Downlink Energy Efficiency Power Allocation for OFDM-based Aerial Systems with Limited Satellite Backhaul

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Abstract—**Emergency communications can benefit from the integrated aerial-satellite systems due to the frequent Line Of Sight (LOS) access for User Equipments (UEs) and the robust satellite backhaul. This paper addresses an energy efficiency power allocation problem for a OFDM-based aerial system with limited satellite backhaul. Due to the non-convex backhaul capacity limit, the optimization problem is solved in two steps. Firstly, a non-negative parameter is introduced to convert the objective function to an equivalent convex form according to the fractional programming. Then, the optimal parameter for maximum Energy Efficiency (EE) without total power and backhaul capacity constraints is obtained. Secondly, by proving that the derivative of $R(P_{tot})$ is equal to the above introduced parameter, which is decreasing in total power limit, P_{tot}, these two constraints can be transformed into a lower bound on the parameter through geometric interpretation. Thus, an energy efficiency power allocation algorithm is proposed. Finally, numerical results show that the circuit power, total power limit and backhaul capacity limit have effect on the system EE.**

Keywords–energy efficiency; backhaul limit; power allocation.

I. Introduction

Large-scale disaster or unexpected emergency may overload or totally paralyze the existing terrestrial network in severe cases. Therefore, an effective Public Protection and Disaster Relief (PPDR) system is crucial to meet the requirements of victims and first responders, which is characterized by rapid deployment, large capacity-coverage, flexibility and scalability [1].

In this regard, ABSOLUTE project is working at establishing an integrated satellite-aerial-terrestrial architecture to guarantee reliable communication in the aftermath of natural disaster [2]. An important element of the integrated architecture is aerial station which is an air balloon or aircraft based Low Attitude Platform (LAP). The LAP goes from 100 meters to 1000 meters height, lifting with a low-complexity Long Term Evolution (LTE) eNB, named Aerial eNodeB (AeNB), and filling the gaps of destroyed LTE base stations. The lower altitude of aerial station compared to that of satellite makes it easier for LAP to support frequent Line Of Sight (LOS) with User Equipments (UEs), which explains the advantage of LAPs for the public safety network.

Emergency communications can greatly benefit from the integrated aerial-satellite systems which could provide UEs with high capacity-coverage thanks to low delay aerial links, while reliable backhauling links to remote networks (headquarters) can be supplied by the satellite segment [3]. On the other hand, due to the limited overload, LAPs are unable to carry enough battery and AeNB thus may face stiff constraints regarding its total energy consumption. Therefore, the Energy Efficiency (EE) is demonstrated as a significant metric to evaluate the power allocation performance of AeNB.

In this paper, we consider a subset of aerial-satellite systems. As illustrated in Figure 1, the system consists of a single air balloon based LAP, keeping a quasi-stationary position over a predefined area and serving a set of UEs. The AeNB is connected to its Evolved Packet Core (EPC) through optical link. The satellite serves as a backhaul to connect aerial system with Headquarter. We assume that aerial system is OFDM-based and design a downlink energy efficiency power allocation algorithm for a OFDM-based aerial system.

Different from previous work on downlink EE maximization for OFDM-based systems under only convex constraints, such as total power limit [4], minimal overall system throughput limit [5] and interference limit [6], we maximize the system EE with the total power constraint, peak power limit, QoS requirement of each UE, as well as non-convex backhaul capacity constraint. The problem is solved in two steps owing to this non-convex constraint. Firstly, according to the properties of fractional programming, a non-negative parameter is introduced to convert the objective function to an equivalent convex form. Then, the optimal parameter for maximum EE without total power and backhaul capacity constraints is obtained. Secondly, by proving that the derivative of $R(P_{tot})$ is the above introduced parameter, which is decreasing in total power limit, P_{tot}, this paper transform these two constraints into a lower bound on the parameter through geometric interpretation. Finally, theoretical analysis is corroborated by numerical experiments.

The remainder of the paper is organized as follows. In Section II, system description and channel model are introduced. In Section III, we present the energy efficiency power allocation algorithm. The simulation results are presented and discussed in Section IV. Finally, Section V concludes the paper.

II. System Description and Channel Model

In this section, we present our system model in detail. Also, a practical channel model is given here.

A. System Description

As illustrated in Figure 1, information is transmitted from the headquarter ground station to the AeNB through the satellite link L_1, L_2, Internet and optical link. In this paper, we omit the detailed signal transmission of these links and see them as a whole backhaul, the capacity of which is denoted as $C_{backhaul}$. We focus on the downlink transmission of the aerial system. We consider a single OFDM-based aerial network with K active UEs. The total bandwidth, B, is divided into B subcarriers, each with $W = \frac{B}{K}$.

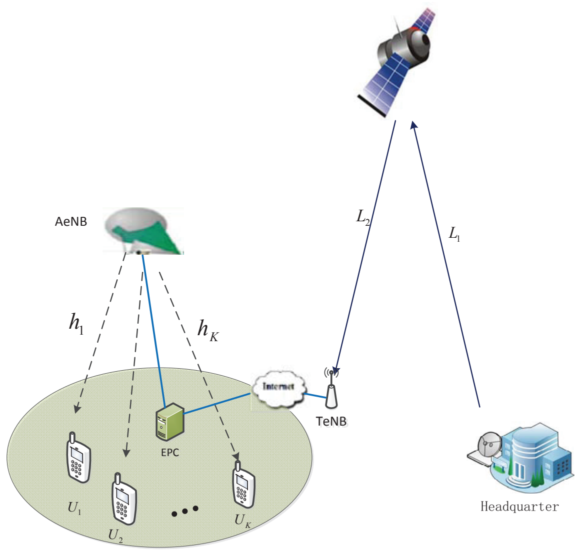

Figure 1. System Model in Integrated Aerial-Satellite Network

Assume that the kth subcarrier is assigned to the kth UE to avoid interference among different UEs. Then, the maximum achievable data rate at the kth UE is accordingly

$$r_k = W\log_2(1 + \frac{|h_k|^2 p_k}{\sigma^2}) = W\log_2(1 + g_k p_k), \quad (1)$$

where h_k and p_k denote the channel fading factor and transmit power from AeNB to the kth UE, respectively. σ^2 is the received noise power and $g_k = \frac{|h_k|^2}{\sigma^2}$ is the Carrier-to-Noise Ratio (CNR) of the kth subcarrier. Then, the overall throughput of the OFDM-based aerial system is given by $R = \sum\limits_{k\in\mathcal{K}} r_k$, where $\mathcal{K} = \{1, 2, ..., K\}$ denotes the set of all UEs.

The total power consumption at AeNB is modeled as [6]

$$P_{tot} = \zeta P + P_c, \quad (2)$$

where ζ is the reciprocal of drain efficiency of power amplifier. $P = \sum\limits_{k\in\mathcal{K}} p_k$ and P_c represent the transmit power consumption and the circuit power consumption incurred active circuit block, respectively.

We use the throughput for unit-energy consumption to measure the system EE, i.e.,

$$\eta_{EE} = \frac{\sum\limits_{k\in\mathcal{K}} W\log_2(1 + g_k p_k)}{\zeta \sum\limits_{k\in\mathcal{K}} p_k + P_c}. \quad (3)$$

The energy efficient power allocation problem considering each UE's QoS, total power limit as well as backhaul capacity constraint can be written as $\mathcal{P}1$:

$$\begin{aligned}
\mathcal{P}1: \quad & \max_{p_k, k\in\mathcal{K}} \frac{\sum\limits_{k\in\mathcal{K}} W\log_2(1 + g_k p_k)}{\zeta \sum\limits_{k\in\mathcal{K}} p_k + P_c} && (4a)\\
\text{s.t.} \quad & W\log_2(1 + g_k p_k) \geqslant r_{k,\min}, \forall k \in \mathcal{K}, && (4b)\\
& \zeta \sum_{k\in\mathcal{K}} p_k + P_c \leqslant P_{Tot}, && (4c)\\
& \sum_{k\in\mathcal{K}} W\log_2(1 + g_k p_k) \leqslant C_{backhaul}, && (4d)\\
& 0 \leqslant p_k \leqslant p_{peak}, \forall k \in \mathcal{K}, && (4e)
\end{aligned}$$

where $r_{k,\min}$ is the traffic-related minimum rate requirement of the kth UE; P_{Tot} and $C_{backhaul}$ denote the maximal total power consumption and backhaul capacity, respectively. The allowed peak power at each subcarrier, p_{peak}, is also considered in this paper.

For simplicity, we assume that $r_{k,\min}, \forall k \in \mathcal{K}$ is achievable under the constraint (4c) and (4d), i.e.,

$$\zeta \sum_{k\in\mathcal{K}} p_{k,\min} + P_c \leqslant P_{Tot} \quad (5)$$

$$\text{and} \sum_{k\in\mathcal{K}} r_{k.\min} \leqslant C_{backhaul} \quad (6)$$

should be satisfied simultaneously, where $p_{k,\min} = \frac{2^{\frac{r_{k,\min}}{W}} - 1}{g_k}$ is the minimum required transmit power to meet the kth UE's QoS. If not, optimization problem $\mathcal{P}1$ is unfeasible.

B. Channel Model

An existing empirical propagation channel model [7] between the AeNB and UEs is adopted in this paper. The large-scale fading taking path loss and shadow fading into account is given as

$$L = \begin{cases} L_{FSL} + \xi_{LOS}, & LOS \\ L_{FSL} + L_s + \xi_{NLOS}, & NLOS, \end{cases} \quad (7)$$

where L_{FSL} is the free space loss in dB as follows:

$$L_{FSL} = 20\log(d_{km}) + 20\log(f_{GHz}) + 92.4, \quad (8)$$

where d_{km} is the propagation distance between transmitter and receiver. f_{GHz} denotes the carrier frequency in GHz. An elevation angel dependent shadowing L_s is a normal distributed random variable. The location variability components ξ_{LOS} and ξ_{NLOS} both follow Log-normal distribution with zero mean.

With respect to the small-scale fading, Rayleigh distribution is added to NLOS link and Rician distribution LOS link.

III. Energy Efficiency Power Allocation

$\mathcal{P}1$ is obviously a non-convex problem since the objective function (4a) and the backhaul capacity constraint (4d) is non-convex [8]. The objective of a fractional programming, as we observed in $\mathcal{P}1$, takes the form of a ratio of two functions which is very challenging to solve directly. According to Isheden et al. [9], a non-negative parametric can be introduced to formulate a parametric optimization $\mathcal{P}2$ which is closely related with $\mathcal{P}1$.

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$$\mathcal{P}2: \max_{p_k \in \mathcal{S}} \sum_{k \in \mathcal{K}} W \log_2(1 + g_k p_k) - \lambda \left(\zeta \sum_{k \in \mathcal{K}} p_k + P_c \right) \tag{9a}$$

$$\text{s.t.} \quad (4c)(4d), \tag{9b}$$

where $\mathcal{S} = \{p_k, \forall k \in \mathcal{K} \mid p_{k,\min} \leq p_k \leq p_{peak}\}$ is the set of individual powers and each has a box constraint. Note that (9a) is convex for a given λ since its formulated as the difference between a convex function and a concave function (linear function for more strictly). But, $\mathcal{P}2$ is still a non-convex optimization problem due to the constraint (4d).

In order to solve problem $\mathcal{P}2$ effectively, we first leave out the constraint (4c) and (4d) and consider the optimal system EE only under the individual power set, which is formulated as

$$\mathcal{P}3: \max_{p_k \in \mathcal{S}} \sum_{k \in \mathcal{K}} W \log_2(1 + g_k p_k) - \lambda \left(\zeta \sum_{k \in \mathcal{K}} p_k + P_c \right). \tag{10}$$

For convenience, we define the optimal value of $\mathcal{P}3$ as a function of λ, denoting as $F(\lambda)$.

The optimal power allocation is achieved at the stationary point for a given λ since (10) is convex, i.e.,

$$\left. \frac{d \sum\limits_{k \in \mathcal{K}} W \log_2(1+g_k p_k) - \lambda \left(\zeta \sum\limits_{k \in \mathcal{K}} p_k + P_c \right)}{d p_k} \right|_{p_k = p_k^*} = 0, \quad \forall k \in \mathcal{K}. \tag{11}$$

Taking the box constraints into account, the optimal power allocation is given as

$$p_k^*(\lambda) = \left[\frac{W \log_2 e}{\lambda \zeta} - \frac{1}{g_k} \right]_{p_{k,\min}}^{p_{peak}}, \forall k \in K. \tag{12}$$

It is obvious that the maximum system energy efficiency without constraints (4c) and (4d) can be achieved by finding the optimal λ^* of $\mathcal{P}3$. According to Isheden et al. [9], the optimal power allocation to obtain the maximum energy efficiency only under the individual power set $\mathcal{S}$ is the same as that of $\mathcal{P}3$ for $\lambda = \lambda^*$, where λ^* satisfies $F(\lambda^*) = \max\limits_{p_k \in \mathcal{S}} \sum\limits_{k \in \mathcal{K}} W \log_2(1 + g_k p_k) - \lambda^* \left(\zeta \sum\limits_{k \in \mathcal{K}} p_k + P_c \right) = 0$. In addition, λ^* is the optimal bit-per-joule, i.e.,

$$\lambda^* = \frac{\sum\limits_{k \in \mathcal{K}} W \log_2(1 + g_k p_k^*)}{\zeta \sum\limits_{k \in \mathcal{K}} p_k^* + P_c}. \tag{13}$$

Since the power allocation has been expressed in (12) for a given λ, we need to determine λ^* with $F(\lambda^*) = 0$. The Dinkelbachs method [9], as described in Algorithm 1, is adopted in this paper to find it.

Until now, the problem $\mathcal{P}3$ without constraints (4c) and (4d) has been solved. In order to solve problem $\mathcal{P}2$, we explore some properties of $R(P_{tot})$, which are described in following Lemma 1. Define $R(P_{tot})$ as the maximum overall system throughput under the given total power consumption P_{tot} and individual power constraint, which is given as

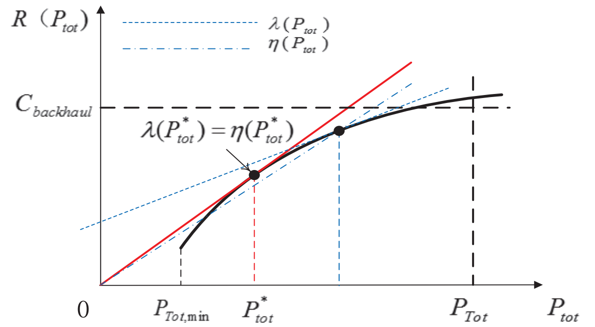

Figure 2. Simple Intuitive Illustration of $R(P_{tot})$

$$R(P_{tot}) \triangleq \max_{p_k \in \mathcal{S}, P_c + \zeta \sum\limits_{k \in \mathcal{K}} p_k \leqslant P_{tot}} \sum_{k \in \mathcal{K}} W \log_2(1+g_k p_k). \tag{14}$$

Due to the individual power constraint in $\mathcal{S}$, $P_{tot} \in [P_{Tot,\min}, P_{Tot,\max}]$, in which $P_{Tot,\min} = \zeta \sum\limits_{k \in \mathcal{K}} p_{k,\min} + P_c$ and $P_{Tot,\max} = \zeta K p_{peak} + P_c$. Since the right hand side of (14) is a convex optimization problem with fixed P_{tot}, $R(P_{tot})$ has a unique value for all allowed P_{tot}. According to the constraint $P_c + \zeta \sum\limits_{k \in \mathcal{K}} p_k \leqslant P_{tot}$, it is obvious that $R(P_{tot})$ is an increasing function in P_{tot}. The curve of $R(P_{tot})$ is plotted in Figure 2.

Lemma 1: $R(P_{tot})$ is continuously differentiable in P_{tot} and $R'(P_{tot}) = \lambda(P_{tot})$ is decreasing in P_{tot}, where $P_{tot} \in (P_{Tot,\min}, P_{Tot,\max})$.

Proof: please refer to Appendix for a proof of Lemma 1.

As illustrated in Figure 2, the slope of the origin-to-$(P_{tot}, R(P_{tot}))$ is $\eta(P_{tot}) = \frac{R(P_{tot})}{P_{tot}}$, which represents the maximum system EE at P_{tot}. By Lemma 1, we have that the tangent at $(P_{tot}, R(P_{tot}))$ is $\lambda(P_{tot})$. It is obvious to see that the slope $\eta(P_{tot})$ first increases and then decreases with growing of P_{tot} as well as the optimal EE is achieved when $\eta(P_{tot}^*) = \lambda(P_{tot}^*)$. When taking the constraints (4c) and (4d) into consideration, it is straightforward that (4c) and (4d) correspond to an upper lower bound on λ, say $\lambda_{P_{T,\min}}$ and $\lambda_{C_{b,\min}}$, respectively. This is because that $\lambda(P_{tot})$ is decreasing in P_{tot} which is described in Lemma 1. Obviously, the system lower bound $\lambda_{\min} = \max(\lambda_{P_T,\min}, \lambda_{C_b,\min})$ and $\lambda_{\max}$ is determined by $P_{Tot,\min}$. Therefore, if the optimal λ^* of $\mathcal{P}3$ falls into the interval $[\lambda_{\min}, \lambda_{\max}]$, it is also the optimal λ^* of $\mathcal{P}2$. If not, $\lambda^* < \lambda_{\min}$ must be satisfied and optimal λ^* of $\mathcal{P}2$ is replaced by $\lambda_{\min}$. $\lambda^* > \lambda_{\max}$ would not occur due to the lower bound of total power consumption.

According to the above analysis, we design an energy efficiency power allocation algorithm in the following Algorithm 1, which is based on Dinkelbachs method.

IV. Numerical Results

In this section, simulation results and discussions are presented to evaluate the effectiveness of our proposed energy

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Algorithm 1 Energy Efficiency Power Allocation Algorithm

Initialize λ satisfying $F(\lambda) \geqslant 0$ and tolerance ε;
while $(|F(\lambda)| > \varepsilon)$ **do**
  Determine $p_k^*(\lambda)$ in (12) and $F(\lambda)$ in (10);
  $\lambda \leftarrow \frac{\sum_{k\in\mathcal{K}} W\log_2(1+g_k p_k^*(\lambda))}{\zeta\sum_{k\in\mathcal{K}} p_k^*(\lambda)+P_c}$;
end while
Calculate $\lambda_{\min} = \max(\lambda_{P_T,\min}, \lambda_{C_b,\min})$;
if $\lambda \geqslant \lambda_{\min}$ **then**
  $\lambda^* = \lambda$;
else
  $\lambda^* = \lambda_{\min}$;
end if
return $p_k^*(\lambda^*)$;

Figure 3. Energy Efficiency Power Allocation Algorithm

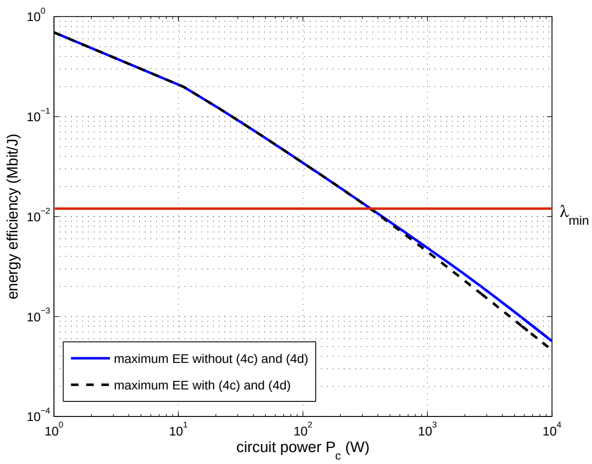

Figure 4. Energy Efficiency versus the circuit power for $\mathcal{P}3$ and $\mathcal{P}2$

efficiency power allocation algorithm. We use Matlab for the simulation. In our simulation, the total bandwidth, 0.5MHz, is equally divided into 20 orthogonal subcarriers and assigned to 20 users, as well as the carrier frequency is selected to 2GHz. The requirement of each user is 100kbit and the peak power at each subcarrier is set to 10W. For simplicity, we set the drain efficiency of power amplifier as 1. The AeNB is assumed to $500m$ high, all users are uniformly distributed in a circle around the AeNB and the radius of which is $3km$. The practical channel factor has been described in (7)(8). According to Holis et al. [7], we choose the Dense Urban environment for our simulation and the probability of LOS is then determined. The power spectrum of the noise equals to -110dBm/Hz.

Figure 4 depicts the impact of static circuit power on the energy efficiency for the problem of $\mathcal{P}3$ and $\mathcal{P}2$, i.e., without and with the consideration of total power and backhaul capacity constraints. The maximum total power and the backhaul capacity are set to 40W and 5Mbit, respectively. Note that the energy efficiency is decreasing with the increase of static circuit power. λ_{min} here is co-determined by the total power and backhaul capacity. When the circuit power is small,

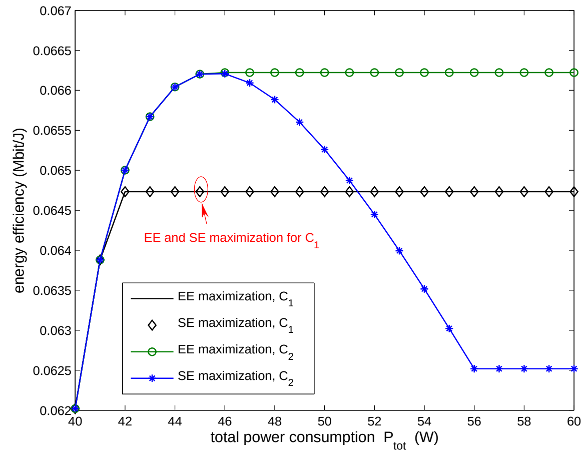

Figure 5. Energy Efficiency versus the total power consumption for different backhaul capacity

these two constraints has no effect on the energy efficiency and this case corresponds to the optimal λ^* of $\mathcal{P}3$ falls into $[\lambda_{\min}, \lambda_{\max}]$. However, when circuit power goes large, the energy efficiency is limited by these two constraints since the optimal λ^* of $\mathcal{P}3$ is less than λ_{min}.

Figure 5 illustrates the energy efficiency versus the total power consumption for different backhaul capacity. The static power is fixed to 35W, so the increase of total power is caused by transmit power only. The backhaul capacity C_1 and C_2 are set to 2.7Mbit and 3.5Mbit, respectively. In Figure 5, EE maximization is our proposed algorithm and SE maximization is achieved by maximizing the system throughput under the same constraints. It can be observed that when the total power is large enough, the energy efficiency of each algorithm approaches a constant value. However, for EE maximization and $C_{backaul} = C_2$ case, this is because that the maximum energy efficiency is obtained and thus resource allocator is not willing to consume more power. For other three cases, the constant value is caused by limited backhaul capacity.

V. Conclusion and Future Work

In this paper, we consider the downlink energy efficiency power allocation for a single OFDM-based aerial system with limited satellite backhaul. Due to the non-convex backhaul capacity limit, the problem is solved in two steps by exploring the properties of fractional programming and the derivative of $R(P_{tot})$. Then, an energy efficiency power allocation algorithm is proposed. Finally, theoretical analysis is corroborated by numerical experiments. The cooperation between multiple aerial systems can be considered in our future work.

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Appendix
Proof of Lemma 1

In Xiong et al. [5], $R(P_{tot})$ with the individual power constraint $p_k \geq 0$ is proved to be differentiable. In this

$$
\begin{aligned}
R'_{+}(P_{tot}) &= \lim_{\Delta P_{tot}^{+} \to 0} \frac{R(P_{tot}+\Delta P_{tot}) - R(P_{tot})}{\Delta P_{tot}} \\
&= \lim_{\Delta P_{tot}^{+} \to 0} \frac{\left(\max\limits_{\Delta p_k \geqslant 0} \sum\limits_{k\in K} W\log_2(1+g_k(\hat{p}_k+\Delta p_k)) \right) - \sum\limits_{k\in K} W\log_2(1+g_k\hat{p}_k)}{\Delta P_{tot}} \\
&= \lim_{\Delta P_{tot}^{+} \to 0} \frac{\max\limits_{\Delta p_k \geqslant 0} \sum\limits_{k\in \mathcal{K}} W\log_2 e \ln\left(1+\frac{g_k \Delta p_k}{1+g_k\hat{p}_k}\right)}{\Delta P_{tot}} \approx \lim_{\Delta P_{tot}^{+} \to 0} \frac{\max\limits_{\Delta p_k \geqslant 0} \sum\limits_{k\in \mathcal{K}} \frac{W g_k \log_2 e}{1+g_k\hat{p}_k}\Delta p_k}{\Delta P_{tot}} \\
&= \lim_{\Delta P_{tot}^{+} \to 0} \frac{\max\limits_{\Delta p_k > 0} \sum\limits_{k\in \mathcal{K}-\mathcal{K}'} \frac{W g_k \log_2 e}{1+g_k\hat{p}_k}\Delta p_k}{\Delta P_{tot}} \overset{(a)}{=} \max_{k\in\mathcal{K}-\mathcal{K}'} \frac{W g_k \log_2 e}{\zeta(1+g_k\hat{p}_k)}. \qquad (15)
\end{aligned}
$$

$$
\begin{aligned}
R'_{-}(P_{tot}) &= \lim_{\Delta P_{tot}^{-} \to 0} \frac{R(P_{tot}+\Delta P_{tot}) - R(P_{tot})}{\Delta P_{tot}} \approx \lim_{\Delta P_{tot}^{-} \to 0} \frac{\max\limits_{\Delta p_k < 0} \sum\limits_{k\in \mathcal{K}-\mathcal{K}'} \frac{W g_k \log_2 e}{1+g_k\hat{p}_k}\Delta p_k}{\Delta P_{tot}} \\
&= \min_{k\in\mathcal{K}-\mathcal{K}'} \frac{W g_k \log_2 e}{\zeta(1+g_k\hat{p}_k)}. \qquad (16)
\end{aligned}
$$

paper, we further prove that $R(P_{tot})$ is differentiable and the derivative of $R(P_{tot})$ is continuous and decreasing. We consider the limit under the constraint $\sum\limits_{k\in\mathcal{K}} \Delta p_k = \Delta P_{tot}$ and let $\hat{p}_k$ denote the optimal power allocation at the kth subcarrier under the total power consumption P_{tot}.

According to the definition of derivative, we derive that (15) is satisfied, in which $\mathcal{K}' = \{k|\Delta p_k = 0, k \in \mathcal{K}\}$. We have $\Delta p_k = 0$ if either of the following two cases occurs, i.e., (a) $\hat{p}_k = p_{\min}$ and $\exists k' \in \mathcal{K}$, $\hat{p}_{k'} > p_{k',\min}$ and the water level at the k'th subcarrier is lower than that of kth subcarrier; (b) $\hat{p}_k = p_{peak}$. $\overset{(a)}{=}$ is obtained since $\max\limits_{\Delta p_k > 0} \sum\limits_{k\in\mathcal{K}-\mathcal{K}'} \frac{W g_k \log_2 e}{1+g_k\hat{p}_k}\Delta p_k = \left(\max\limits_{k\in\mathcal{K}-\mathcal{K}'} \frac{W g_k \log_2 e}{\zeta(1+g_k\hat{p}_k)}\right)\left(\sum\limits_{k\in\mathcal{K}-\mathcal{K}'} \Delta p_k\right)$.

If $\mathcal{K}' = \phi$, $\max\limits_{k\in\mathcal{K}} \frac{W g_k \log_2 e}{\zeta(1+g_k\hat{p}_k)}$ is equivalent to $\min\limits_{k\in\mathcal{K}} \frac{1}{g_k} + \hat{p}_k$. According to $\hat{p}_k = \frac{W\log_2 e}{\lambda_+(P_{tot})\zeta} - \frac{1}{g_k}$, we know that $\min\limits_{k\in\mathcal{K}} \frac{1}{g_k} + \hat{p}_k = \frac{W\log_2 e}{\lambda_+(P_{tot})\zeta}$, where $\frac{W\log_2 e}{\lambda_+(P_{tot})\zeta}$ is the water level under P_{tot}. Hence, we have $R'_+(P_{tot}) = \lambda_+(P_{tot})$. If case (a) or (b) occurs, $\mathcal{K}' \neq \phi$. In this case, $\min\limits_{k\in\mathcal{K}-\mathcal{K}'} \frac{1}{g_k} + \hat{p}_k = \frac{W\log_2 e}{\lambda_+(P_{tot})\zeta}$ is also satisfied. However, $\frac{W\log_2 e}{\lambda_+(P_{tot})\zeta}$ here denotes the water level without regard to the kth ($k \in \mathcal{K}'$) subcarrier. As a whole, $\frac{W\log_2 e}{\lambda_+(P_{tot})\zeta}$ denotes the water level at the kth ($k \in \mathcal{K} - \mathcal{K}'$) subcarrier which would increase if the value of P_{tot} grows ΔP_{tot}.

Similarly, (16) is derived. In this case, we have $\Delta p_k = 0$ if either of the following two cases occurs, i.e., (c) $\hat{p}_k = p_{peak}$ and $\exists k' \in \mathcal{K}$, $\hat{p}_{k'} < p_{peak}$ and the water level at the k'th subcarrier is higher than that of kth subcarrier; (d) $\hat{p}_k = p_{k,min}$. Analogously, $R'_-(P_{tot}) = \lambda_-(P_{tot})$ is satisfied and $\frac{W\log_2 e}{\lambda_-(P_{tot})\zeta}$ here represents the water level at the kth ($k \in \mathcal{K}'$) subcarrier which would decrease if the value of P_{tot} reduces ΔP_{tot}.

Obviously, water level $\frac{W\log_2 e}{\lambda_+(P_{tot})\zeta} = \frac{W\log_2 e}{\lambda_-(P_{tot})\zeta}$ holds for any given $P_{tot} \in (P_{Tot,\min}, P_{Tot,\max})$. Then, we have $R'_+(P_{tot}) = R'_-(P_{tot}) = \lambda(P_{tot})$. The existence of the limit indicates that $R(P_{tot})$ is differentiable in P_{tot}. Clearly, the growing of water level is continuous as the value of P_{tot} gets larger. Therefore, $\lambda(P_{tot})$ is continuously decreasing since its inversely proportional with water level. This completes the proof of lemma 1.

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