Multi-port Power Amplifier Calibration Estimation Technique Based on ICA Algorithm

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***Abstract*—A multi-port power amplifier (MPA) is a multi-input and multi-output system that is capable of amplifying several input signals simultaneously by a set of shared amplifiers without mutual interference. In a practical MPA, the component imperfections reduce the port isolation, which introduces leakage or cross-port interference among the output ports. In this paper, an independent component analysis (ICA) based technique is developed to estimate a calibration matrix that minimizes the effects of the imperfections. The MPA output signals are used in the estimation of the calibration matrix. Simulation studies show that the proposed calibration matrix estimation technique can lead to a significant improvement in the MPA output port isolation.**

***Keywords*-Multi-port power amplifier; Calibration; Independent component analysis; JADE;**

I. Introduction

Multi-beam satellite communication systems can achieve a high antenna gain and thus improve their effective isotropically radiated power and gain/noise temperature. In such a system, users may move from one geographic area to another serviced by different beams. As a result, traffic may not be distributed uniformly among the beams and it fluctuates over time. A multi-port power amplifier (MPA) based architecture [1]-[3] is an effective approach to enable flexible power allocation in multiple beam systems to address capacity variations among the beams during the lifespan of the satellite.

An MPA is a multi-input and multi-output system that is capable of amplifying multiple input signals simultaneously using shared amplifiers. Ideally, the MPA amplifies the multiple input signals and outputs them separately via different output ports without any mutual interference. In practice, component imperfections in the MPA reduce the port isolation, which introduces leakage or cross-port interference among the output ports.

The component imperfections can be reasonably well controlled over temperature and time at low frequencies [4], but it is difficult and costly at microwave frequencies and above. Some methods have been presented to linearize the nonlinearity of the PAs in the MPA [5], [6]. Assuming that the PAs are linear, a type-based calibration technique, which exploits the uniqueness of the statistics for a given communication signal, has been presented [7] to compensate for the imperfections in other MPA components. In this paper, we develop a technique based on the independent component analysis (ICA) algorithm [8], [9] to estimate a calibration matrix to compensate for MPA imperfections. This technique does not require any prior information about the MPA input signals except that they are independent. The ICA is a statistical and computational approach with many applications, including source separation and biomedical signal processing. Here, the proposed technique uses a conventional ICA algorithm, i.e., the joint approximate diagonalization of eigenmatrices (JADE) [8], to estimate the input-output transfer function of the MPA from its output signals. A calibration matrix, i.e. the inverse of the MPA transfer function, is applied to the input signals before feeding them to the MPA to compensate for the impairments in the MPA. Computer simulations are used to demonstrate the performance of the proposed technique.

The paper is organized as follows: Section II describes the MPA model. Section III presents the ICA based calibration matrix estimation technique. Simulation results are given in Section IV. Section V concludes the paper.

II. Multi-port Power Amplifier Model

An N-port MPA system has N input ports and N output ports. It is composed of an input hybrid matrix (IHM), a set of N shared power amplifiers (PA), and an output hybrid matrix (OHM). The IHM is made up of 3dB, 90° hybrid couplers, and has N input ports and N output ports. The PAs operate in their linear region, and each of the PA inputs is connected to one output port of the IHM. The PA outputs are connected to the inputs of the OHM that is identical in structure to the IHM. Fig. 1 shows a functional block diagram of a 4-port MPA.

Ideally, the MPA's input signals are summed together by the IHM with different phase relations to generate N summed signals. Each summed signal is amplified by one PA, and then fed to one input port of the OHM. The OHM combines its input signals coherently to generate the amplified original input signals. If there are no component mismatches/imperfections, the transfer function of the MPA,

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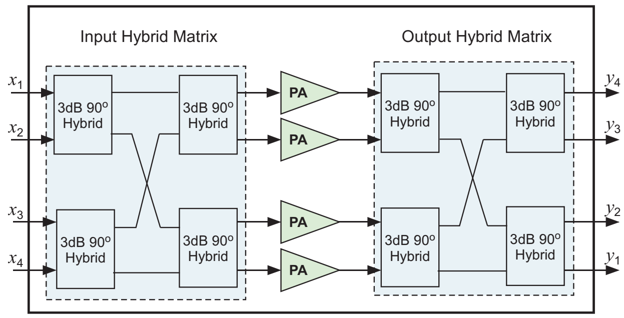

Figure 1. Functional block diagram of a 4-port MPA.

denoted by an $N \times N$ square matrix $\mathbf{A}$, is an anti-diagonal matrix. In this paper we reverse the output port order such that $\mathbf{A}$ is represented by a diagonal matrix $G\mathbf{I}$, where G represents a complex gain of the MPA, and $\mathbf{I}$ is an $N \times N$ identity matrix. Without loss of generality, we assume $G = 1$.

The input and output relationship of the MPA can be expressed by

$$\mathbf{y} = \mathbf{A}\mathbf{x} \tag{1}$$

where $\mathbf{x} = [x_1,\ x_2,\ \cdots,\ x_N]^{\mathrm{T}}$ denotes the input signal vector, $\mathbf{y} = [y_1,\ y_2,\ \cdots,\ y_N]^{\mathrm{T}}$ denotes the output signal vector, and the superscript "T" denotes the vector transpose. It should be noted that $\mathbf{y}$ and $\mathbf{x}$ are functions of time with the time index t being implied for ease of presentation.

In the ideal MPA, each input signal is co-amplified by the shared PAs and then combined coherently for output via individual output ports without mutual interference due to the diagonal transfer function. However, in reality, the component mismatches/imperfections destroy the diagonal property of the MPA transfer function, yielding a transfer function represented by a full square matrix with non-zero complex-valued elements

$$\mathbf{A} = \begin{bmatrix} a_{1,1} & a_{1,2} & \cdots & a_{1,N} \\ a_{2,1} & a_{2,2} & \cdots & a_{2,N} \\ \vdots & & \ddots & \vdots \\ a_{N,1} & a_{N,2} & \cdots & a_{N,N} \end{bmatrix}$$

$a_{k,l}$ $(k, l = 1, 2, \cdots, N)$, which represents the transfer characteristics from the lth input port to the kth output port. Normally, $|a_{k,k}| \approx 1$ for $k = 1, 2, \cdots, N$, while $|a_{k,l}| << 1$ for $k \neq l$, where $|\cdot|$ denotes the complex norm operator. Therefore, the inverse of $\mathbf{A}$ always exists.

III. ICA based Estimation Technique of the Calibration Matrix

The objective of the MPA calibration is to eliminate the leakage among different ports by restoring the diagonal property of its transfer function. Figure 2 illustrates a simplified functional block diagram of an N-port MPA with the calibration function. The MPA output signals are sampled in parallel, and used in the digital signal processor (DSP) to estimate the calibration matrix.

The output of the MPA with calibration can be expressed by

$$\mathbf{y} = \mathbf{A}\mathbf{C}\mathbf{x} \tag{2}$$

where $\mathbf{C}$ denotes the calibration matrix. If $\mathbf{AC} = \mathbf{I}$, the MPA is perfectly calibrated, which occurs when $\mathbf{C} = \mathbf{A}^{-1}$.

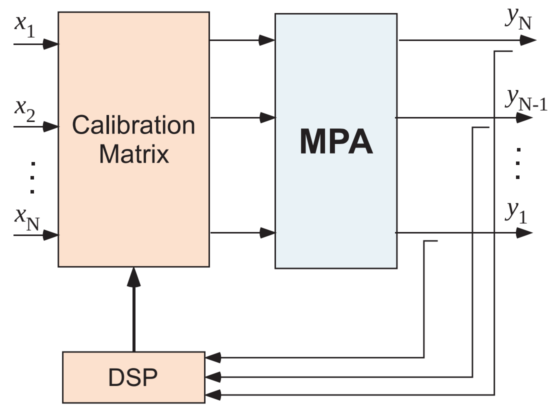

Figure 2. Simplified functional block diagram of the MPA with calibration function.

To determine $\mathbf{C}$, let's assume that x_ns are independent non-Gaussian signals with zero mean and unit variance. That is, $\mathrm{E}[\mathbf{x}\mathbf{x}^H] = \mathbf{I}$, where $\mathrm{E}[\cdot]$ is the mathematical expectation operator and the superscript "H" denotes Hermite transpose. At the MPA output, y_n's become correlated due to the signal leakage.

The ICA-based estimation technique consists of three steps. In the first step, we pre-whiten y_n's by multiplying a full ranked matrix $\mathbf{M}$, yielding a new vector $\mathbf{z} = \mathbf{M}\mathbf{y}$, where $\mathbf{z} = [z_1,\ z_2,\ \cdots,\ z_N]^{\mathrm{T}}$, such that z_n's are mutually uncorrelated and all have unit variance, i.e., $\mathbf{z}$ becomes $\mathrm{E}[\mathbf{z}\mathbf{z}^{\mathrm{H}}] = \mathbf{I}$. This transformation is always possible and can be accomplished by classic principal component analysis or singular value decomposition. $\mathbf{M}$ is constructed from the eigenvalues and eigenvectors of the covariance matrix of $\mathbf{y}$ [9]. After transformation, we have

$$\mathbf{z} = \mathbf{M}\mathbf{y} = \mathbf{M}\mathbf{A}\mathbf{x} = \mathbf{B}\mathbf{x} \tag{3}$$

where $\mathbf{B} = \mathbf{MA}$ is an orthogonal matrix due to the assumptions on $\mathbf{x}$: it holds $\mathrm{E}[\mathbf{z}\mathbf{z}^{\mathrm{H}}] = \mathbf{B}\mathrm{E}[\mathbf{x}\mathbf{x}^{\mathrm{H}}]\mathbf{B}^{\mathrm{H}} = \mathbf{I}$. According to Eq. (3), $\mathbf{e} = \mathbf{B}^{-1}\mathbf{z} = \mathbf{B}^{\mathrm{H}}\mathbf{z}$ will be the estimate of $\mathbf{x}$, where $\mathbf{e} = [e_1,\ e_2,\ \cdots,\ e_N]^{\mathrm{T}}$.

In the second step, the orthogonal matrix $\mathbf{B}$ is determined by the JADE algorithm as a unitary maximizer of the cost function

$$\zeta(\mathbf{B}) = \sum_{i,k,l=1}^{N} |cum(e_i, e_i^*, e_k, e_l^*)|^2$$

where $cum(e_i, e_i^*, e_k, e_l^*)$ denotes the 4th-order cumulants of $\mathbf{e}$ [8]. Thus $\mathbf{B}$ can be obtained as

$$\mathbf{B} = \arg\max_{\mathbf{B}} \zeta(\mathbf{B}) \tag{4}$$

Eq. (4) is equivalent to minimize the sum of all the squared cross-cumulants, i.e. distinct indices in the first and second terms as $cum(e_i, e_j^*, e_k, e_l^*)$.

For the complex N-dimensional random vector $\mathbf{e}$, its 4th-order cumulants are associated with a quadricovariance denoted by $\mathbf{Q}$, defined as a linear matrix-to-matrix mapping $\mathbf{R} \to \mathbf{S} = \mathbf{Q}(\mathbf{R})$ by

$$s_{ij} = \sum_{k,l=1}^{N} cum(e_i, e_j^*, e_k, e_l^*) r_{kl} \tag{5}$$

where $\mathbf{R}$ and $\mathbf{S}$ are $N \times N$ matrices with entries r_{ij} and s_{ij}, respectively. There exist N^2 real numbers λ_n and N^2 orthogonal matrices $\mathbf{R}_n$, satisfying $\mathbf{Q}(\mathbf{R}_n) = \lambda_n \mathbf{R}_n$, $n = 1, 2, \cdots, N^2$. Note that $\mathbf{Q}$ is actually a 4th-order tensor and $\mathbf{R}_n$'s are the eigenmatrices of $\mathbf{Q}$ associated to its eigenvalues λ_n. It has been proved [8] that the quadricovariance $\mathbf{Q}$ has exactly rank N so that only N out of all λ_ns are non-zero. The cost function in Eq. (4) becomes

$$\zeta(\mathbf{B}) = \sum_{n=1}^{N} \lambda_n \| diag(\mathbf{B}\mathbf{R}_n\mathbf{B}^{\mathrm{H}}) \|^2 \tag{6}$$

where $\|diag(\cdot)\|$ is the norm of the vector built from the diagonal elements of the matrix argument. According to Eq. (6), the unitary matrix $\mathbf{B}$ can be obtained by a joint diagonalization of the N eigenmatrices $\mathbf{R}_n$.

However, it is well known that the JADE algorithm has two inherent ambiguities: permutation ambiguity and scaling ambiguity. That is, $\mathbf{A}$ can be estimated as

$$\mathbf{A} = \mathbf{M}^{-1}\mathbf{B}\mathbf{D}\mathbf{P} \tag{7}$$

where $\mathbf{D}$ is an $N \times N$ nonsingular diagonal matrix that stands for the scaling ambiguity, and $\mathbf{P}$ is an $N \times N$ permutation matrix that is simply obtained from an identity matrix with its rows and columns being re-ordered.

In the third step, we will solve the ambiguity issue. From the assumption that $\mathbf{A}$ has much larger diagonal elements than off-diagonal elements, we can obtain $\mathbf{P}$ that re-orders $\mathbf{M}^{-1}\mathbf{B}$ so that the larger elements are on its diagonal position. The scaling factor in $\mathbf{D}$ can be estimated from the eigenvalues of the first step as the diagonal elements in an ideal $\mathbf{A}$ are 1. Having determined the estimate of $\mathbf{A}$ of Eq. (7), the calibration matrix can be derived as $\mathbf{C} = \mathbf{A}^{-1}$.

To achieve a better estimation performance and to track variations of the MPA characteristics, the above 3-step procedure can be repeated as follows:

(i) Set $k = 0$ and $\mathbf{C}_k = \mathbf{I}$;

(ii) Estimate a new $\mathbf{C}$ from the MPA output signals using the above 3-step procedure;

(iii) Update the calibration circuit according to $\mathbf{C}_{k+1} = \mathbf{C}_k\mathbf{C}$;

(iv) Increment $k = k + 1$ and go to (ii).

IV. Computer Simulations

To validate the ICA based MPA calibration technique, computer simulations using Matlab have been performed for a 4-port MPA. It is assumed that the hybrids in the IHM and the OHM have a gain error within ± 1dB and a phase error within $\pm 10°$, and the PAs have a gain variation within ± 1dB and a phase variation within $\pm 10°$. The actual errors are generated from a uniform random number generator. The amplitude and phase in the resulting MPA transfer function used in our simulations is listed below, respectively:

$$\begin{bmatrix} 1.04 & 0.13 & 0.13 & 0.18 \\ 0.07 & 0.96 & 0.16 & 0.09 \\ 0.18 & 0.11 & 1.11 & 0.06 \\ 0.12 & 0.16 & 0.05 & 1 \end{bmatrix}$$

and

$$\begin{bmatrix} 1.3° & -122.4° & -9.1° & -149.8° \\ -112.6° & -13.6° & 25.0° & -16.1° \\ 173.2° & -62.5° & 11.3° & -174.2° \\ 95.7° & 167.1° & -87.3° & 0° \end{bmatrix}$$

Without loss of generality, one element is normalized to 1 in order to have a unique solution. The four input signals are QPSK, 16QAM, 8PSK, and 16QAM modulated with variances of 1, 2.25, 1, and 2.25, respectively. These signals are square root raised cosine pulse-shaping filtered with rolloff factors of 0.25, 0.35, 0.35, and 0.25. The simulation uses 8 samples per symbol. 10^5 samples, i.e. 12,500 symbols, are used in the simulation to calculate the cumulants. The 3-step procedure is repeated 10 times.

The error vector magnitude (EVM) is used to measure the quality of the MPA output signals. It is defined as

$$\mathrm{EVM} = \sqrt{\frac{\sum_{l=1}^{L} |D_{ideal,l} - D_{meas,l}|^2}{\sum_{l=1}^{L} |D_{ideal,l}|^2}} \times 100\% \tag{8}$$

where $D_{meas,l}$ is the value of the lth received symbol, $D_{ideal,l}$ is the ideal value of the lth symbol, and L is the total number of symbols used in the calculation. The EVM is essentially a measure of the interference to signal ratio.

Figure 3 illustrates the constellation of the MPA output signals without calibration. It clearly shows that the signal constellation scatters widely due to the cross port interference caused by the MPA imperfections. Figure 4 shows the constellations of the MPA output signals after calibration. We observe that the proposed ICA-based technique significantly reduces the cross-port interference and restores

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the signal constellations. The EVM values before and after calibration are summarized in Table I, which shows that they are reduced from 24% to 0.6% for QPSK, from 28% to 0.7% for 8PSK, and from 26% to 2% for 16QAM, respectively.

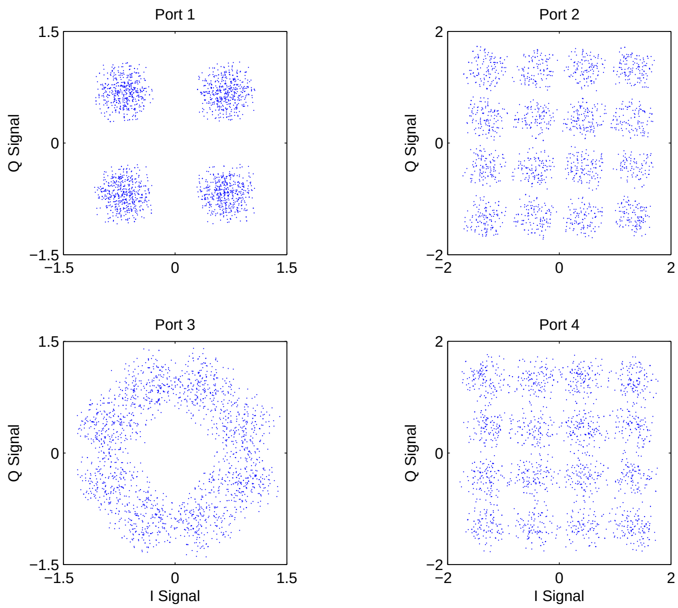

Figure 3. Signal constellations before calibration.

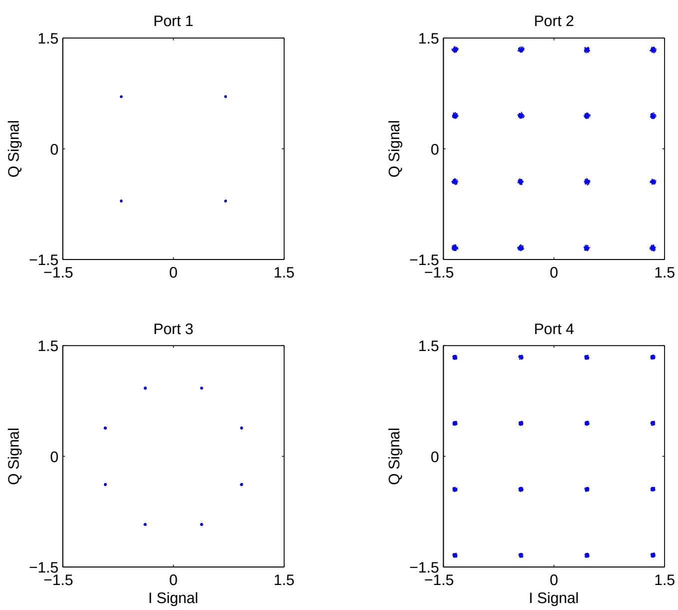

Figure 4. Constellations after calibration.

V. Conclusions

An MPA calibration technique based on the independent component analysis algorithm has been proposed in this paper. It applies a calibration matrix to the MPA input signals before feeding them to the MPA in order to precompensate for the impairments in the MPA. The calibration matrix is estimated using the JADE algorithm. Simulation results show that the proposed technique can significantly reduce the cross-port interference in the MPA output ports, which in turn greatly improves the system performance.

TABLE I
EVM values before and after calibration.

Port#	Before (%)	After %)
1 (QPSK)	24.10	0.63
2 (16QAM)	25.65	2.12
3 (8PSK)	28.26	0.74
4 (16QAM)	27.11	1.94

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