Assessing the System Condition Based upon Limited Maintenance Data of the Taipei Metro System and Estimating its Remaining Lifetime

Tzu-Chia Kao and Snow H. Tseng
Department of Electrical Engineering,
National Taiwan University,
#1, Sec. 4, Roosevelt Rd, Taipei 10617, Taiwan
e-mails: b03901004@ntu.edu.tw, stseng@ntu.edu.tw

Abstract—**In this study, we analyze the maintenance data provided by Taipei Rapid Transit Corporation; the data consists of recent Taipei metro system maintenance record. However, it is unclear whether the lifetime information of the system is contained within the partial-lifespan maintenance record of a still-young metro system. Furthermore, the Taipei metro system is renowned for its performance and well maintenance, which further complicates the analysis. Based on limited maintenance records, the research objective is to explore the feasibility of extracting reliable information that is indicative of the current stage of life and the remaining lifetime of the metro system.**

Keywords-degradation; maintenance; metro; MRT; performance analysis; data analysis.

I. INTRODUCTION

In line of the recent reported research study [1], we further analyze the maintenance record of the Taipei Mass Rapid Transit (MRT) system, which is a well-maintained system renown worldwide for its tidiness, stability and efficiency. The Taipei metro system of Taipei Rapid Transit Corporation (TRTC) began operation on March 28, 1996; it has been operating for 24 years. Since the MRT TRTC is relatively young, most of the equipment has not yet been replaced; the records of repair and maintenance are detailed and are stored in digital form. The goal of this research is to determine whether it is possible to acquire status information of the system from the limited time-span maintenance records. Such information may be crucial for improving the performance of the Taipei MRT system. More generally, this research may shed light on enhancing performance of other metropolitan MRT systems.

Here we investigate the feasibility of extracting information from the Taipei MRT maintenance record and determine whether it is possible to acquire reliable information of the system performance from the limited time-span maintenance records. If the maintenance data indeed contains such information of the current system status, our goal is to assess the current stage of life of the Taipei MRT system and determine the remaining lifetime.

The performance and degradation of metropolitan metro systems have generally been the focus of the daily public. Various studies have been reported, including technical issues of the MRT. Rail track condition monitoring is an important technical concern of the MRT system [2]. However, constant monitoring of the MRT system is not available, typically the usual maintenance is performed once a month or less. Track condition has attracted much attention since it is a potential threat to the railway system. Studies to prevent such problem have been reported [3]-[5]. The general goal of the research and technical modifications is to improve the performance and reliability of a mass rapid transit system.

Comparing information of other metropolitan mass transport systems may be helpful. As reported in [6], train R36 of the New York metropolitan subway serviced from 1964 to 2003, a total of 39 years. R160s were used to replace 45-year-old trains. News reported that the oldest trains of the New York City Subway were planned to serve for 58 years, and later this type of train were actually found too old with very high failure rate and not appealing to the general public [7]. The subway train lifetime was estimated to be around 40 to 50 years. For example, since 1987, some lines of Singapore Mass Rapid Transit (SMRT) have been operating for 30 years. Thus, the actual wear-out period of a metro system, assuming they are similar, may roughly lie between 20 years (the oldest TRTC asset), and 40 years (New York City Subway). However, all of these metropolitan metro systems are different in various aspects, including: model, company, maintenance, management culture, etc. It is natural that the characteristics of these MRT systems are not the same or even far from similar. With multiple factors entangled, the problem to accurately assess the system performance and age may be very challenging.

Assessment and quantification of the system current status are essential to improving performance; to accurately assess the system current status is often nontrivial. The degradation curve is commonly employed for estimation of the system current status. Analysis of the reliability relies on the failure rate and maintenance records [8]. Based on the status of the system, possible improvement of the maintenance and performance can be assessed. To study the maintenance and performance characteristics, various approaches have been reported [9]-[17], including the popular bathtub curve analysis [18]-[23]. Typically the Bathtub-shaped curve is employed for system performance analysis [24]. Analysis based upon the bathtub curve has been extensively applied to various problems; modifications to improve applicability have been reported [11], [25], [26]. It is possible that the bathtub-shaped curve could be affected by human factors; for example, if the asset retired in its early stage, the curve may not rise up within the wear-out period and may even descend. If properly maintained, the curve may not rise in the wear-out period, similar to the situation in airline industry. However, few MRT systems in reality exhibit degradation behavior similar to the bathtub-shaped curve model [27]. It is possible such bathtub curve may not be the ideal model for analyzing the metro system performance.

The rest of this paper is organized as follows: Section II describes the goal of this research project. Section III describes the research method. Section IV summarizes the data analysis. A summary is presented in Section V, a conclusion in Section VI, and lastly, a description of the Future Work in Section VII, followed by an acknowledgement.

II. Research Goal

We propose a thorough investigation of the Taipei MRT data. Available data consists of 11 MRT systems: electric multiple unit (EMU) propulsion, EMU air conditioner, EMU communication, switcher, platform door, 22kV switchboard, automated fare collection (AFC) door, Wenhu Line traffic control computer, transmission system, elevator, and escalator. By analyzing the dataset with various approaches, such as deep learning, the research objective is to analyze the current status of the system, and identify possible tendencies or features that may be indicative of the system performance.

III. Methods

The bathtub-shaped curve model [24] is commonly employed to assess the system condition. It consists of a break-in trend as the system condition improves, followed by a plateau regime where the system condition is stable; after the stable regime, the system condition withers with increased malfunction rate, followed by a steep increase of malfunction rate as the system breaks down. Together, the bathtub-shaped curve represents the various stages of an ideal system.

The bathtub-shaped degradation curve is a theoretical model used in many problems. It is an idealized evolution trend of a system. The feasibility of applying such bathtub-shaped curve may depend on the application. Specifically, the system condition may not follow the same degradation curve, also, each equipment system may exhibit different characteristics depending on the specific application. Furthermore, each equipment in the Taipei metro system consists of various brands and various models that may possess different intrinsic characteristics. Together, all these factors intertwined complicates the situation and it is very challenging to dis-entangle the complex problem.

Ideally, analysis of the maintenance data would yield a simple bathtub-shaped degradation curve for each equipment. However, most systems do not follow the same degradation curve, not to mention each equipment may exhibit different characteristics. Since each equipment is maintained by human, the degradation curve is unlikely to be a simple universal bathtub-shaped curve. By analyzing the maintenance data, our goal is to decipher the feasibility to assess the MRT current stage of life based on available data ranging over a limited time-span.

IV. Data Analysis

We use the maintenance records provided by TRTC. The AFC gates statistics are shown in Figure 1; the average malfunction rate is calculated and shown in Figure 2. As shown in Figure 1, the equipment consists of mixed brand, model, age; thus, the total number of malfunctions is not representative of individual equipment.

Based on the data analysis, it is found that the failure rate has been declining each year. It is speculated that this declination is due to the TRTC's maintenance becoming better and better. On the other hand, because of the difference on the age and amount of the equipment, the total number of malfunctions is not a direct indication of the failure rate of a specific equipment.

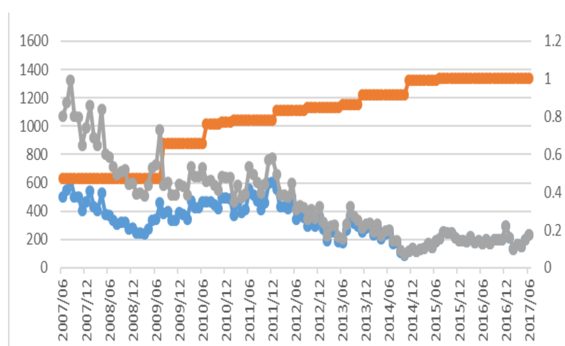

Figure 1. The malfunction rate of AFC gates vs. time. The failure rate decreases monotonically with time. (Blue line): the total number of incidences per month; (orange line): the total number of equipment (as the MRT system is expands, the total number of equipment increases; (gray line): the monthly malfunction rate.

With sufficient maintenance data of escalators and elevators, we try to quantify the failure rate *vs.* the age of each individual equipment. On the other hand, for some equipment that seem to have no difference among each other, e.g., EMU, platform door, switcher and 22kV switchboard, we use the original method of analyzing the failure times against failure date. As for the Wenhu line Central control computers, AFC gates and transmission system, we cannot decipher the replaced new ones from others, so we employ the latter method.

The following data of the number of escalators, elevators, equipment of individual station and the operation time of individual equipment are acquired from the Internet data and the records given by TRTC, and may differ from the actual number.

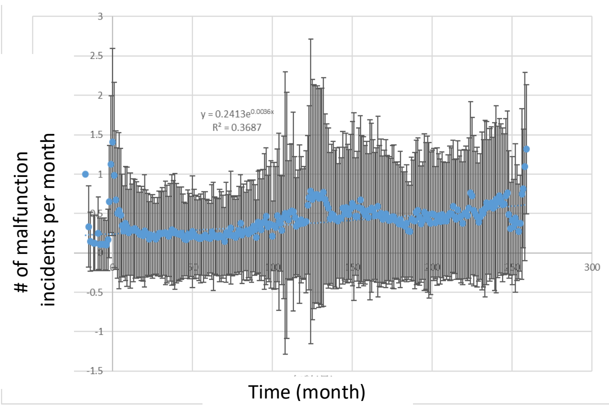

Figure 2. The malfunction rate as a function of time.

Notice that the malfunction rate of Figure 2 rises on both ends of the curve. The malfunction rate increases with time as the system becomes frail and old. Also, the data points on the left and right side of the curve each belongs to different groups of systems. It is speculated that the systems of the left side may have received adequate maintenance from the early years and therefore exhibits a lower malfunction rate; whereas the systems of the right side received adequate maintenance in its later years and therefore exhibits a higher malfunction rate.

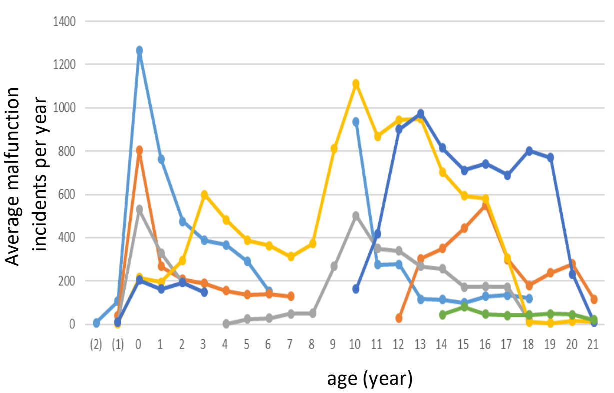

Figure 3. The average malfunction incidents each year for different subway lines.

We also tried categorizing the maintenance record according to different subway lines (Figure 3). Notice that the data span varies for each subway line, since some subway stations were built much earlier and some were built later. However, there are possible pitfalls with such categorization: each subway line has different number of stations; even along the same subway line, the age of each station also differs. For example, for the Songshan-Xintien Line, its stations roughly can be divided into two groups: stations that have been established for more than ten years, or less than ten years. If each sub-group has a characteristic peak, then the overall malfunction rate of Songshan-Xintien Line may exhibit two peaks due to each sub-groups. Analyzing the same escalator all employed the same time should be more appropriate.

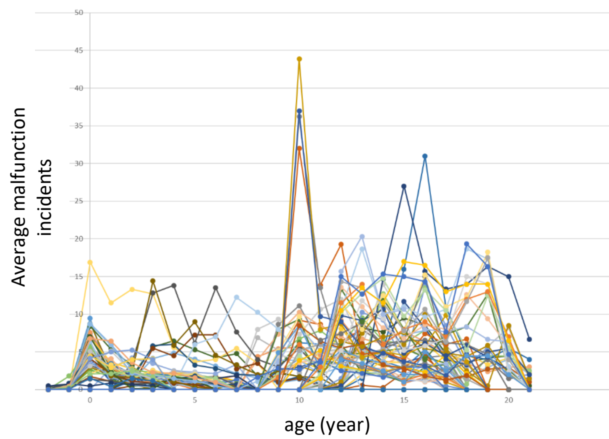

Figure 4. The average malfunction incidents per month as a function of station age. Each curve is acquired by: the total number of escalator malfunction incidents divided by the total number of escalators in each station.

As shown in Figure 4, the malfunction incidents as a function of station age is sporadic. It is infeasible to add up all these curves of different stations, because some stations have more escalators and the weighting differs. Nevertheless, though sporadic, the average malfunction incident rate is clearly lower for younger stations.

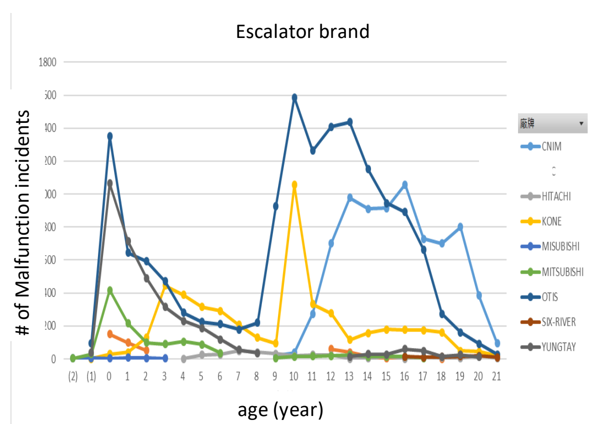

Figure 5. The average malfunction incidents per escalator vs. age (year) for different brands. The trend varies for different brands.

In Figure 5, the trend varies with different brands. In addition, the number of escalators of each brand is not the same and therefore the comparison may not be conclusive.

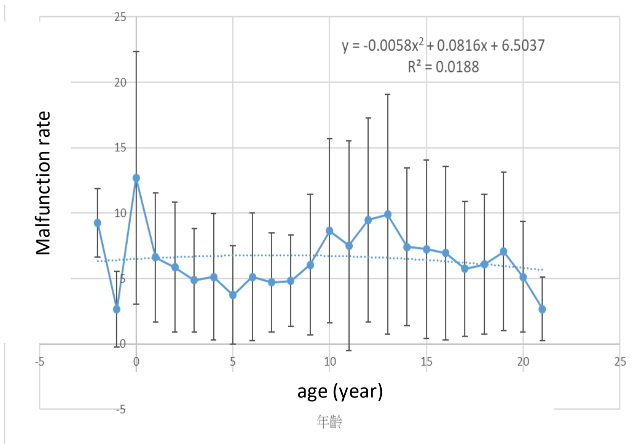

Figure 6. The elevator average malfunction rate vs. age (year).

Figure 6 is obtained by averaging the malfunction rate of each elevator. There is some increase followed by decrease in the middle. Overall, there is no clear trend of increase or decrease.

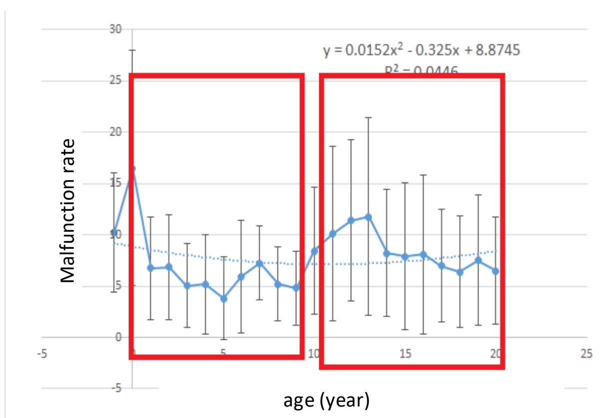

Figure 7. The elevator average malfunction rate vs. age (year).

The elevator average malfunction rate vs. age (year) is shown in Figure 7. The data was reorganized; the trend as shown can be separated into two groups (the two red boxes as indicated in Figure 7). It seems that the younger group of elevators (left red box) and older group (right red box) of elevators both exhibit a decrease in the malfunction rate with time. It is suggested that the maintenance technique may have improved or maintenance has been employed more frequently, so that the malfunction rate degrease with time, regardless of the specific age of the elevator.

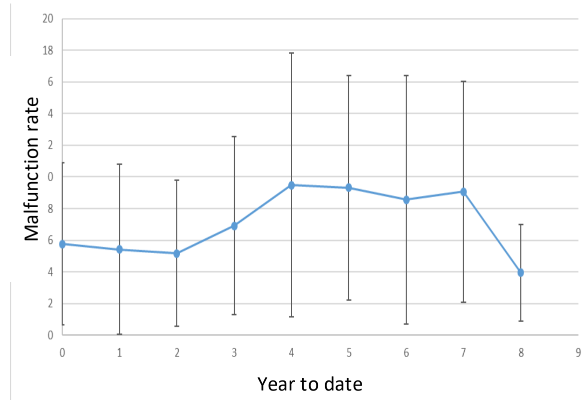

Figure 8. The average elevator malfunction rate vs. number of years to date.

Figure 8 is the malfunction rate plotted against the number of years to date. The overall variation does not show a monotonic trend. It is suggested that the malfunction rate of the elevator is not directly dependent upon the elevator age.

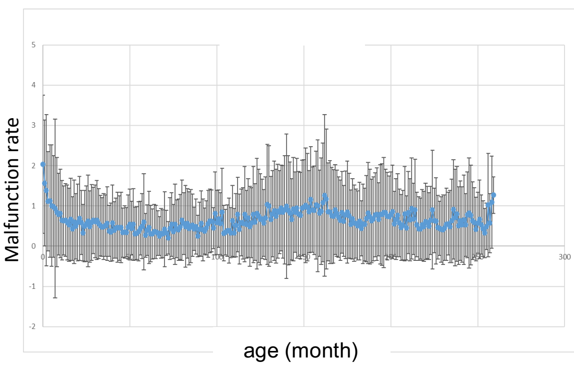

Figure 9. The escalator average malfunction rate vs. age (year).

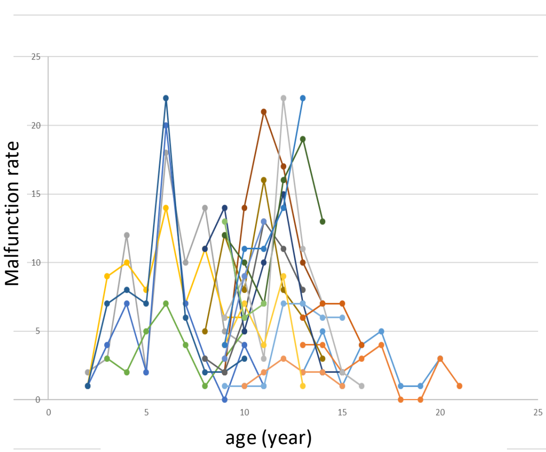

Figure 10. The malfunction rate as a function of age for 20 elevators. The variation is large and sporadic.

The elevator average malfunction incidents vs. age (year) is depicted in Figure 9; the trend is similar to Figure 8. The variation between each month is not pronounced; in addition, increase, decrease, occurs sporadically. Overall, the characteristics are not apparent.

Figure 10 is the malfunction rate as a function of age for 20 elevators. The plotted data exhibits large variations with sporadic behavior. It is difficult to come up with an average trend to represent the general behavior of these elevators.

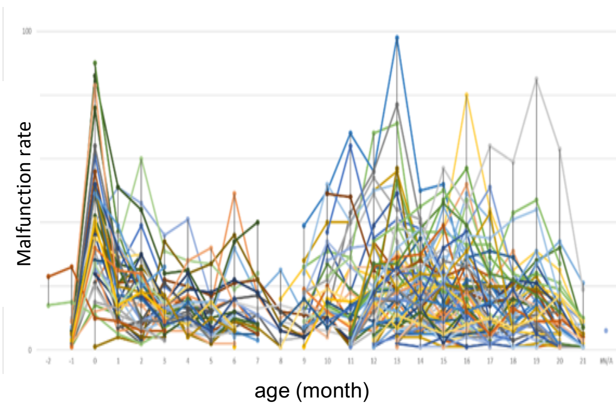

Figure 11. the number of elevator malfunction incidents vs. age.

The number of elevator malfunction incidents as a function of age is shown in Figure 11. As compared to the escalator, there is not apparent increase of the malfunction rate similar to the elevator case (Figure 2). Then we analyze the maintenance record of the MRT transmission system. Though the total number of transmission system is given, the age of each transmission system cannot be deciphered.

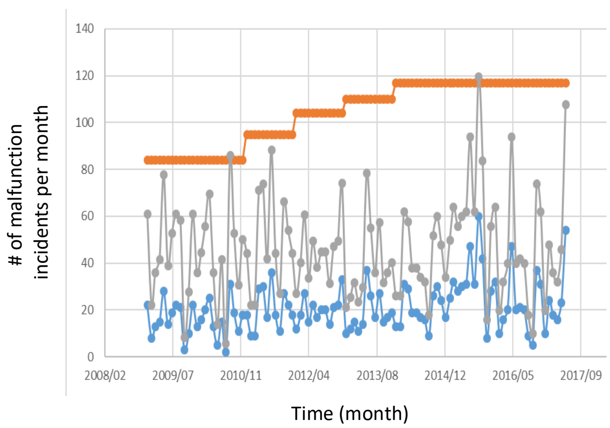

Figure 12. The total malfunction incidents per month as a function of time. (Blue line): the total malfunction incidents; (orange line): the total number of transmission systems; (gray line): the ratio of malfunction incidents over the total number of transmission systems.

As shown in Figure 12, the total number of transmission systems increases with time, and the number of malfunction incidents also increased slightly with time. The malfunction ratio (gray line) does not show an overall increase with time.

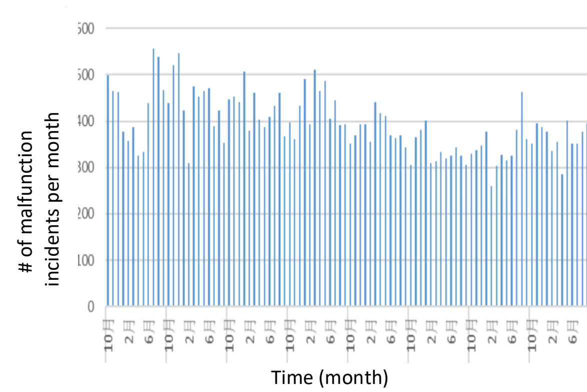

Figure 13. The total number malfunction counts of the escalators per month.

As shown in Figure 13, the total number of malfunction counts per month appears to be steady. However, since the total number of equipment varied with time, this steadiness of the escalator malfunction rate is not conclusive. More data is required to ascertain its general trend.

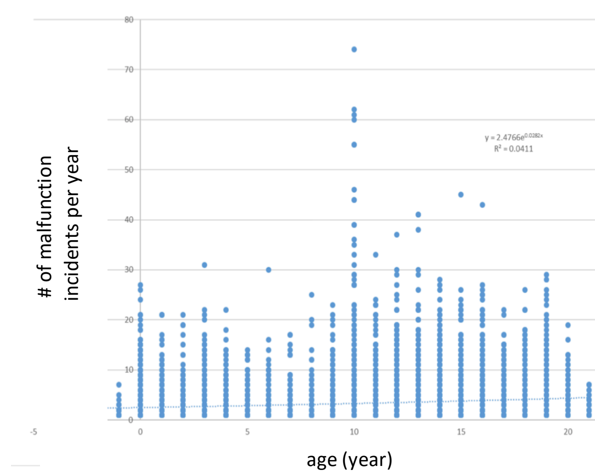

Figure 14. The number malfunction incidents of individual escalator per year as a function of age. The malfunction seemed to increase slightly with age.

The malfunction count per year for an individual escalator is plotted in Figure 14. The decrease near age of 20 years may be due to incomplete data. Overall, the malfunction count per year is steady and gradually rises with time.

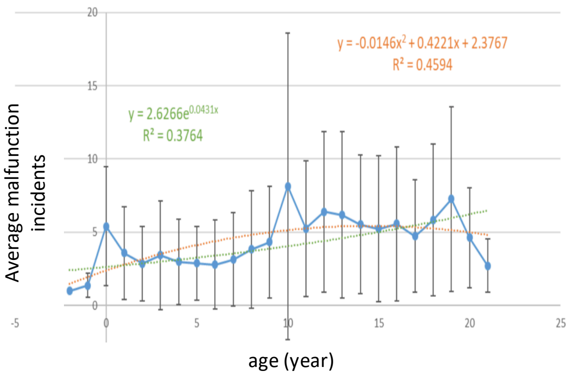

Figure 15. The average escalator malfunction rate. This is the average malfunction rate acquired from Figure 14. Notice that the overall malfunction rate gradually increases with age.

Figure 15 is the average escalator malfunction rate, showing a trend of gradual increase with the escalator age. However, the overall malfunction rate should increase with age; we anticipate the general trend increases if the data range is expended to 40 years or more.

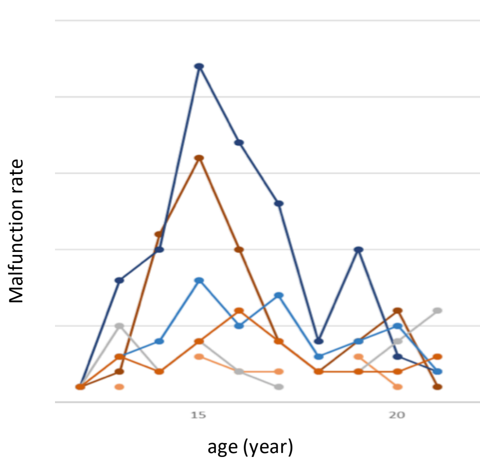

Figure 16. Malfunction rate of individual escalators vs. age. It is shown that the individual variation is pronounced; the relationship of individual malfunction rate and age is not decisive.

Figure 16 is the malfunction rate for six individual escalators. Notice that the individual variation is pronounced, suggesting that malfunction rate is not solely dependent upon the age of the escalator.

Trend of the maintenance data showed that the failure rate has been declining every year until it reached low and stable end tail. Possible factors are analyzed; this mostly likely is due to the improvement of MRT maintenance. On the other hand, since the age of each equipment and the number of samples for each equipment are not consistent, the total failure rate is not a fair representation of a specific individual equipment. Thus, our goal is to analyze the age's effect of the individual equipment.

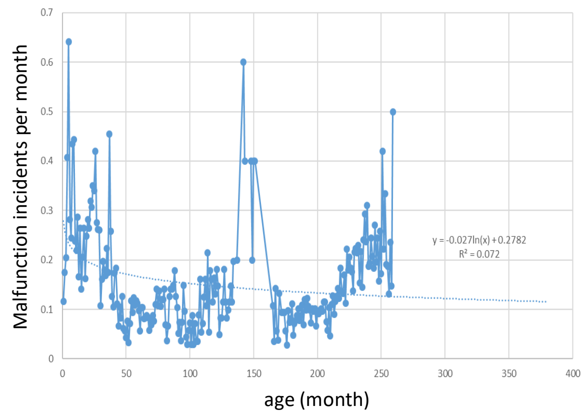

Figure 17. The average malfunction rate of the AFC gate *vs*. time.

With sufficient maintenance data of escalators and elevators, we quantify the failure rate vs. the age for each individual equipment with correlation analysis. On the other hand, for some equipment that seem to have no variation among one another, e.g., EMU, platform door, switcher and 22kV switchboard, we propose to calculate the malfunction rate vs. date of incident. As shown in Figure 17, the Wenhu line Central AFC gate failure rate vs. date is analyzed. The spikes at the center is likely due to lack of sufficient data points to reveal detail trend. Overall, the malfunction rate decreases and later increases, exhibiting behavior similar to a bathtub curve.

Then we analyze the maintenance data of the escalator system, elevator system, and EMU air conditioner of the metro system. These three systems are essential components of the metro system. The elevator and escalator are used regularly by the commuting people.

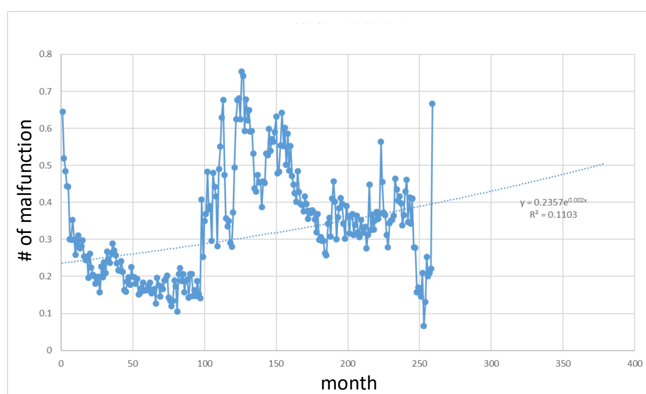

Figure 18. The reported average number of malfunctions each month of the Taipei metro station escalators.

There are 1144 escalators that have been recorded. From Figure 18, it is apparent that the malfunction rate increases with time. Nevertheless, there are two major age groups of escalators; it is unclear the trend of the younger group and older group of escalators are the same. Data analysis suggests that these two group should be analyzed separately instead of as a whole.

The maintenance records of the elevator, escalator, and EMU air conditioner are analyzed. As shown in Figure 18, the maintenance record of the escalator is depicted; trend of the malfunction rate is irregular. Maybe due to limited span of the maintenance data, the specific stage of life-time is not apparent; further analysis is required.

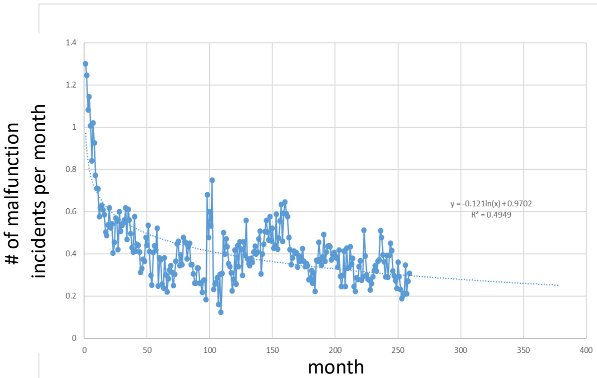

Figure 19. The reported average number of malfunctions each month of the Taipei metro station elevators.

Next, the maintenance record of the elevator system is shown in Figure 19. The maintenance record decreases with time monotonically. This monotonic trend appears to be a good match with the ideal bathtub-shaped curve.

Lastly, the maintenance record of the EMU air conditioner is shown in Figure 20, also exhibiting a decrease in its malfunction rate. However, compared to the smooth trend of Figure 8, the EMU air conditioner maintenance data is more volatile. Though the maintenance data of the three systems differ, they all roughly decrease with time, suggesting that the system is still young whereas the performance is still improving. Or, the system is being well-maintained such that the malfunction rate does not reflect the system performance accurately.

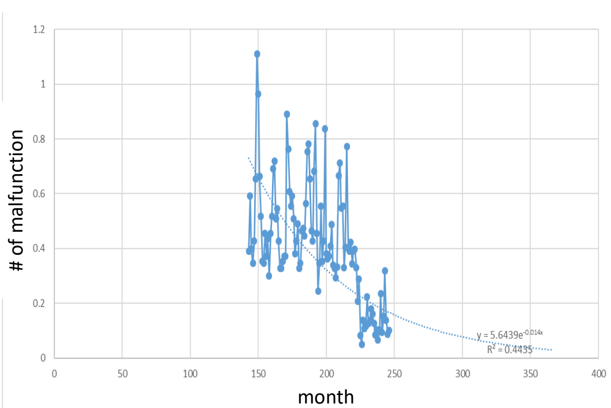

Figure 20. The reported average number of malfunctions each month of the Taipei metro EMU air conditioners.

The maintenance record provided by TRTC consists of only number of malfunction incidents per month. However, the severity of each malfunction may be drastically different, from as simple as the replacement of light bulb, up to power combustion resulting in system complete breakdown. Yet, in the available maintenance records, there is no information describing the severity of the malfunction event. Statistical analyses of these three systems (escalator, elevator, and EMU air conditioner) exhibit no apparent degradation of the system. Trends of the maintenance records suggest that the system condition improves with time, which is not the typical characteristics of a withering system.

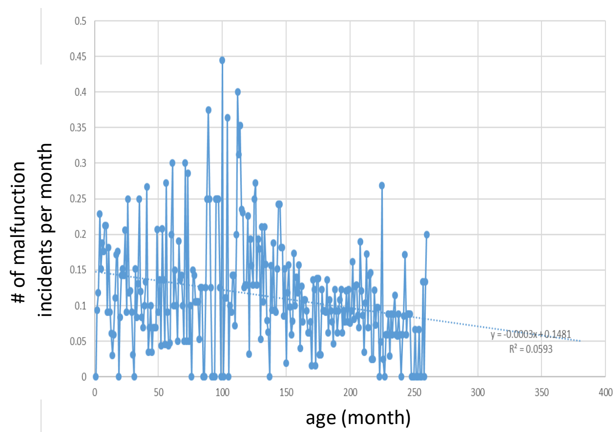

Figure 21. The average Taipei metro station elevators malfunction rate as a function of age (month). The general trend of the sporadic data decreased slightly.

More data point is required to determine the general trend of the malfunction rate. As shown in Figure 21, the trend decreases slightly with age; additional data point would be helpful to pinpoint the overall malfunction rate trend. Similarly, in Figure 22, more data is required to depict the trend of the malfunction rate of Muzha Line MRT platform screen door.

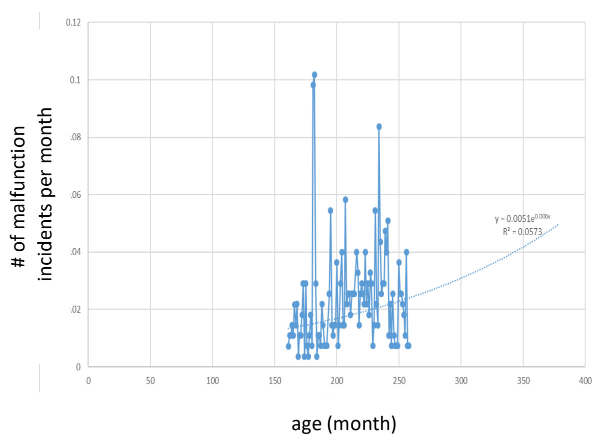

Figure 22. The average malfunction rate of the Muzha Line MRT platform screen door. The maintenance data showed a slight increase in malfunction rate with time.

V. Discussion

Taipei Metro is still very young compared to other metropolitan metro systems in the world. For instance, the first subway route of New York City began operating in 1904 [6]; Tokyo Metro began operating in 1927. As mentioned in related data survey, New York Subway replaces its trains about every 40 years, far older than TRTC's age. Based on the available Taipei metro maintenance record, the objective of this research is to assess the current condition of the metro system, and furthermore, if possible, to estimate the remaining lifetime of each system.

By means of data analysis, our goal is to identify characteristics indicative of the current status of the metro system, remaining lifetime, and extrapolate its future trend. Data analysis of the Taipei metro system maintenance record revealed general trend and characteristics of the system condition and performance, however, information regarding the total lifespan of the system, current stage-of-life, and remaining lifetime have not been ascertained. Possible reasons include: i) the system is still in its early stage, ii) the regular maintenance altered the natural deterioration trend, iii) the maintenance records being insufficient both in quantity and variety. As a result, the estimation of system remaining lifetime based on the limited maintenance data is very challenging.

On the other hand, using the count of malfunction incidents reported each month to assess the performance of Taipei Metro system is not very sensitive or informative. There are various types of malfunction, small glitches in the system, or, as large as a system failure, each of these incident reports are treated and weighed equally as a single event. Thus, the recorded failure rate is not an accurate measure of the system performance. Furthermore, each equipment consists of various brands and models, it may be infeasible to statistically come up with a universal degradation relationship for such complex systems.

Furthermore, with essential components replaced by new ones through regular maintenance is like renewing the system lifetime, which further complicates the estimation of system lifetime. As for when an equipment is due to be replaced is determined by a wide range of factors, including: economic consideration, new technology, the public opinion for a trendy new system, etc. Oftentimes the equipment may be operating properly, yet, it is retired and replaced for non-technical reasons such as politics or public opinion, etc. Thus, for pragmatic concerns, lifespan of the equipment is not the critical factor for retirement/replacement.

On a more fundamental level, it is undetermined that the desired information can be extracted from the maintenance record. The specific information may be entangled and infeasible to be extracted from the limited maintenance records. More importantly, the desired information may not be fully contained within the dataset. The maintenance record may cover only a small fraction of the entire system lifespan. Therefore, the accuracy to estimate the system lifetime based upon the provided information may be limited.

Even though the malfunction rate may not be a sensitive and reliable measure of the Metro system present status, it can still provide useful information: i) the maintenance is considered appropriate if the failure rate continues to stay low; ii) the maintenance needs to be renovated if the failure rate continues to increase. We suggest recording more parameters that are indicative of the equipment status, such as: recording the number of passengers using the equipment, the operating time, the number of passengers effected by the failure incident, the cost of repair, or even the weather condition such as humidity or earthquake. Recent research [7] suggested that system performance can be better assessed with more information used to monitor the system; the analysis outcome can better reflect the system status and remaining lifetime.

Based on the available maintenance record of the Taipei metro system, statistical analysis indicates that no signs of deterioration or withering. The data analysis falls short to yield information regarding the total lifespan, the current stage of life, and the remaining lifetime. If data with longer span and more variables is available, it is possible that such information can be ascertained. On the other hand, data analysis shows that the maintenance of the Taipei metro system is adequate.

Overall, TRTC is doing a fine job in terms of maintenance and repair, which is clearly reflected in its low malfunction rate. Based on the maintenance record, it is reasonable to extrapolate and predict that the Taipei Metro system can maintain its current level of performance for a good 20 to 60 years without significant degradation. Since "trendiness" cannot be achieved via maintenance, if the Metro system is renovated every few years with new technology and trendy products such as slick new cable cars with new technologies; brighter flat displays with higher contrast, smart tint windows, etc., it is possible that Taipei Metro system can maintain a constant modern slick appearance that differs from other metropolitan metro systems (such as Tokyo, New York, Boston, etc.) that appears to be aged. As the maintenance data has shown, the Taipei Metro system is performing well and data analysis suggests that it would likely continue its performance at its current standard.

VI. Conclusions

The maintenance record provided by TRTC contains only number of malfunction incidents per month; the record contained no severity information or detailed description of the specific malfunction. Thus, the simple malfunction rate record falls short to provide an informative description of the status of the Taipei metro system, which involves enormous number of factors including human. Such complex system exhibits sophisticated characteristics that can hardly be characterized with a single variable. Thus, it is unrealistic to decipher the system status with just a single variable.

Nevertheless, we believe rich information is embedded within the maintenance records and can be harnessed with the appropriate analysis tools. Possibly with big data analysis, one can decipher more information that may help enhance the MRT performance. Instead of a single parameter (number of malfunction incidents per month), with more recorded data (i.e., longer timespan, wider variety of variables, and detailed description of each malfunction incident), combined with big data analysis, it would be more feasible to yield information indicative of the system condition to better monitor the system condition.

VII. FUTURE WORK

In order to extract information indicative of the system status, we propose the following:

First, based upon the original maintenance records, calculate the duration between: i) when the equipment was first engaged in operation, and ii) the failure date. This time interval represents the duration of malfunction-free operation, which is also the time for a malfunction to take place. By analyzing the malfunction-free duration instead of the number of malfunctions each month may yield more realistic relationship.

Second, calculate the failure times per month to acquire the failure rate for each individual equipment. (For the equipment related to EMU, the failure rate is acquired by the failure time divided by the total mileage.)

Third, average each equipment's failure times to get the average failure rate. Use a statistical software to calculate the regression curve, and extrapolate to compare with other metro system performance data.

The above three steps are to be performed repeatedly. Each step is employed to process another set of data. The difference between each dataset is compared and analyzed for its regression behavior and general trend. Based on experience analyzing the Taipei MRT dataset, we believe the above approach would enable extraction of crucial information that is indicative of the system status.

ACKNOWLEDGMENT

We thank TRTC for providing maintenance records for analysis and very helpful and responsive with all of our questions. This research is supported by the Taiwan National Science Council Grant MOST-106-2112-M-002-008 and MOST-107-2112-M-002-011.

REFERENCES

[1] S. H. Tseng and T.-C. Kao, "Investigating the Feasibility to Estimate System Performance Based Upon Limited Data of the Taipei Metro System," in *The Fourteenth International Conference on Systems*, Valencia, Spain, S. S. Compte, Ed., March 24-28, 2019, pp. 69-72.

[2] X. K. Wei, F. Liu, and L. M. Jia, "Urban rail track condition monitoring based on in-service vehicle acceleration measurements," *Measurement,* vol. 80, pp. 217-228, Feb 2016, doi: 10.1016/j.measurement.2015.11.033.

[3] M. Molodova, M. Oregui, A. Nunez, Z. L. Li, and R. Dollevoet, "Health condition monitoring of insulated joints based on axle box acceleration measurements," *Engineering Structures,* vol. 123, pp. 225-235, September 2016, doi: 10.1016/j.engstruct.2016.05.018.

[4] G. Lederman, S. H. Chen, J. Garrett, J. Kovacevic, H. Y. Noh, and J. Bielak, "Track-monitoring from the dynamic response of an operational train," *Mechanical Systems and Signal Processing,* vol. 87, pp. 1-16, Mar 2017, doi: 10.1016/j.ymssp.2016.06.041.

[5] R. Jiang *et al.*, "Network operation reliability in a Manhattan-like urban system with adaptive traffic lights," *Transportation Research Part C-Emerging Technologies,* vol. 69, pp. 527-547, August 2016, doi: 10.1016/j.trc.2016.01.006.

[6] Metropolitan Transportation Authority. "New York City Transit - History and Chronology." Retrieved Jan. 1st, 2018, from http://web.mta.info/nyct/facts/ffhist.htm

[7] D. Rivoli. "Ancient subway trains on C and J/Z lines won't be replaced until 2022, documents say." http://www.nydailynews.com/new-york/ancient-subway-trains-won-replaced-2022-article-1.2323289

[8] H. Yin, K. Wang, Y. Qin, Q. Hua, and Q. Jiang, "Reliability analysis of subway vehicles based on the data of operational failures," *EURASIP Journal on Wireless Communications and Networking,* journal article vol. 2017, no. 1, p. 212, December 15 2017, doi: 10.1186/s13638-017-0996-y.

[9] Z. G. Li, J. G. Zhou, and B. Y. Liu, "System Reliability Analysis Method Based on Fuzzy Probability," *International Journal of Fuzzy Systems,* vol. 19, no. 6, pp. 1759-1767, December 2017, doi: 10.1007/s40815-017-0363-5.

[10] A. Z. Afify, G. M. Cordeiro, N. S. Butt, E. M. M. Ortega, and A. K. Suzuki, "A new lifetime model with variable shapes for the hazard rate," *Brazilian Journal of Probability and Statistics,* vol. 31, no. 3, pp. 516-541, Aug 2017, doi: 10.1214/16-bjps322.

[11] T. Kamel, A. Limam, and C. Silvani, "Modeling the degradation of old subway galleries using a continuum approach," *Tunnelling and Underground Space Technology,* vol. 48, pp. 77-93, April 2015, doi: 10.1016/j.tust.2014.12.015.

[12] D. Brancherie and A. Ibrahimbegovic, "Novel anisotropic continuum-discrete damage model capable of representing localized failure of massive structures: Part I: theoretical formulation and numerical implementation," *Engineering Computations,* vol. 26, no. 1-2, pp. 100-127, 2009, doi: 10.1108/02644400910924825.

[13] R. Tahmasbi and S. Rezaei, "A two-parameter lifetime distribution with decreasing failure rate," *Computational Statistics & Data Analysis,* vol. 52, no. 8, pp. 3889-3901, April 2008, doi: 10.1016/j.csda.2007.12.002.

[14] C. F. Daganzo and N. Geroliminis, "An analytical approximation for the macroscopic fundamental diagram of urban traffic," *Transportation Research Part B-Methodological,* vol. 42, no. 9, pp. 771-781, Nov 2008, doi: 10.1016/j.trb.2008.06.008.

[15] C. Kus, "A new lifetime distribution," *Computational Statistics & Data Analysis,* vol. 51, no. 9, pp. 4497-4509, May 15 2007, doi: 10.1016/j.csda.2006.07.017.

[16] C. D. Lai, M. Xie, and D. N. P. Murthy, "A modified Weibull distribution," *IEEE Transactions on Reliability,* vol. 52, no. 1, pp. 33-37, March 2003, doi: 10.1109/tr.2002.805788.

[17] O. O. Aalen and H. K. Gjessing, "Understanding the shape of the hazard rate: A process point of view," *Statistical Science,* vol. 16, no. 1, pp. 1-14, Feb 2001.

[18] S. K. Maurya, A. Kaushik, S. K. Singh, and U. Singh, "A new class of distribution having decreasing, increasing, and bathtub-shaped failure rate," *Communications in Statistics-Theory and Methods,* vol. 46, no. 20, pp. 10359-10372, 2017, doi: 10.1080/03610926.2016.1235196.

[19] Q. H. Duan and J. R. Liu, "Modelling a Bathtub-Shaped Failure Rate by a Coxian Distribution," *IEEE Transactions on Reliability,* vol. 65, no. 2, pp. 878-885, Jun 2016, doi: 10.1109/tr.2015.2494374.

[20] W. J. Roesch, "Using a new bathtub curve to correlate quality and reliability," *Microelectronics Reliability,* vol. 52, no. 12, pp. 2864-2869, Dec 2012, doi: 10.1016/j.microrel.2012.08.022.

[21] J. Navarro and P. J. Hernandez, "How to obtain bathtub-shaped failure rate models from normal mixtures," *Probability in the Engineering and Informational Sciences,* vol. 18, no. 4, pp. 511-531, 2004.

[22] S. Rajarshi and M. B. Rajarshi, "Bathtub distributions - a review," *Communications in Statistics-Theory and Methods,* vol. 17, no. 8, pp. 2597-2621, 1988, doi: 10.1080/03610928808829761.

[23] M. V. Aarset, "How to identify an bathtub hazard rate," *IEEE Transactions on Reliability,* vol. 36, no. 1, pp. 106-108, Apr 1987, doi: 10.1109/tr.1987.5222310.

[24] K. L. Wong, "The bathtub does not hold water any more," *Quality and Reliability Engineering International,* vol. 4, no. 3, pp. 279-282, 1988, doi: 10.1002/qre.4680040311.

[25] H. T. Zeng, T. Lan, and Q. M. Chen, "Five and four-parameter lifetime distributions for bathtub-shaped failure rate using Perks mortality equation," *Reliability Engineering & System Safety,* vol. 152, pp. 307-315, August 2016, doi: 10.1016/j.ress.2016.03.014.

[26] D. N. P. Murthy and R. Jiang, "Parametric study of sectional models involving two Weibull distributions," *Reliability Engineering & System Safety,* vol. 56, no. 2, pp. 151-159, May 1997, doi: 10.1016/s0951-8320(96)00114-7.

[27] G. A. Klutke, P. C. Kiessler, and M. A. Wortman, "A critical look at the bathtub curve," *IEEE Transactions on Reliability,* vol. 52, no. 1, pp. 125-129, 2003, doi: 10.1109/TR.2002.804492.