

# The Effect of Radar Cross Section and Speed of Target on the Detection of MIMO Radar

Raed S. M. Daraghma

Department of Electrical and Electronics Engineering  
Palestine Technical University, Palestine  
Email: R.daraghmeh@ptuk.edu.ps

**Abstract**— In this paper, we address the problem of Direction of Departure (DOD) and Direction of Arrival (DOA) estimation for Multiple Input Multiple Output (MIMO) radar. The work presented studies the effect of Radar Cross Section (RCS), Signal to Noise Ratio (SNR) and speed of targets on the performance of the MIMO radar. Analysis can be used to find the direction of multiple types of targets such as CAPON, MUSIC and parallel factor (PARAFAC). To differentiate the meaning of targets, varying targets of different types, such as bicycle, bird, man, ship and jet have been considered. After defining suitable values for each type of target in 2D space, the performance of each type is discussed by using the MATLAB program.

**Keywords**- MIMO Radar; Target Localization; Parallel Factor (PARAFAC); Direction of Arrival.

## I. INTRODUCTION

Some of the most important RADAR applications are the detection performance and high resolution of the moving target localization. Radar Cross Section (RCS), range, location and velocity are utility parameters of the moving target [1][2]. To improve the accuracy of target detection and estimation, antenna arrays have been used. MIMO radar uses multiple transmitter and multiple receiver elements. Generally, unlike the phased-array systems, MIMO radar has several advantages compared to the conventional phased array systems: higher resolution, more degrees of freedom, improved parameter specification, better spatial coverage and detection diversity gain. MIMO radars can be classified into two categories: (1) MIMO radar with widely separated antennas scheme and (2) MIMO radar using collocated antennas, which is similar to phase array radar [3][4]. In the literature, there are many configurations of MIMO radar according to the location of the transmitting and receiving elements. Widely separated antennas represent one of these configurations. In this scheme, the separation between transmitter and receiver should be large enough to receive the uncorrelated echoes from the different targets. The main advantage of this scheme is that the spatial diversity of the targets RCS enhances the radar performance [5]. In this paper, a bistatic MIMO radar technique with transmission spatial diversity is proposed, and the estimation performance is analyzed. Moreover, the angles with respect to receiver can be determined using the proposed technique. In addition, the maximum number of targets that can be identified with this technique is discussed in this paper. MIMO radar can deal with multiple targets. Linearly

independent waveforms are transmitted at the same time via multiple antennas. These independent waveforms are linearly combined at the targets with different phases, after which the signal waveforms reflected from different targets are linearly independent of each other, which allow for the application of CAPON, MUSIC and PARAFAC algorithms [6]. In this work, we focus on the application of MIMO radar to the estimation of DOA and the DOD of multiple targets exist in the same range bin for bistatic MIMO radar system. We are particularly interested to optimize the average angular error for different types of targets.

The paper is organized as follows. In Section 2, the previous work on the subject is summarized. The MIMO radar signal model is presented in Sections 3 and 4. The performance of MIMO radar is evaluated through simulations via MATLAB in Section 5. Some concluding remarks are given in Section 6.

## II. RELATED WORK

There are many existing methods to localize the moving target. The following parameters: the Angle of Arrival (AOA), the Angle of Departure (AOD), the speed of the target and the RCS are the most used to localize a moving target. Many techniques have been proposed such as CAPON technique [6], the MUSIC (Multiple Signal Classification) technique [7], and parallel factor analysis (PARAFAC) [8].

In this paper, we select a number of different moving targets: simple and complex targets with different RCS and speeds. From the existing work on the application of CAPON, MUSIC and PARAFAC to the localization of different targets, we notice the importance of the types of targets and the effect of changing the speed of targets. This paper focuses on comparing the performance criterion for different types of targets as well as the impact of the number of antennas on the performance of three different techniques mentioned above.

## III. MIMO RADAR SIGNAL MODEL

In this section, we consider that the Coherent Processing Interval (CPI) consists of  $Q$  consecutive pulse periods. The Swelling II target model [8] was assumed, where RCS coefficient is varying from pulse to pulse. The targets are located in the far-field. The RCS coefficients are assumed to vary independently from pulse to pulse, and the propagation medium is non dispersive.

The baseband received signal at the output of the receive array after synchronization can be written as:

$$X_q = B(\phi) \sum_q A^T(\Theta) S + W_q, \text{ where } q = 1, \dots, Q \quad (1)$$

where  $X_q \in \mathbb{C}^{M_r \times L}$  collects the  $L$  samples received by  $M_r$  antennas for the  $q^{\text{th}}$  pulse period.

$\sum_q = \text{diag}(c_q)$ , with  $c_q = [\delta_{1q}, \dots, \delta_{kq}]$  where  $\delta_{kq} = \alpha_{kq} e^{j(q-1)x_q}$ , i.e.,  $x_q$  is the Doppler frequency of the  $k^{\text{th}}$  target [9]. The RCS coefficients  $\alpha_{kq}$ ,  $k = 1, \dots, K$ , are varying independently from pulse to pulse, and  $W_q \in \mathbb{C}^{M_r \times L}$  is the noise interference term. MIMO radar transmits mutually orthogonal waveforms. We assume that  $1/LS^H = I_M$ . After right multiplication of (1) by  $\frac{1}{L}S^H$ , the matched filter output is:

$$Y_q = B(\phi) \sum_q A^T(\Theta) + Z_q, \text{ where } q = 1, \dots, Q \quad (2)$$

Where

$$Y_q = 1/LXS^H \in \mathbb{C}^{M_r \times M_t} \text{ and } Z_q = 1/LWS^H \quad (3)$$

Let us factorize (3):

$$Y_q = (A(\phi) \odot B(\Theta)) C_q^T + Z_q \text{ where, } q = 1, \dots, Q \quad (4)$$

where

$$Y_q = \text{vec}(Y_q), Z_q = \text{vec}(Z_q) \quad (5)$$

which can be written in the compact form:

$$Y = (A(\phi) \odot B(\Theta)) C^T + Z \quad (6)$$

where  $Y = [Y_1, \dots, Y_Q]$  and  $Z = [Z_1, \dots, Z_Q]$  are of size  $M_t M_r \times Q$  and  $C^T = [C_1^T, \dots, C_Q^T]$  is of size  $K \times Q$ .

From [10] the CAPON estimator can be written as:

$$P(\phi, \theta) = \frac{1}{(a(\phi) \odot b(\theta))^H R_{YY}^{-1} (a(\phi) \odot b(\theta))} \quad (7)$$

$$R_{YY} = \frac{1}{Q} YY^H$$

The MUSIC estimator can be written as:

$$P_{MUSIC}(\phi, \theta) = \frac{1}{a(\phi) \odot b(\theta)^H E_Y E_Y^H a(\phi) \odot b(\theta)} \quad (8)$$

where  $E_Y$  is the  $M_t M_r \times (M_t M_r - K)$

matrix contains the noise eigenvectors of  $R_{YY}$ .

The third estimator PARAFAC was derived in [11].

PARAFAC implies the transmit and receive angles relative to the same target are automatically paired.

#### IV. DATA MODEL

In this section, we consider the multiple pulses, multiple arrays case. The MIMO radar system has the following parameters :

- $M_t$  transmit array.
- $M_r$  receive array.
- $K$  targets in a far field.
- $Q$  transmitted pulses, and the RCS is varying independently from pulse to pulse (Swelling II case)
- $\delta_{kq}$  is the reflection coefficient of the  $K^{\text{th}}$  target during the  $q^{\text{th}}$  pulse.
- $\{\Theta\}_{k=1}^K, \{\Phi\}_{k=1}^K$  are the DODs and DOAs with respect to transmit and receive array, respectively.  
 $A(\Theta) = [a(\Theta_1), \dots, a(\Theta_K)]$  is the transmit steering vector relative to  $K$  targets,  $B(\phi) = [b(\phi_1), \dots, b(\phi_K)]$  is receiving steering vector relative to  $K$  targets.

#### V. SIMULATION RESULTS

In this section, MATLAB program simulation results are presented to verify the above analysis and compare the performance of the three techniques (Capon, MUSIC and PARAFAC). Localization of the multiple targets for a Uniform Linear Array (ULA) configuration at the transmitter and receiver can be achieved by the above algorithms [8].

We generate the matrices  $S$ ,  $A$  and  $B$  as explained in the previous section.  $S$  is generated by  $[S]_m = (1+j/2) [H_N]_m$ , where  $H_N$  is the  $N \times N$  Hadamard matrix, and  $N$  is fixed to 256.

The Signal To Noise Ratio (SNR) at the receiver is defined as:  $\text{SNR} = 10 \log \left( \frac{\sum_{q=1}^Q \|B \sum_q A^T S\|_F^2}{\|W\|_F^2} \right)$  dB, where Additive White Gaussian Noise (AWGN) is assumed, and  $\|\cdot\|_F$  is the Frobenis norm. We consider ULA transmit and receive arrays with  $\lambda/2$  interelement spacing for both arrays [8]. For the Swerling II target model, each column of the matrix  $C \in \mathbb{C}^{Q \times K}$  is generated from a complex Gaussian

distribution with zero mean and variance  $\sigma^2_{\delta k}$ , where  $\alpha_k$  is sample drawn from a complex Gaussian distribution with zero mean and variance  $\sigma^2_{\delta k}$  and the Doppler frequency  $x_k$  is generated by:

$$x_k = \frac{2\pi v_k T_p}{\lambda}$$

where  $v_k$  is the target velocity,  $T_p = 5 \times 10^{-6}$  is the pulse duration in seconds,  $\lambda = 3 \times 10^8 / f_c$ , with  $f_c = 1$  GHz.

This subsection analyzes the impact of the number of targets on the performance detection. The performance criterion is the absolute value of the difference between the true angle and estimated angle, averaged over transmit and receive angles and over all targets. In a first experiment (Figure 1), we consider seven types of targets. The variance and speed of each target was given in Table I. We simulate the presence of two to six targets, starting from  $K=2$  with DODs  $[10^0, 20^0]$  and DOAs  $[0^0, 30^0]$  until  $K=6$ , DODs  $[10^0, 20^0, 30^0, 40^0, 50^0, 60^0]$  and DOAs  $[0^0, 30^0, 5^0, 15^0, 25^0, 30^0]$ . The number of pulses is  $Q = 100$ , number of samples for each transmitted pulses is  $L = 512$ ,  $SNR = 10$  dB, and Swerling II model is chosen. We plotted the performance of the CAPON method, and we compared the performance of the different types of targets via Monte Carlo simulation. From Figure 1, it is clear that a better angular resolution is achieved when the target is "Man" and the worst angular resolution is achieved when the target is "Car". From Figure 1, we observe that the global performance of all types of targets seriously degrade when the number of targets is increased. In Figure 2, we simulate the presence of two to six targets. The other parameters are the same as in Figure 1, but, in this case we have plotted the performance of the MUSIC technique. We compare the performance of the different types of targets via Monte Carlo simulation. From Figure 2, it is clear that a better angular resolution is achieved when the target is "Boat" and the worst angular resolution is achieved when the target is "Fighter". From Figure 2, we observe that the global performance of all types of targets seriously degrades when the number of targets is increased.

In Figure 3, we simulate the presence of two to six targets. The other parameters are the same as in Figure 1, but, in this case, we have plotted the performance of the parallel factor (PARAFAC) technique, and we compared the performance of the different types of targets via Monte Carlo simulation. From Figure 3, it is clear that the best angular resolution is achieved when the target is "Car" and the worst angular resolution is achieved when the target is "Jet". From Figure 3, we observe that the global performance of all types of targets seriously degrades when the number of targets is increased.

This subsection analyzes the impact of signal to noise ratio on the performance detection. In a second experiment (see Figure 4), we simulate the presence of three targets ( $K=3$ ) characterized by DODs  $[10^0, 20^0, 30^0]$  and DOAs  $[-10^0, -20^0, -30^0]$ . The number of pulses  $Q = 100$ , the number of samples for each transmitted pulse  $L = 512$ , the number of transmit and

receive sub arrays is fixed to 5,  $SNR \in (0, 2, 4, 6, 8, 10)$  dB, and the Swerling II model is chosen. We plotted the performance of the CAPON method, and we compared the performance of the different types of targets via Monte Carlo simulation. From Figure 4, it is clear that the best angular resolution is achieved when the target is "Bird" and the worst angular resolution is achieved when the target is "Bicycle". As expected, from Figure 4, we observe that the performance of all types of targets improves when the signal to noise ratio increases. In Figure 5, we simulate the presence of three targets. The other parameters are the same as in Figure 4, but, in this case, we have plotted the performance of the MUSIC technique, and we compared the performance of the different types of targets via Monte Carlo simulation. From Figure 5, it is clear that a better angular resolution is achieved when the target is "Bird" and the worst angular resolution is achieved when the target is "Bicycle". In Figure 6, we simulate the presence of three targets. The other parameters are the same as in Figure 4, but, in this case, we have plotted the performance of the parallel factor (PARAFAC) technique and we compared the performance of the different types of targets via Monte Carlo simulation. From Figure 6, it is clear that the best angular resolution is achieved when the target is "Bird" and the worst angular resolution is achieved when the target is "Jet".

TABLE I. RCS AND SPEED FOR DIFFERENT TYPES OF TARGETS

Target Type	Radar cross section of target ( in $m^2$ )	Speed of target ( $m/s^2$ )
Bicycle	2	10
Man	1	6.5
Car	100	100
Fighter	40	125
Boat	0.02	20
Jumbo Jet	100	40
Bird	0.01	150

## VI. CONCLUSION

In this paper, we have considered the detection and localization of moving target in bistatic MIMO radar with widely separated antennas, where multiple antennas transmit linearly independent waveforms and multiple antenna receive the reflected signal. We can significantly improve the estimation accuracy of the bistatic MIMO radar techniques as well as enhance their performance. The main problems

encountered in MIMO radar detection are radar cross section and speed of the target. To illustrate the impact of these two parameters on the performance of MIMO radar, several types of targets and three popular techniques (CAPON, MUSIC and PARAFAC) were considered for comparison. From the simulation results, we have shown that irrespective of the radar cross section and speed of target a high performance (low angular error) can be obtained when the signal to noise ratio increases. On the contrary, low performance can be obtained when the number of targets increases.

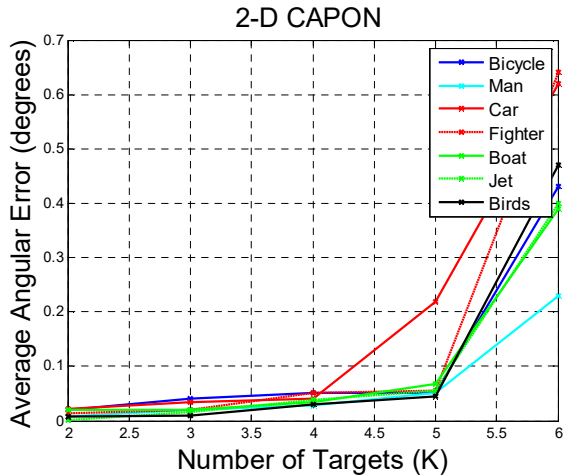


Figure 1. Average angular error with number of targets (2-D capon case)

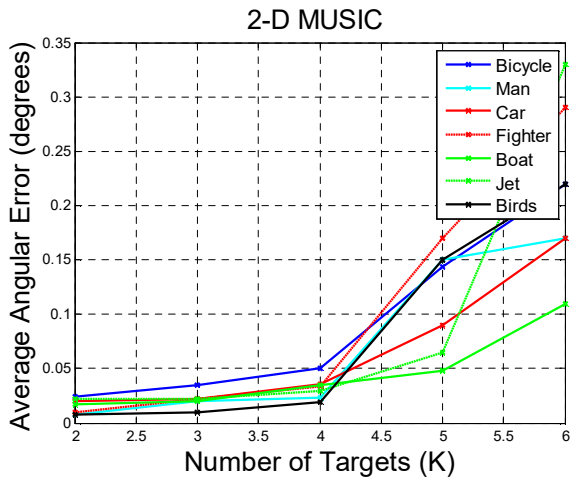


Figure 2. Average angular error with number of targets (2-D capon case)

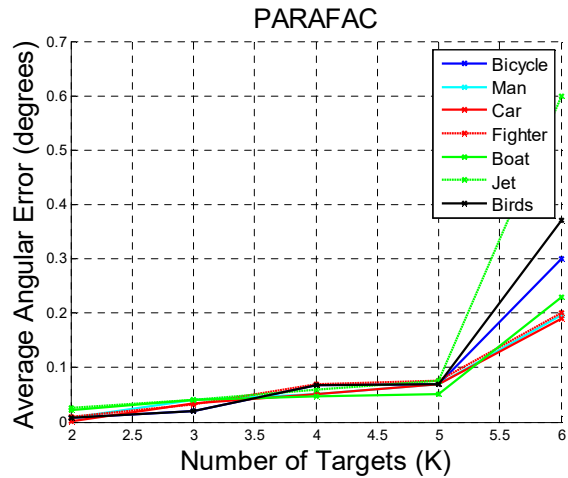


Figure 3. Average angular error with number of targets (PARAFAC case)

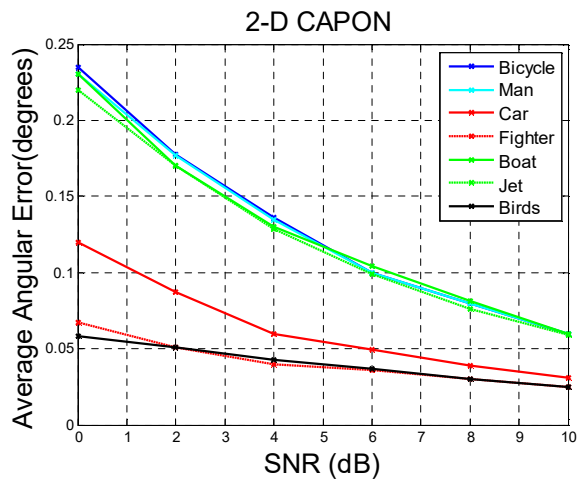


Figure 4. Average angular error with signal to noise ratio for each target (2-D Capon case)

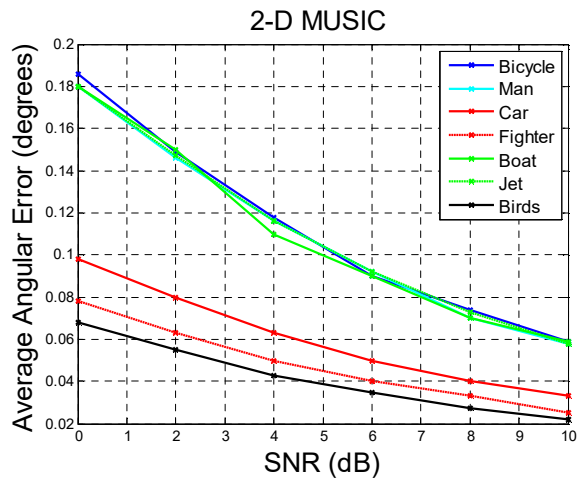


Figure 5. Average angular error with signal to noise ratio for each target (2-D MUSIC case)

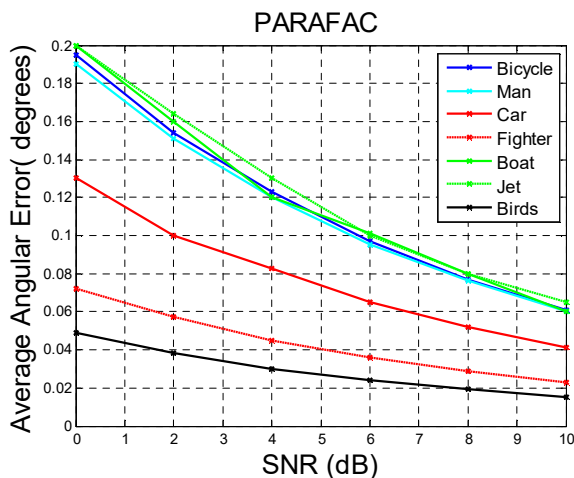


Figure 6. Average angular error with signal to noise ratio for each target (PARAFAC case)

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