order to select the next pair of individuals during the process of establishing the new generation, see Figure 12, two groups of randomly chosen individuals are formed, the size of the group is configurable and given by the arity value. Then, the best individual from each arena is selected, circumventing the disadvantage of random selection. This method is depicted in Figure 11.

We describe in brief the creation strategy of the new generation. Some parameters are freely configurable, in order to assure best performance. Thus, the *crossover rate,* i.e., the threshold of the probability that a crossover is performed, has to be set in advance. Then, a crossover is performed if a randomly generated number is less than the crossover rate. Same is true regarding the mutation rate.

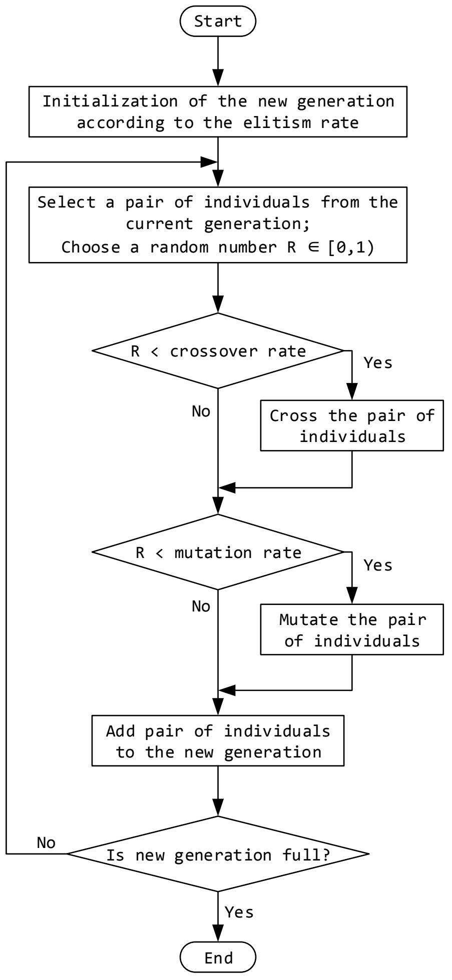
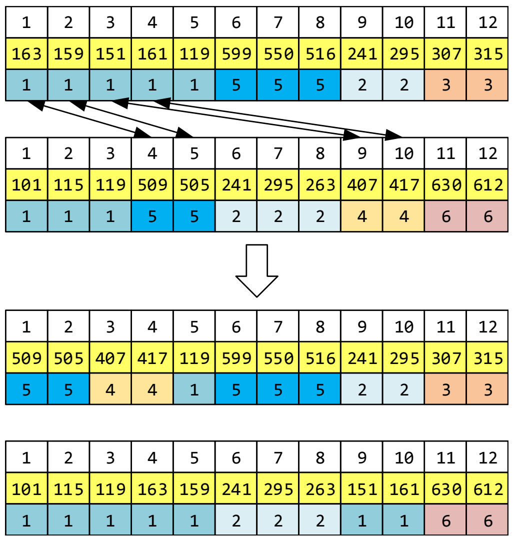

Start
Initialization of the new generation according to the elitism rate
Select a pair of individuals from the current generation; Choose a random number R ∈ [0,1)
R < crossover rate
Yes
No
Cross the pair of individuals
R < mutation rate
Yes
No
Mutate the pair of individuals
Add pair of individuals to the new generation
Is new generation full?
No
Yes
End

Figure 12: Example of the creation of a new generation.

The crossover policy is quite straightforward. The position and length of the genes to be crossed over are randomly generated and the two descendants have receive the interchanged genes of their parents. In our case, this policy has been improved, such that by in the end, the number of the bins of at least one descendant is using, is lower than (or if this is not possible equal to) the number of the bins of their parents. This way, the reduction of the number of bins an individual is using, is enforced by the crossover policy itself.

The mutation policy is also very intuitive. In addition to the mutation rate, which defines in the end, whether mutation is applied after the crossover phase or not, the exchange rate indicates whether a slot (IC) is to be renewed. Analogously, the mutation policy can be configured such that the number of bins the descendant is using is reduced or in worst case, kept constant, i.e., using the Bin Reduction Mutation (BRM) policy.

A simplified flowchart regarding the constitution of a new generation based on the current one is shown in Figure 12. First, based on the elitism rate, the best individuals of the current generation are cloned into the next generation. This way, the genes of the most promising individuals are saved for the next generation, this way, initializing the new generation. Next, either randomly or by tournament selection a new pair of individuals is selected from the current generation. According to a randomly generated number, crossover and mutation is performed on the pair of the individuals or on the single individuals, respectively. The pair of individuals generated this way is added to the set of individuals forming the new generation. The process of filling up the set of the new generation with new pairs is continued until the size of the new generation equals the size of the current generation.



1	2	3	4	5	6	7	8	9	10	11	12
163	159	151	161	119	599	550	516	241	295	307	315
1	1	1	1	1	5	5	5	2	2	3	3

1	2	3	4	5	6	7	8	9	10	11	12
101	115	119	509	505	241	295	263	407	417	630	612
1	1	1	5	5	2	2	2	4	4	6	6

1	2	3	4	5	6	7	8	9	10	11	12
509	505	407	417	119	599	550	516	241	295	307	315
5	5	4	4	1	5	5	5	2	2	3	3

1	2	3	4	5	6	7	8	9	10	11	12
101	115	119	163	159	241	295	263	151	161	630	612
1	1	1	1	1	2	2	2	1	1	6	6

Figure 13: Example of bin reduction crossover policy.

Next, we illustrate based on a representative example in Figure 13, the Bin Reduction Crossover (BRC) policy. The BRC policy enhances the classical crossover policy, such that it uses the inside knowledge regarding the allocation of the ICs to the respective bins. As already mentioned, this allocation is fixed, it cannot be changed during the selection process. The general strategy of the BRC algorithm is to get rid of bins sparsely represented in exchange of ICs from much better represented bins. The two candidates dispose of ICs stored in some common bins, depicted in blue tones and and of unshared boxes, depicted in red tones. Some ICs can be found in both individuals, for example those with ID = 119, 241, 295. The basic idea is to move ICs from the common bins from one individual to another and in return to try to reduce the number of bins by returning ICs from sparsely bins. In our example, the ICs with ID = 136, 159, 151, 161 are moved from the first individual to the second, and the ICs with ID = 509, 505, 407, 417 are returned in order to balance the content of the individuals. This way, after the crossover is fulfilled, the second individual has genes from three boxes, which is a substantial reduction of bins used by the second individual. The overall number of bins composing the two individuals have been reduced this way by one, which corresponds to the envisaged objective.

An overview of establishing a new generation is depicted in Figure 14, which uses UML sequence for his flow chart. It uses implementation-related presentation and gives a glimpse of the Java code.

C. *Performance Results*

The benchmarks were performed on a Intel® Core^{TM} i5-6500 CPU (quad core CPU 3.2 GHz, 16 GB RAM) running on Windows 10 and Eclipse 3.7.0 using Java SE Runtime Environment 1.6.0_22. The genetic algorithm was implemented using the Apache Common Library, version 3.0. The test data is a subset of the production environment and contained 5518 ICs in 261 boxes, having 28 measurements on average. The restricted test data is a subset of the production environment and contained 27,590 ICs in 1,305 boxes. Due to the incomplete set of production data, only the two most critical modules are considered for selection. After taking into account the attributes corresponding to the specifications of the two modules regarding the ICs (article, number of pins, etc.) only eleven boxes contain ICs to be considered for the selection process. We term *pre-selection* the method to restrict the number of boxes by excluding those boxes which do not contain selectable elements. In this way, the search area can be drastically reduced and thus, the performance of the selection algorithm can be substantially improved.

We use a generation size of 500 individuals, an elitism rate of 10 percent and an arity value of 5. The number of generation is limited to 1000 and the runtime of the selection algorithm is limited to 300 seconds. The other parameters like the crossover rate and the mutation rate are configured on a case by case basis. Regarding the fitness function, the following configuration parameters have proved themselves as good choice: attribute weight = 1; measurement weight = 2; bin weight = 5. This means especially, that fulfilling the bin constraints is the most difficult one. In summary, we use the configuration parameters given in Table I for the performance tests.

TABLE I: SETTINGS OF CONFIGURATION PARAMETERS.

Configuration parameter	Value
Population size	500 individuals
Generation limit	1000 generations
Crossover policy	Bin reduction
Crossover rate	78 %
Mutation rate	13 %
Runtime limit	300 seconds
Elitism rate	10 %
Arity	5

The prediction of the results of the selection algorithm is hardly possible, since we use random strategies to generate the initial population, to select the parents for the next generation, to determine the crossover and mutation policy. Moreover, parameters like the elitism rate and the arity have to be configured. Hence, the interaction between many factors that

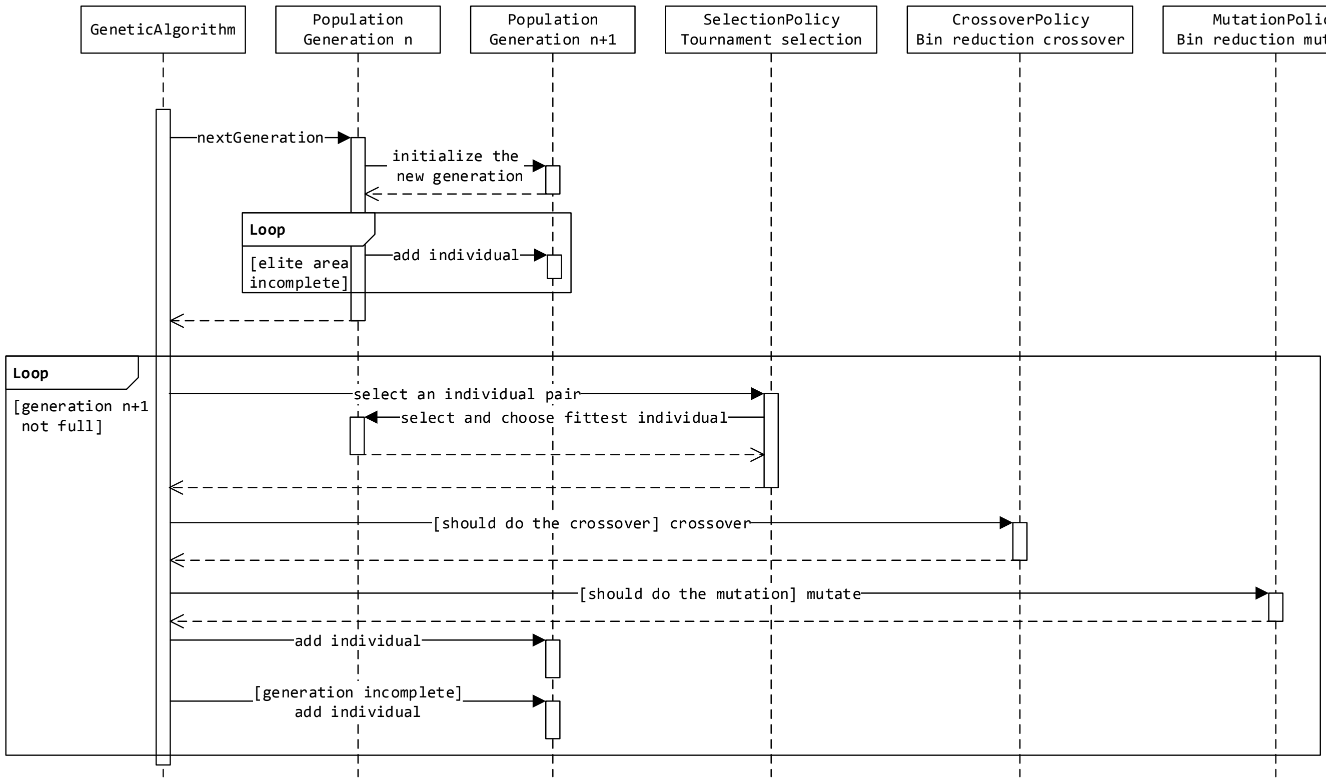

GeneticAlgorithm
Population Generation n
Population Generation n+1
SelectionPolicy Tournament selection
CrossoverPolicy Bin reduction crossover
MutationPolicy Bin reduction mutation
nextGeneration
initialize the new generation
Loop
[elite area incomplete]
add individual
Loop
[generation n+1 not full]
select an individual pair
select and choose fittest individual
[should do the crossover] crossover
[should do the mutation] mutate
add individual
[generation incomplete] add individual

Figure 14: UML sequence diagram of a new generation.

can influence the success and performance of the selection algorithm is not obvious.

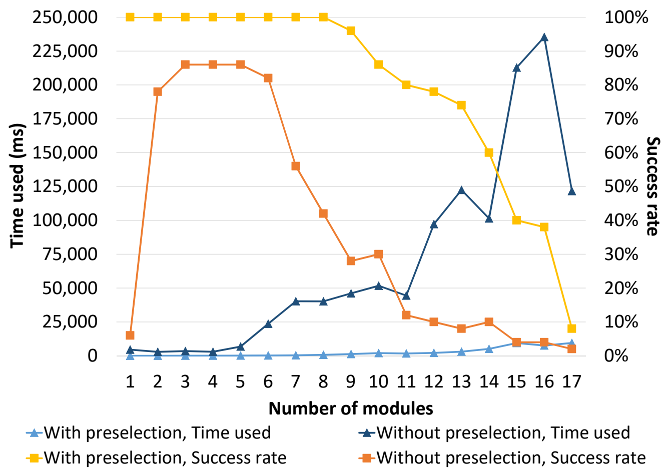


Figure 15: Apache: Success rate and time used depending on the number of modules with and without pre-selection.

It is not the aim of this study to deliver the possible best solution in an acceptable time frame and to improve the performance of the algorithm. Instead, our objective is to deliver an acceptable solution, i.e., a solution that fulfills the required constraints, for the industry to a crucial problem regarding their production problems. For example, there is no technological benefit of tightening the measurement constraints; the bin constraint was set up in such a way that seeking a lower value is not possible due to the fixed number of ICs of a work order and to the maximal capacity of the bins. Hence, the acceptable solution is also the best possible solution.

Nevertheless, we tried to improve the selection algorithm by testing the influence of the parameters, we find out to be decisive. This was also the case for the parameters of the fitness function as described above.

As already mentioned, we have an algorithm in place, which can find

a) a suboptimal solution in a heuristic way,

b) determine exactly whether the group of bins contain ICs which satisfies the specification of a particular work order.

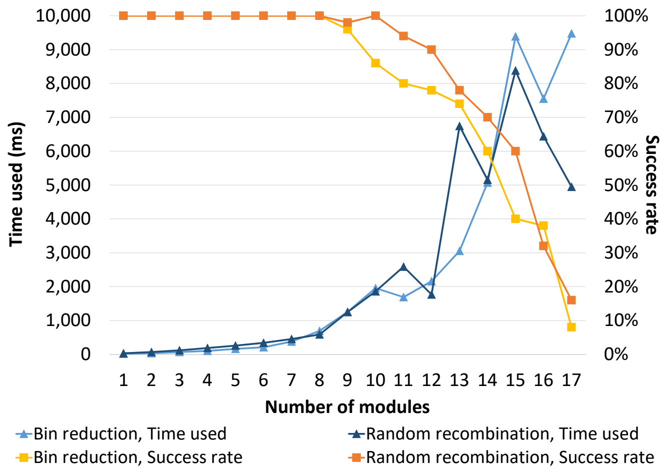


Figure 16: Apache: Success rate and time used when selecting the random recombination crossover policy or the bins reduction policy.

Figure 15 shows that the selection algorithm using pre-selection delivers the expected results, finding individuals having 19 modules. The *success rate,* i.e., the probability that the selection algorithm reaches with an individual the given number of modules, is over 60 percent and thus, high enough for practical systems. The pre-selection strategy is very straightforward and easy to implement. Thus, no practical system would renounce to it. Nevertheless, when neglecting the benefit of reducing the search space by using pre-selection, the results of the genetic algorithm are not always as promising as with pre-selection. In order to evaluate worst-case scenarios, we used work that posed a lot of difficulties to select with the heuristic algorithm in place. As illustrated in Figure 15, the success rate to select 19 modules as in the previous case, is at 60 percent. This means especially, that the successful run of the genetic algorithm heavily depends on the random numbers that were generated.

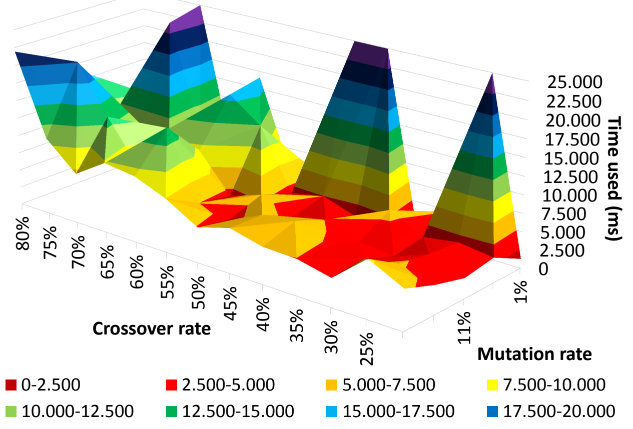


Figure 17: Apache: Time used depending on the crossover rate and the mutation rate.

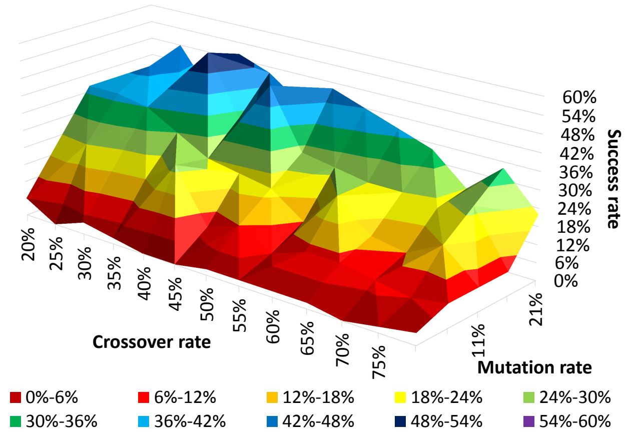


Figure 18: Apache: Success rate depending on the crossover rate and the mutation rate.

Figure 16 shows the difference between the random recombination crossover policy and the bins reduction policy. The boxes reduction crossover policy tries to reduce the number of boxes of the new individuals by focusing on the common bins of the parents. As a conclusion, using business logic over general approach, the general approach is as expected slower and has a lower success rate. This is the price to pay for using a more general solution over a customized one.

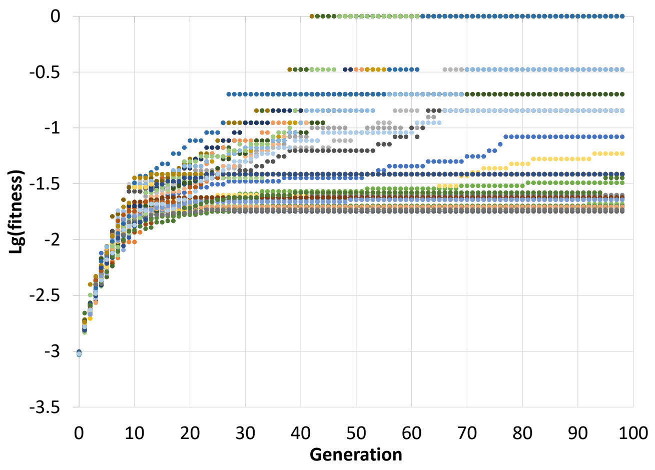


Figure 19: Logarithmic representation of the values of the fitness function depending on the number of generations (50 attempts).

Figures 17 and 18 show the influence of the crossover rate and the mutation rate to the success rate and the wall clock time. As not obvious at first glance, a smaller crossover rate and a higher mutation rate gives better values for the success rate. Keeping the crossover rate and the mutation rate low, better run time performance is achieved. Generally speaking, high mutation rate can destroy the structure of good chromosomes, if used randomly [32]. The above remark does not hold in our case, since we do not exchange ICs randomly, but according to our strategy to minimize the number of bins. We use for the three-dimensional graphics in this paper the best performing strategy, i.e. the bin reduction policy and pre-selection.

The tendency of the convergence of the fitness function is visualized in Figure 19. The graph shows that in the end all 50 threads converge after some generations, but only a subset to the envisaged value. Each dot indicates the best fitness value of a individual within one thread corresponding to a specific generation. Recall that the maximum value of the fitness function is per definition equal to 1, the higher the value of the fitness function, the better the solution. The values of the fitness functions are discrete, $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, ...\}$. We use a decimal logarithmic representation, as a consequence the value 0 of $lg(fitness)$ indicates that a solution has been found. The best thread found a solution within 43 generations and the slowest thread - which is not represented in the graphic - found a solution after 228 generations. There are 8 threads in total which found a solution, the rest - although they converge - were not successful.

V. Resuming on Multi-Objective Optimization

Our preferred implementation framework is *Apache Commons Mathematics Library*, version 3.0 [33]. However, a very similar combinatorial grouping problem, the *Bin Packing Problem (BPP)* is investigated [34], by using the off the shelf *jMetal* framework [35]. The (one-dimensional) BPP [36] is defined as follows: given an unlimited number of bins with an integer capacity $c > 0$ each, a set of n items, $N = \{1, 2, ..., n\}$, and an integer weight $w_i, 0 < w_i \leq c$ for each item $i \in N$ assign each item to one bin, such that the total weight of the items in each bin does not exceed c and the number of bins used is minimized.

Luo et al. [34] extend the base implementation of jMetal to problems with dynamic variables. This was necessary, since the number of the genes in chromosomes is fixed in the base implementation of jMetal. However, the number of the genes in the specific implementation of the chromosomes for BPP – termed group based representation – is fluctuating; they vary in length depending on how many bins are used in every solution. Accordingly, the adopted implementation of BPP includes specific adaptations and enhancements of the basic primitives of jMetal, including those for chromosomes, crossover and mutation. The need for dynamic variables is justified by difficulties to use other solutions due to the fitness function.

In order to evaluate the performance of their algorithms – termed GABP –, Luo et al. [34] use public bench data as well as self-created big data sets. The performance of GABP does not differ very much from some of the known implementation of BPP. The main benefit of GABP is the implementation in a generic framework. However, the problem described in this article, the *Matching Lego(R)-Like Bricks Problem (MLBP)* is new to our knowledge, we are now aware of any implementation of a similar problem. The nearest problem to the MLBP seems to be BPP.

It seems that Luo et al. [34] used the fitness function as given below (termed cost function) [37] for their *group-based encoding scheme*:

$$f_{BPP} = \frac{1}{N_u} \cdot \sum_{i=1}^{N_u} (\frac{fill_i}{c})^k \tag{12}$$

with N_u being the number of bins used, $fill_i$ the sum of sizes of the objects in the bin i, c the bin capacity, and k a constant, $k > 1$. In other words, the cost function to maximize is the average over all bins used, of the k-th power of the bin's utilization of it's capacity. The authors state that experiments show that $k = 2$ gives good results. As Falkenauer and Delchambre [37] point out that one of the major purpose of the fitness function is to guide the algorithm in the search.

Although, both BPP and MLBP yield a reduction of the number of bins, the fitness function of the algorithms are different. The strategy at BPP is to pack the bins as full as possible, i.e., a bad use of the capacity of the bins leads to the necessity of supplementary bins [37]. On the contrary, the suggestion for the fitness function for MLBP is given by the need to fulfill the constraints.

By comparing the formula (2) for the fitness function of MLBP with the formula (12) as above, it is obvious that both formulas are very similar, both are expected to be maximized, contain summation over parameters of the respective problems and the possibility to optimize the execution time of the respective algorithms through clever setting of constants. In this respect, both MLBP and BPP substantially benefit from an ingeniously designed fitness function to ensure fast convergence towards the optimization goals [37].

Although, in concept MLBP is merely a constraint satisfaction problem, the implementation as described in this article, can be easily adapted to simulate an optimization problem. Within the current algorithm, the number of required bins is fixed. This assumption is perfectly reasonable from an industrial perspective, since the number of the objects in the bins is more or less the same and as the number of objects needed for a work order is known, the minimal number of bins necessary to fulfill the work order is thus determined. Besides, the maximum number of bins that can be accessed by a machine is fixed, less beans means just a vacant working place. However, by setting the bin constraints to a lower value, – while keeping the other constraints unchanged – and by modifying the exit criteria accordingly, the algorithm will loop – delivering better solutions according to the guidelines of the fitness function – until the new exit criteria are met. Thus, a local extremum relating to the fitness function is found. The local extremum is not automatically a solution to the constraint satisfaction problem, this has to be validated. The exit criteria only ensure that the best local value of the fitness function and corresponding physical entities are found. In this respect, the MLBP and BPP are closely related problems.

There are other systems, which provide frameworks of evolutionary algorithms, such as EvA2, OPT4j, ECJ, MOEAT, for a discussion and bibliography see [37]. Moreover, a remarkable attempt to obtain a deeper understanding of the structure of the BPP by using Principal Component Analysis and repercussions on the performance of the heuristic approaches to solve them, was undertaken [38].

The aim of jMetal was to set up a Java-based framework in order to develop meta heuristics for solving Multi-Objective Optimization Problems (MOOP). jMetal provides a rich set of Java classes as base components, which can be reused for the implementation of generic operators, thus making a comparison of different meta heuristics possible [39] [40].

Unfortunately, the Apache Commons Mathematics Library deployed in our use case, does not support multi-objective optimization mechanisms. This means especially, that multi-objective optimization have to be simulated by single-objective optimization. For example, optimization criteria for MLBP are a) reducing the number of bins, b) fulfillment of the measurement constraints, c) fulfillment of the attribute constraints. These criteria are independent of each other and ideally within the multi-objective optimization they can be optimized independently, such that an improvement of a criterion does not lead to a degradation of another one. For practical purposes, the fitness function, see formula (2), can be used for simulating multi-objective optimization by choosing adequate weight function. Thus, by choosing the values $w = 5$ for the criterion (a), $w = 2$ for criterion (b) and $w = 1$ for criterion (c) we achieve fast convergence and hence reduced execution time, but for example by improving criterion (a) we cannot avoid the degradation of criterion (b) or (c). By using frameworks which support multi-objective mechanism, better convergence of the genetic algorithm is expected.

The jMetal project was started in 2006 [39] and since then it underwent significant improvements and major releases [41], such that the redesigned jMetal should be useful to researchers of the multi-objective optimization community, such as evolutionary algorithm, evolution strategies, scatter search, particle swarm optimization, ant colony optimization, etc., [42]. Improvements regarding a new package for automatic tuning of algorithm parameter settings have been introduced [43] in order to facilitate accurate Pareto front approximations.

In addition, jMetal in conjunction with Spark – which is becoming a dominant technology in the Big Data context – have been used to solve Big Data Optimization problems by setting up a software platform. Accordingly, a dynamic bi-objective instance of the Traveling Salesman Problem based on near real-time traffic data from New York City has been solved [44] [45].

VI. Pursuing considerations on metaheuristics: An excerpt

This section is a logical continuation of the last two preceding sections, it extents the approach of the use case in Section IV by considering the jMetal framework presented in Section V as the natural and fruitful terrain for further enhancement and advanced development.

A. General Considerations

We definitely do not intend to analyze in depth the Apache framework versus the jMetal framework; we restrict our analysis to the algorithm used in Section IV, which is ported to the jMetal framework. The comparison and test results should be interpreted accordingly, i.e., some other use case could lead to different conclusions. Moreover, the Apache and the jMetal frameworks are templates, which the developers can freely modify according to their needs, hence minor modifications in the templates could lead to unexpected major improvements.

```
//  gGA.java
//
//  Author:
//       Antonio J. Nebro <antonio@lcc.uma.es>
//       Juan J. Durillo <durillo@lcc.uma.es>
//
//  Copyright (c) 2011 Antonio J. Nebro, Juan J. Durillo
...
package jmetal.metaheuristics.singleObjective.
     geneticAlgorithm;
...
    while (evaluations < maxEvaluations) {
       ...
      // Copy the best two individuals to the offspring
           population
      offspringPopulation.add(new Solution(population.get
           (0))) ;
      offspringPopulation.add(new Solution(population.get
           (1))) ;
      // Reproductive cycle
        ...
        // Crossover; Mutation; Evaluation of the new
             individuals; Replacement: the two new
             individuals are inserted in the offspring
             population
        ...
      // The offspring population becomes the new current
           population
      ...
    } // while
...
```

Listing 1: Copy only the two best individuals to the offspring population.

The main objectives of this section are:

a) compare the multi bjective, jMetal framework based genetic algorithm versus the single-objective genetic, Apache framework based algorithm presented in Section IV,

b) compare the jMetal framework based genetic algorithm with other heuristic algorithms,

c) record the performance behavior of the investigated algorithms,

d) determine pros and cons of the Apache framework versus the jMetal framework and the different heuristic algorithms.

Unfortunately, the Apache framework does not support multi-objective optimization, hence in order to get around it, we switch to the jMetal framework, which has been deliberately developed to allow such comparisons. The best way to get directly familiar with jMetal or Apache in addition to studying the scientific literature [34] [35], [39]–[45] , is to read coding [46] [47].

Validation is the methodology to investigate whether individual bricks (ICs) or set of bricks satisfy the specified constraints. The validation fields – i.e., the structure that holds the information necessary to perform the decision regarding the creation of the next generation – is used both in the Apache and the jMetal framework based implementation of the genetic algorithm. The validation results are calculated during the validation process, i.e., during the process where it is checked whether the individual bricks or set of bricks fulfill the given constraints. This way, the overall quality of an individual during the reproduction process can be very well determined. As soon as the validation fields are fully filled, the information in this structure can be used to calculate the fitness value of the individuals and subsequently, it can be used to determine the unsuitable bricks and exchange them during the crossover and mutation process. As already presented, the fitness value contains the final score of an individual during the validation process, in other words, it represents the degree to which the individual is more closely to fulfill the specification of the work order. For example, if a brick breaks one of the constraints regarding the attribute values, it has to be removed during the mutation process, since the attributes are specific properties of each brick and unlike the measurement constraints, they are not influenced by the other bricks assigned to an individual.

```
// GeneticAlgorithm.java
// Licensed to the Apache Software Foundation (ASF) under
     one or more contributor license agreements.
...
package org.apache.commons.math3.genetics;
...
//Implementation of a genetic algorithm. All factors that
     govern the operation of the algorithm can be cond for
     a specific problem.
...
public class GeneticAlgorithm {
...
//Evolve the given population. Evolution stops when the
     stopping condition is satisfied.
...
        while (!condition.isSatisfied(current)) {
            current = nextGeneration(current);
            generationsEvolved++;
        }
...
}
```

Listing 2: Apache Commons Math. Stopping Condition

Summing up, the validation results of an individual contain in addition to the information regarding the final score, also details regarding the genes (bricks) which violate the constraints. This latter kind of information is necessary for further processing. Normally, the validation results of the parents should be determined before the mutation step is performed. In the Apache framework a pair of two individuals is provided and the crossover and mutation operations are performed directly on them. Unfortunately, this is not the case for the jMetal framework, it does not provides pairing, just individuals. As a consequence, since the validation fields are not part of the original framework, the validation results are not properly calculated and some – but not necessary all – of the mutations cannot be performed and are skipped. In order to overcome this anomaly, we modified the original implementation of the mutation procedure used in the Apache context, by adding an additional validation in order ensure that before mutation is performed, the parents dispose of the appropriate validation results.

Unfortunately, jMetal has only one predefined stopping condition, i.e., the algorithm can stop only if the number of evaluations, i.e., in our case the number of generations surpasses some threshold – i.e., *maxEvaluations* – compare Listing 1, line 11. In contrast, the Apache framework allows additional to the condition above, multiple and flexible stopping conditions:

a) exit after some predefined time,

b) exit if the envisaged result has been found, etc., see Listing 2, line 12 for an example,

c) exit according to any user defined conditions, due to the flexibility of the Apache environment.

```
// ElitisticListPopulation.java
// Licensed to the Apache Software Foundation (ASF) under
     one or more contributor license agreements.
 ...
package org.apache.commons.math3.genetics;
...
//Population of chromosomes which uses elitism (certain
     percentage of the best chromosomes is directly copied
     to the next generation).
 ...
public class ElitisticListPopulation extends
     ListPopulation {
    // percentage of chromosomes copied to the next
        generation
    private double elitismRate =10.0;
...
    // Creates a new ElitisticListPopulation instance.
     ...
     // Start the population for the next generation.
     ...
    public Population nextGeneration() {
        // initialize a new generation with the same
            parameters
...
        // index of the last "not good enough" chromosome
        for (int i=boundIndex; i<oldChromosomes.size(); i
            ++) {
            nextGeneration.addChromosome(oldChromosomes.
                get(i));
        }
...
}
```

Listing 3: Apache Commons Math. Elitism Rate

The jMetal framework also uses a predefined type of population, characterized by the elitism rate, which specifies the percentage of individuals with the highest fitness value to be taken over unmodified to the new generation. This way, the genetic algorithm will put directly some of his best solutions unmodified to the next generation. This strategy has been successfully used with the Apache framework to optimize the run time of the genetic algorithm. Unfortunately, the elitism rate is hard coded and cannot be configured in the jMetal framework. Only the two best solutions are forwarded unmodified to the next generation, compare Listing 1, line 17,18. This means by considering our population size of 500 individuals, an elitism rate of 0.4%. We obtained best performance results within the Apache framework with this rate set to 10 in the benchmarks of Section IV, see Listing 3. In contrast to the jMetal framework, where it is not possible to create or define a new population type, the Apache framework can use a lot of other population types apart from the class *ElitisticListPopulation*, for example population types based on random selection of the individuals or based on keeping diversity selection. These new population types can be set up either by the user itself, or the development team of the Apache framework. Of course, random selection can be a promising approach, since some of the best genes can be hidden in bad individuals and those individuals might evolve later. The strategy to diversify means especially, that no similar individuals will be directly selected for the next generation. Obviously, the concept of similarity has to be specified accordingly, for example, by trying to have as much bins involved in the generation of the initial population, we avoid to have a restricted set of bins in the initial population that has no solution at all.

As mentioned above, the jMetal framework is less flexible than the Apache framework, i.e., the possibility to configure the jMetal framework is less pronounced. Hence, we cannot compare the performance of those frameworks directly, instead we are obliged to switch to those less efficient configuration parameters that work for both frameworks in order to be able to compare similar runs. We intend to do performance benchmarks as follows:

a) Apache versus jMetal (Apache and jMetal best performing configuration);

b) Apache versus jMetal (Apache and jMetal same configuration);

c) jMetal single objective genetic algorithm versus jMetal multi-objective genetic algorithm;

d) jMetal genetic algorithm versus jMetal other heuristic algorithms.

B. *Used Methods and Strategies*

Evolutionary algorithms such as the Non-dominated Sorting Genetic Algorithm-II (NSGA-II) [28], [43], [48] and Strength Pareto Evolutionary Algorithm 2 (SPEA-2) [49], [50] have become standard approaches, although some approaches based on Particle Swarm Optimization (PSO) [51], simulated annealing, and Scatter Search (SS) [52] are best known. Durillo [39] gives a very good overview of jMetal, describing some of the evolutionary algorithms implemented, for example NSGA-II, SPEA-2, Pareto Archived Evolution Strategy (PAES), Optimal multi-objective Optimization based on Particle Swarm Optimization (OMOPSO) [53], Archive based hYbrid Scatter Search (AbYSS), etc. The PSO algorithm has been used successfully not only for continuous optimization problems, but also to a combinatorial optimization of a real university course timetabling problem [54], see [55] for a more recent study regarding non-continuous search spaces.

Besides jMetal there are further frameworks for meta-heuristic optimization, for example Opt4J [56]. This framework is modular, i.e., it supports the optimization of complex optimization tasks by allowing its decomposition into subtasks that may be designed and developed separately. Additional optimization frameworks, java-based and non-java-based are listed on the SourceForge site [57], where also the latest release can be found. Additionally, a framework that works as a top-level layer over generic optimization frameworks that implement a high number of metaheuristics proposed in the technical literature, such as jMetal and Opt4J is presented in [58]. JECoLi, another novel Java-based library for the implementation of metaheuristic optimization algorithms with a focus on Genetic and Evolutionary Computation based methods is introduced [59].

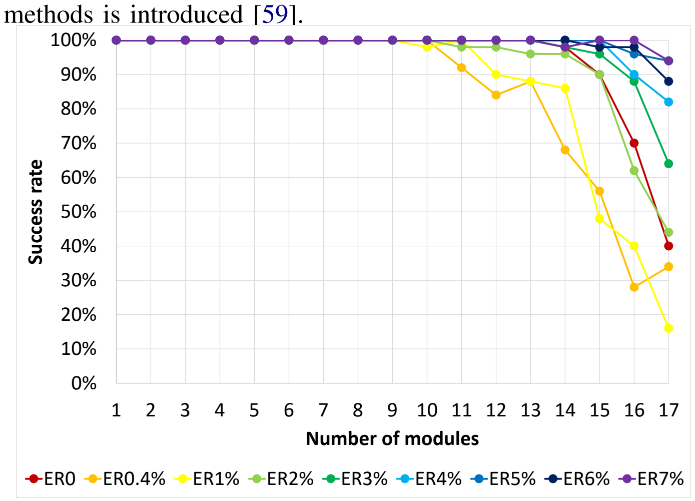


Figure 20: Success rate depending on the elitism rate.

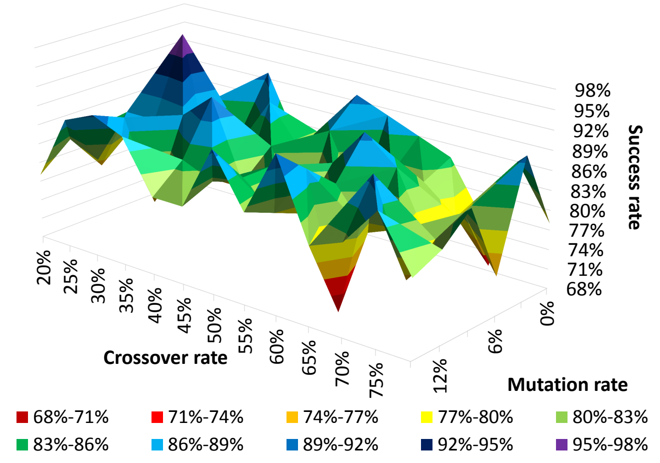


Figure 21: jMetal single-objective optimization: Success rate depending on the crossover rate and the elitism rate.

C. Performance Results

This subsection is aligned with Subsection IV-C as far as the testing strategy is concerned, such that performance and accuracy comparison are possible. Accordingly, the test scenarios in this subsection are similar to those in Figure 15, Figure 16, Figure 17, Figure 18 as done for the Apache Framework. The configuration parameter for the test settings are as in Table I.

1) jMetal Single-Objective Genetic Algorithm: Actually, the naive expectation regarding the similitude between the performance behavior of the Apache Framework and the single-objective optimization part of the jMetal Framework has not been met. Furthermore, for the Apache Framework, the elitism rate is not so sensitive regarding the fluctuation of the success rate, whereas for the jMetal Framework this is true for the mutation rate.

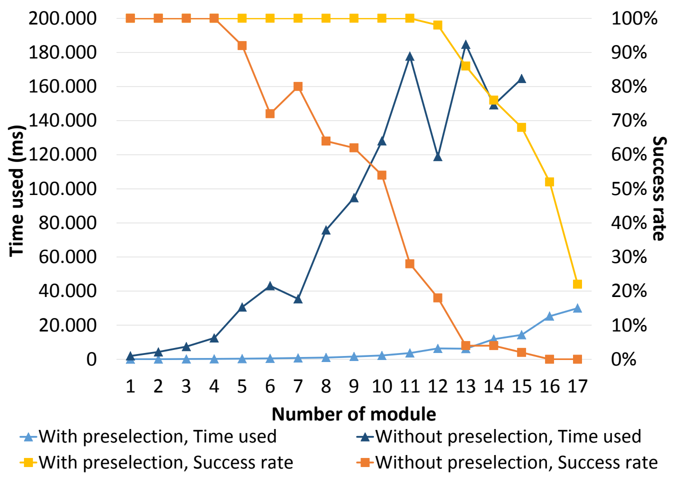


Figure 22: jMetal single-objective optimization: Success rate and time used depending on the number of modules with and without pre-selection.

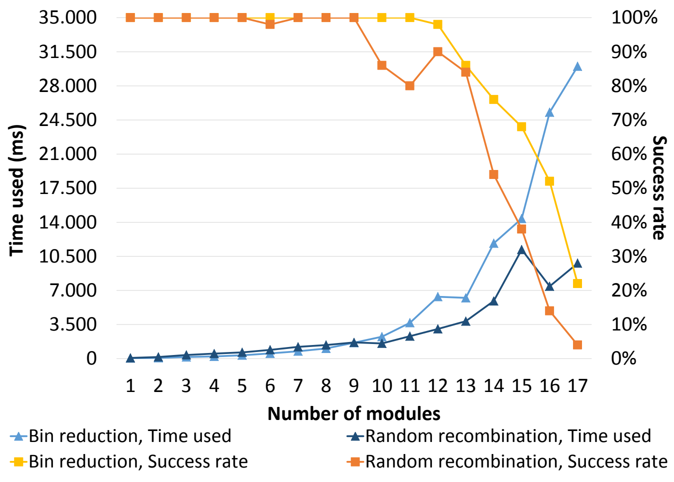


Figure 23: jMetal single-objective optimization: Success rate and time used when selecting the random recombination crossover policy or the bins reduction policy.

Figure 22 corresponds to Figure 15 and Figure 23 corresponds to Figure 16 respectively, generated for genetic algorithm using the Apache Framework. For a detailed background description see the corresponding explanation in Subsection IV-C. Figure 24 corresponds to Figure 17 and Figure 25 corresponds to Figure 18 respectively, generated for genetic algorithm using the Apache Framework.

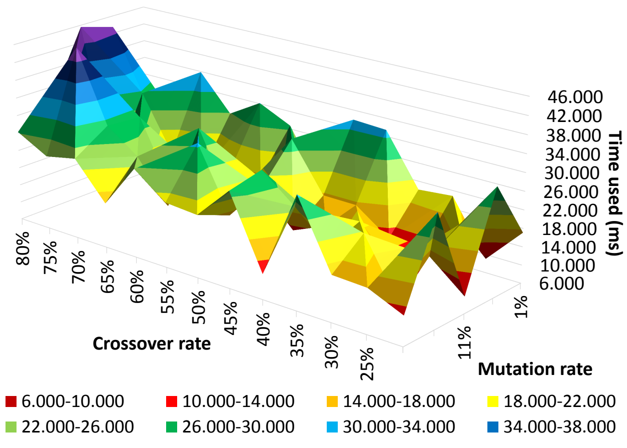


Figure 24: jMetal single-objective optimization: Time used depending on the crossover rate and the mutation rate.

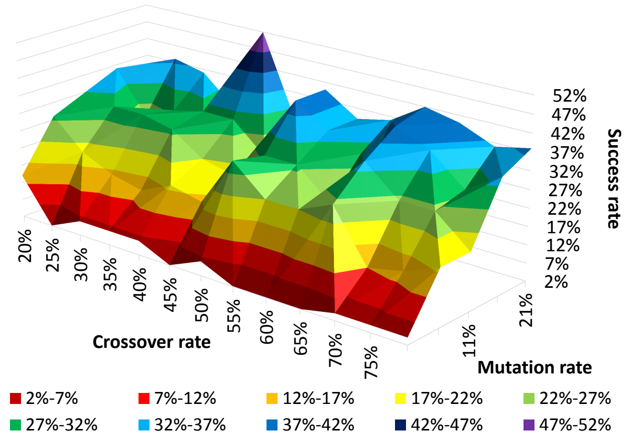


Figure 25: jMetal single-objective optimization: Success rate depending on the crossover rate and the mutation rate.

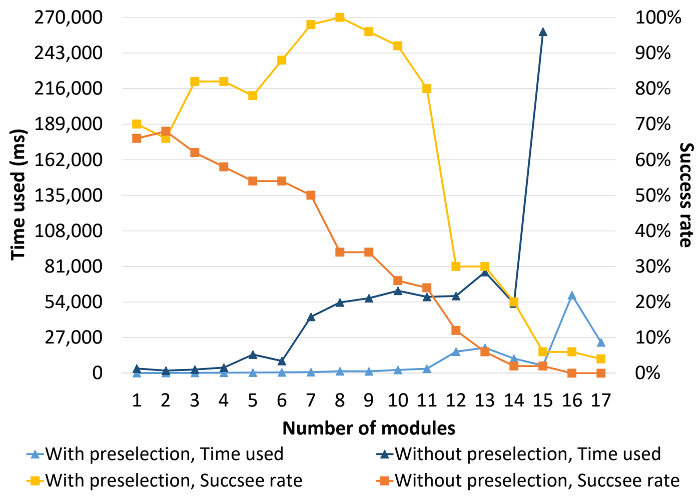


Figure 26: jMetal multi-objective optimization using NSGA-II : Success rate and time used depending on the number of modules with and without pre-selection.

Unexpectedly, the elitism rate has a major influence on the success rate, even for lower values, see Figure 20 for a test scenario with the elitism rate $\in [0\%, 7\%]$. The other configuration parameters for the performance tests are given in Table I. The optimal value for the elitism rate is around %7, unfortunately the elitism rate is not configurable in the official release of jMetal. As already mentioned, to circumvent this shortcoming, we have adapted the original open source code accordingly. As is Figure 21, increasing the elitism rate does not improve the success rate, moreover an appropriate tuple of crossover rate and elitism rate seems to be the most effective.

For a detailed description of the corresponding performance results, see the explanation in Subsection IV-C. A low crossover rate and low elitism rate gives the success rate at over 95%. Increased crossover rates and increase elitism rates also deliver success rates at over 90%. Unfortunately, high success rates also deteriorates considerably its run time aspects.

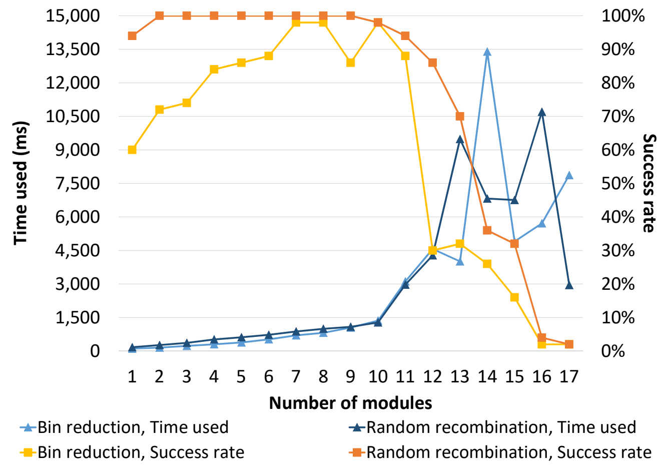


Figure 27: jMetal multi-objective optimization using NSGA-II : Success rate and time used when selecting the random recombination crossover policy or the bins reduction policy.

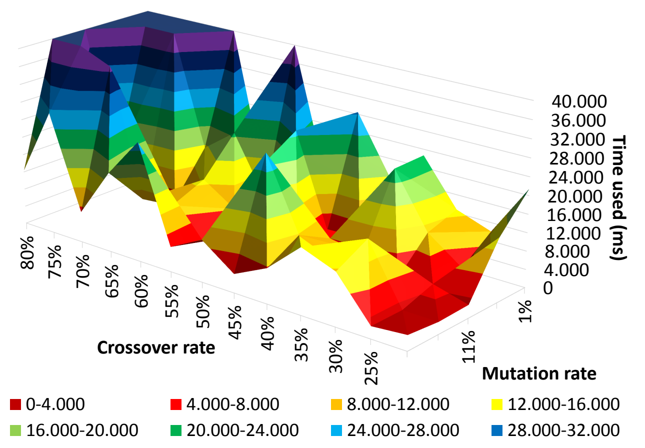


Figure 28: jMetal multi-objective optimization using NSGA-II : Time used depending on the crossover rate and the mutation rate.

2) jMetal multi-objective Genetic Algorithm: Unfortunately, NSGA-II does not use the concept of elitism rate to select offsprings, it uses Pareto front and distance instead. Hence we cannot compare directly the similar tests for single-objective optimization based on crossover rate and elitism rate, see Figure

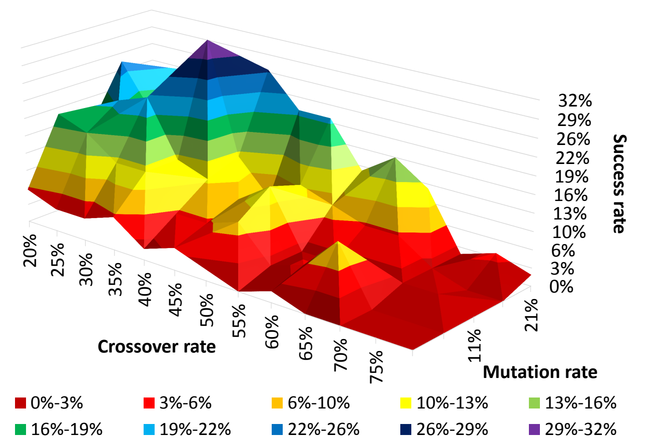


Figure 29: jMetal multi-objective optimization using NSGA-II : Success rate depending on the crossover rate and the mutation rate.

Figure 26 corresponds to Figure 15 and Figure 27 corresponds to Figure 16 respectively, generated for genetic algorithm using the Apache Framework. For a detailed background description see the corresponding explanation in Subsection IV-C. Figure 28 corresponds to the Figure 17 and Figure 29 corresponds to Figure 18 respectively, generated for the genetic algorithm using the Apache Framework.

3) Other evolutionary algorithms: To conclude, we examine the scatter search template adapted to a hybrid - single objective optimization to the multi-objective domain - called Archive-Based hYbrid Scatter Search (AbYSS) [60]–[63]. AbYSS follows the scatter search structure, but uses mutation and crossover operators from evolutionary algorithms. It incorporates typical concepts from the multi-objective field, such as Pareto dominance, etc.

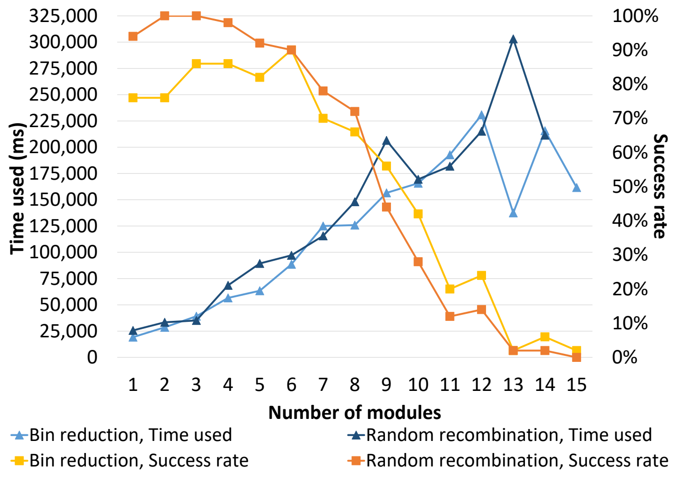


Figure 30: jMetal AbYSS hybrid optimization : Success rate and time used when selecting the random recombination crossover policy or the bins reduction policy.

We generate appropriate tests – as represented in Figure 30, see Figure 16 for the Apache counterpart – concerning the success rate and time used by using the random recombination crossover policy and the bins reduction policy respectively. According to [61] the results of their tests show that AbYSS not only reaches very accurate solutions, but also it scales well with increasingly sized instances. Unfortunately, the performance and accuracy of our tests is lower than expected, such that the AbYSS algorithm finds no more than 15 modules, whereas the number of possible solution is 17, see Figure 30 for the Apache Framework genetic algorithm counterpart. We cease further testing.

4) Final conclusions: Summing up, low crossover and low elitism rates or high crossover and high elitism rates deliver good results for the success rate. On the contrary, high success rates are always achieved with high timed used rates. The success rate is not very sensitive regarding the mutation rates as long as they are low or moderate. Satisfactory results have been achieved with elitism rates exceeding 3%. It seems that the higher success rate in the single-objective optimization case is due to a very well balanced fitness function, whereas if no time for performance improvement tests, the multi-objective path delivers satisfactory results with lower implementation effort. The run time performance of our application implemented using the Apache Framework genetic algorithm is order of magnitudes better than the corresponding implementation based on the jMetal Framework.

VII. Conclusion and Future Work

The main challenge, which led to the results of this paper, was to investigate whether a real-life combinatorial problem, which popped up at a semiconductor company, can be solved within a reasonable time. Moreover, the solution should be flexible, such that the solution is not restricted to the existing specification of the modules.

We established an abstract formal model, such that the implementation of the use case is fully functional within the boundaries of this model. In this sense, new constraints can be added to the existing ones, no inside knowledge regarding the structure of the module is needed, as it is the case for the heuristic algorithm in place.

We set up a genetic algorithmic approach based on the Apache Commons Mathematics Library, implemented it and validated the results. Some decisive policies like the crossover and mutation policy have been additionally implemented and new optimizations like the bin reduction crossover policy have been set up to improve the convergence of the genetic algorithm. The performance results were satisfactory for an industrial application.

The current implementation does not combine good genes (taking into account all the constraints, like measurement, etc.) of the parents. Instead, the crossover strategy is based on random decisions. Additional research is necessary in this direction to find a good balance between a more general suitability (random decisions) and good convergence (adjusted crossover policy). For the time being, only the fitness function contains proprietary information regarding the production process, any other decision is aleatoric.

However, improvements such as the bin reduction crossover policy and improvement of the bin preserving mutation policy based on internal knowledge have in most cases advantageous effects on the selection algorithm. Unfortunately, the advantages of these improvements, like run time reduction and improved accuracy, are achieved by less flexibility. Whenever the external circumstances change, the corresponding policies have to be adapted accordingly.

The implemented basic framework is very flexible, it has many configuration possibilities like the elitism rate and the arity. As a consequence of the random variables, many convergence tests with various configuration assignment have to be performed, in order to ensure satisfying results. Furthermore, of crucial importance for the successful completion of the algorithm is the design of the fitness function, especially the values of the weights.

The convergence tests show that not every execution will succeed to find the best solution delivered, this is due to the random numbers used through out the genetic algorithm. This is exemplified by the success rate. Thus, the selection algorithm based on genetic strategies always delivers local maxima, which may substantially differ from the global one. Our attempt to find the optimal solution using MATLAB for a reduced set of ICs failed due to the long execution time.

As already mentioned, the fitness function plays an outstanding role during the selection of the best candidates for the next generation. This means especially, that two candidates having the same value or fitness function are considered of the same quality. This assertion is not accurate enough, due to the use of three different weights, whose interdependence can hardly be anticipated. Each weight represents the significance of one aspect of the quality of a candidate. To circumvent this dilemma, Pareto optimality [29] can be used to solve the challenge of the multi-objective function. In this case, a new framework [39] is needed, since Apache Commons Mathematics Library does not support multi-objective mechanism. Genetic algorithms, if configured properly, can be used to solve our constraint satisfaction problem. The delivered solution may substantially differ from the optimal one.

The current problem is not defined as an optimization problem, the constraints of a work order are either satisfied or not. Accordingly, two different solutions of the same work order, which satisfy the constraints, are of the same quality. However, the genetic algorithm is based internally on an optimization process – the higher the value of the fitness function, the better the solution. The constraints are used as exit criterion for the genetic algorithm. In this way, the optimization is stopped arbitrarily, considering that a better solution is out of scope.

Moreover, during the production process multiple work orders have to be honored simultaneously. The current strategy at the semiconductor company adopted a sequential one. We can reformulate the problem as an optimization problem: Given a list of work orders, find the maximum number of work orders that can be satisfied simultaneously.

The elapsed time till the genetic algorithm of MLBP converges is in range of seconds, by all means satisfactory for the investigated industrial application. As expected, not all the execution threads converge to the same solution, and not all the threads find an optimal solution, as shown in some cases less than 60 percent. Therefore, starting a bunch of threads within the genetic algorithm increases the chance towards better solutions.

To summarize, regarding the configuration possibilities, the jMetal framework is less flexible than the Apache framework, i.e., the Apache framework allows much more alternatives for settings, in contrast in order to achieve greater adjustability in the jMetal framework, the source code has to be adapted accordingly. The jMetal framework predefines a lot of heuristic algorithms, but they have to be refined and improved in order to reach the profoundness of the Apache framework.

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