Quantum Mechanics Needs Interpretation

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***Abstract*—Since the beginning, quantum mechanics has raised major foundational and interpretative problems. Foundational research has been an important factor in the development of quantum cryptography, quantum information theory and, perhaps one day, practical quantum computers. Many believe that, in turn, quantum information theory has bearing on foundational research. This is largely related to the so-called epistemic view of quantum states, which maintains that the state vector represents information on a system and which has led to the suggestion that quantum theory needs no interpretation. I will argue that this and related approaches fail to take into consideration two different explanatory functions of quantum mechanics, that of accounting for classically unexplainable correlations between classical phenomena and that of explaining the microscopic structure of classical objects. The epistemic view provides no answer to what constitutes the main question of interpretation: How can the world be for quantum mechanics to be true? I will then review three different approaches to understanding quantum mechanics, namely, Bohmian mechanics, Everett's relative states, and Cramer's transactional interpretation. I will show that these approaches answer the above question, as well as other foundational ones. This paper is written from the perspective that different logically consistent interpretations, far from leading to confusion, in fact contribute to increased understanding of the theory.**

I. Introduction

Although the answer was intended to be clear, this paper's title was formulated interrogatively in my contribution to the ICQNM 2009 conference [1]. Explicit consideration of several interpretative schemes now motivates the positive formulation.

Ever since it was proposed more than 80 years ago, quantum mechanics has raised great challenges both in foundations and in applications.[1] The latter have been developed at a very rapid pace, opening up new vistas in most branches of physics as well as in much of chemistry and engineering. Substantial progress and important discoveries have also been made in foundations, though at a much slower rate. The measurement problem, long-distance correlations, and the meaning of the wave function are three of the foundational problems on which there has been and still is lively debate.

It is fair to say that foundational studies have largely contributed to the burgeoning of quantum information theory, one of the most active areas of development of quantum mechanics in the past 25 years. Quantum information is dependent on entanglement, whose significance was brought to light through the Einstein-Podolsky-Rosen (EPR) argument [6]. The realization that transfer protocols based on quantum entanglement may be absolutely secure has opened new windows in the field of cryptography [7]. And the development of quantum algorithms thought to be exponentially faster than their best classical counterparts has drawn great interest in the construction of quantum computers [8]. These face up extraordinary challenges on the experimental side. But attempts to build them are likely to throw much light on the fundamental process of decoherence and perhaps on the limits of quantum mechanics itself [9], [10].

Along with quantum information theory came also a reemphasis of the view that the wave function (or state vector, or density operator) properly represents knowledge, or information [11], [12], [13]. This is often called the *epistemic view* of quantum states. On what the wave function is knowledge of, proponents of the epistemic view do not necessarily agree. The variant most relevant to the present discussion is that rather than referring to objective properties of microscopic objects (such as electrons, photons, etc.), the wave function encapsulates probabilities of results of eventual macroscopic measurements. The Hilbert space formalism of quantum mechanics is taken as complete, and its objects in no need of a realistic interpretation. Additional constructs, like value assignments [14], Bohmian trajectories [15], multiple worlds [16], or transactions [17] are viewed as superfluous at best.

Just like foundational studies have contributed to the development of quantum information theory, many investigators think that the latter can help in solving the foundational and interpretative problems of quantum mechanics. A number of proponents of the epistemic view believe that it considerably attenuates, or even completely solves, the problems of quantum measurement, of long-distance correlations, and of the meaning of the wave function. These problems will be summarized briefly in Sec. II, and the way the epistemic view deals with them will be presented in Sec. III. I will then argue, in Sec. IV, that the epistemic view and related approaches fail to take into consideration that quantum mechanics has two very different explanatory functions: that of accounting for classically unexplainable correlations between classical phenomena, and that of explaining the microscopic structure of classical objects [18], [19]. In Sec. V, I will ask the question of what it means to interpret quantum mechanics, or any scientific theory for that matter. Drawing from the so-called semantic view of theories, I will argue than interpreting quantum mechanics means answering the question, "How can the world be for quantum mechanics to be true?" [20].

[1] Relevant reviews and paper collections are, for instance, [2], [3], [4], [5].

The next three sections will examine how three interpretative schemes of quantum mechanics, namely, Bohmian mechanics, Everett's many worlds, and Cramer's transactional interpretation, answer the above question and attempt to solve the foundational problems. Concluding remarks will be made in the last section.

II. Three problems in quantum mechanics

Although the way to apply quantum mechanics to practical situations was never a matter of dispute, the meaning of the formalism has been problematic from the outset. The first problem concerned the ψ function that appears in Schrödinger's fundamental equation. Is it something like the electric and magnetic fields we are familiar with? Schrödinger first proposed that the absolute square of ψ is proportional to the electron's charge distribution [21]. But this was quickly found untenable. Born then proposed his probabilistic interpretation, according to which the absolute square of ψ represents the probability to find the electron at a given place. This much, an instance of what is now known as Born's rule, is universally accepted. But it is still a matter of debate whether ψ represents an individual system or a statistical ensemble of systems [22], and whether it is a real field or has a strictly operational significance.

The second problem also arose very early in the development of quantum mechanics, and concerns the question of measurement. Broadly speaking, the problem is the following. Suppose we want to describe, in a completely quantum-mechanical way, the process of measuring a physical quantity Q pertaining to a microscopic system. For simplicity, assume that the spectrum of Q is discrete and nondegenerate, that $\mathbf{x}$ stands for the coordinates of the microscopic system, and that the normalized eigenfunction $\phi_i(\mathbf{x})$ corresponds to the eigenvalue q_i. For the process to be fully described by quantum mechanics, the measurement apparatus should also be considered as a quantum system, which comes to interact with the microscopic system. Let $\alpha_0(\xi)$ denote the initial wave function of the apparatus. Here ξ stands for the one-dimensional *pointer coordinate* of the apparatus. The myriad of other apparatus coordinates, representing all its microscopic degrees of freedom, are not explicitly represented.

The interaction between the microscopic quantum system and the apparatus will represent a faithful measurement of Q if the combined system evolves like

$$\phi_i(\mathbf{x})\alpha_0(\xi) \to \phi_i(\mathbf{x})\alpha_i(\xi), \tag{1}$$

where $\alpha_i(\xi)$ represents a state of the apparatus wherein the pointer shows the value α_i (with $\alpha_i \neq \alpha_j$ if $i \neq j$).[2]

It is instructive to see how the evolution (1) can be realized explicitly. Let the interaction between the microscopic system and the apparatus take place in the interval $0 < t < T$. In that time interval, take the Hamiltonian as

$$H = gQP_\xi, \tag{2}$$

where g is a real constant and P_ξ is the momentum operator conjugate to the pointer's position operator. We have neglected, here, terms in the Hamiltonian specifically connected with the microscopic system or the pointer (for instance, $\mathbf{P}\cdot\mathbf{P}/2m$), which is a good approximation if g is sufficiently large and T is sufficiently small. If the initial combined wave function is given by $\phi_i(\mathbf{x})\alpha_0(\xi)$, the final wave function will be obtained straightforwardly [23] as

$$\begin{aligned}\phi_i(\mathbf{x})\alpha_0(\xi) &\to \exp\left\{-\frac{iT}{\hbar}H\right\}\phi_i(\mathbf{x})\alpha_0(\xi)\\ &= \exp\left\{-\frac{iT}{\hbar}gQP_\xi\right\}\phi_i(\mathbf{x})\alpha_0(\xi)\\ &= \phi_i(\mathbf{x})\exp\left\{-\frac{iT}{\hbar}gq_iP_\xi\right\}\alpha_0(\xi)\\ &= \phi_i(\mathbf{x})\alpha_0(\xi - gTq_i)\\ &= \phi_i(\mathbf{x})\alpha_i(\xi).\end{aligned} \tag{3}$$

In its final state α_i, the pointer is moved by a distance gTq_i from its initial state. We assume that the initial wave packet $\alpha_0(\xi)$ is sufficiently narrow for all the $\alpha_i(\xi)$ to be essentially non-overlapping.

If the Schrödinger equation is universally valid, the combined evolution of the microscopic system and macroscopic apparatus is unitary (assuming, unrealistically, that they form together a closed system). But then, an initial state involving the superposition of several eigenstates of an observable of the microscopic system evolves into a final state involving a superposition of macroscopically distinct states of the apparatus (or of the apparatus and environment in more realistic situations). Explicitly,

$$\left\{\sum_i c_i\phi_i(\mathbf{x})\right\}\alpha_0(\xi) \to \sum_i c_i\phi_i(\mathbf{x})\alpha_i(\xi). \tag{4}$$

Obviously, we never see a macroscopic apparatus in a superposition of states corresponding to different pointer readings. The discrepancy between this observation and the unitary evolution expressed in (4) constitutes the measurement problem.

To solve the problem, von Neumann suggested a long time ago that the unitary evolution breaks down somewhere in the measurement process [24]. Specifically, von Neumann postulated that in measurement interactions like (4), the right-hand side abruptly collapses into one of its components. This process is fundamentally indeterministic, and the probability that the superposition collapses into $\phi_j(\mathbf{x})\alpha_j(\xi)$ is taken to be equal to $|c_j|^2$. Von Neumann did not propose any specific mechanism accounting for the collapse of the wave function, but interesting suggestions along these lines were made in subsequent years [25].[3]

A third problem that quantum mechanics has to deal with is the one of long-distance correlations [13], [29]. Consider the realization of the EPR setup in terms of two spin 1/2 particles

[2] For simplicity, we will always assume that the system's states $\phi_i(\mathbf{x})$ don't change in a measurement.

[3] Decoherence theory no doubt helps in making the measurement problem sharper, but the present author shares the view that it is by itself insufficient to solve the problem. For recent perspectives see [26], [27], [28].

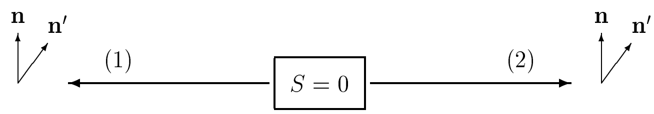

Fig. 1. Two particles prepared in the singlet state and leaving in opposite directions.

(labelled 1 and 2), depicted in Fig. 1. The state vector $|\chi\rangle$ of the compound system is taken to be an eigenstate of the total spin operator with eigenvalue zero. In this case

$$|\chi\rangle = \frac{1}{\sqrt{2}} \{|+;\mathbf{n}\rangle|-;\mathbf{n}\rangle - |-;\mathbf{n}\rangle|+;\mathbf{n}\rangle\} . \quad (5)$$

Here the first vector in a (tensor) product refers to particle 1 and the second vector to particle 2. The vector $|+;\mathbf{n}\rangle$, for instance, stands for an eigenvector of the $\mathbf{n}$-component of the particle's spin operator, with eigenvalue $+1$ (in units of $\hbar/2$). The unit vector $\mathbf{n}$ can point in any direction, a freedom which corresponds to the rotational symmetry of $|\chi\rangle$.

Suppose Alice measures the $\mathbf{n}$-component of the spin of particle 1 and obtains the value $+1$. Then she can predict with certainty that if Bob measures the same component of the spin of particle 2, he will obtain the value -1. It then seems that the state of particle 2 changes immediately upon Alice's obtaining her result, and this no matter how far apart Alice and Bob are. Since the word "immediately", when referring to spatially separated events, is not a relativistically invariant concept, such a mechanism seems to imply instantaneous action at a distance, and is certainly not easy to reconcile with the theory of special relativity.

The interpretation of the wave function, the measurement of a quantum observable, and long-distance correlations are problems that an interpretation of quantum mechanics should clarify.

III. The epistemic and related views

In the epistemic view of quantum states, the wave function represents knowledge, or information. Let us examine the arguments that advocates of the epistemic view offer to solve the foundational and interpretative problems of quantum mechanics. I should point out that they do not all attribute the same strength and generality to these arguments. Some advocates believe that the problems are completely solved by the epistemic view, while others are of the opinion that they are just attenuated. This distinction, however, is not crucial to our purpose, and I will simply give the arguments as they are typically formulated.

The problem that is directly addressed by the epistemic view is the one of the interpretation of the wave function (or state vector, or density operator). Just as the name suggests, the state vector is normally interpreted as representing the state of quantum systems. As we have seen, some believe that the state pertains to an individual system, others to a statistical ensemble of systems. But the epistemic view, which goes back at least to writings of Heisenberg [30], claims that it represents neither. It denies that the (in this context utterly misnamed) state vector represents the state of a microscopic system. Rather, it represents knowledge about the probabilities of results of measurements performed in a given context with a macroscopic apparatus, in other words, information about "the potential consequences of our experimental interventions into nature" [13]. This is often set in the framework of a Bayesian approach, where probability is interpreted in a subjective way.

Now how does the epistemic view deal with the measurement problem? It does so by construing the collapse of the wave function not as a physical process, but as a change of knowledge [31]. Insofar as the wave function is interpreted as objectively describing the state of a physical system, its abrupt change in a measurement implies a similar change in the system, which calls for explanation. If, on the other hand, and in line with a Bayesian view, the wave function describes knowledge of conditional probabilities (i.e., probabilities of future macroscopic events conditional on past macroscopic events), then as long as what is conditionalized upon remains the same, the wave function evolves unitarily. It collapses when the knowledge base changes (this is Bayesian updating), thereby simply reflecting the change in the conditions being held fixed in the specification of probabilities.

The epistemic view also offers an explanation of long-distance correlations like the ones produced in EPR setups. We recall that when Alice obtains the value $+1$ when she measures the $\mathbf{n}$-component of her spin, she can predict with certainty that Bob will obtain -1 when he measures the $\mathbf{n}$-component of his spin. But according to the epistemic view, what changes when Alice performs a measurement is Alice's knowledge. Bob's knowledge will change either if he himself performs a measurement, or if Alice sends him the result of her measurement by conventional means. Hence no information is transmitted instantaneously, and there is no physical collapse on an equal time or spacelike hypersurface.

Related to the epistemic view is the idea of *genuine fortuitousness* [32], [33], a radically instrumentalist view of quantum mechanics. The idea "implies that the basic event, a click in a counter, comes without any cause and thus as a discontinuity in spacetime" [33, p. 405]. Indeed

> [i]t is a hallmark of the theory based on genuine fortuitousness that it does not admit physical variables. It is, therefore, of a novel kind that does not deal with things (objects in space), or measurements, and may be referred to as the theory of no things. (p. 410)

Such approaches to the interpretation of quantum mechanics are to be contrasted with realist views that we will examine later.[4]

[4]The "correlations without correlara" view of quantum mechanics [34], also known as the Ithaca Interpretation, shares with the epistemic view the idea that no reality is attributed to individual properties of quantum systems. However, correlations do have physical reality and the Ithaca interpretation strives to eliminate knowledge from the foundations.

IV. Two explanatory functions

To examine how appropriate the epistemic and related views of quantum mechanics are, it is important to properly understand the explanatory role of quantum mechanics as a physical theory. Although all measurements are made by means of macroscopic apparatus, quantum mechanics is used, as an explanatory theory, in two different ways: it is meant to explain (i) nonclassical correlations between macroscopic objects and (ii) the small-scale structure of macroscopic objects [18], [19]. That these two functions are distinct is best shown by contrasting the world in which we live with a hypothetical, closely related one [20].

Roughly speaking, the hypothetical world is defined so that (a) for all practical purposes, all macroscopic experiments give results that coincide with what we find in the real world, and (b) its microscopic structure, if applicable, is different from the one of the real world. Let us spell this out in more detail.

In the hypothetical world large scale objects, i.e., objects much larger than atomic sizes, behave just like large scale objects in the real world. The trajectories of baseballs and airplanes can be computed accurately by means of classical mechanics with the use of a uniform downward force, air friction, and an appropriate propelling force. Waveguides and antennas obey Maxwell's equations. Steam engines and heat pumps work according to the laws of classical thermodynamics. The motion of planets, comets, and asteroids is well described by Newton's laws of gravitation and of motion, slightly corrected by the equations of general relativity.

Close to atomic scales, however, these laws may no longer hold. Except for one restriction soon to be spelled out, I shall not be specific about the changes that macroscopic laws may or may not undergo in the microscopic realm. Matter, for instance, could either be continuous down to the smallest scales, or made of a small number of constituent particles like our atoms. The laws of particles and fields could be the same at all scales, or else they could undergo significant changes as we probed smaller and smaller distances.

In the hypothetical world one can perform experiments with pieces of equipment like Young's two-slit setup, Stern-Gerlach devices, or Mach-Zehnder interferometers. Let us focus on the Young type experiment. It makes use of two macroscopic objects which we label E and D. These symbols could stand for "emittor" and "detector" if it were not that, as we shall see, they may not emit or detect anything. At any rate, E and D both have on and off states and work in the following way. Whenever D is suitably oriented with respect to E (say, roughly along the x axis) and both are in the on state, D clicks in a more or less random way. The average time interval between clicks depends on the distance r between D and E, and falls roughly as $1/r^2$. The clicking stops if, as shown in Fig. 2, a shield of a suitable material is placed perpendicularly to the x axis, between D and E.

If holes are pierced through the shield, however, the clicking resumes. In particular, with two small holes of appropriate size and separation, differences in the clicking rate are observed for small transverse displacements of D behind the shield. A plot of the clicking rate against D's transverse coordinate displays maxima and minima just as in a wave interference pattern. No such maxima and minima are observed, however, if just one hole is open or if both holes are opened alternately.

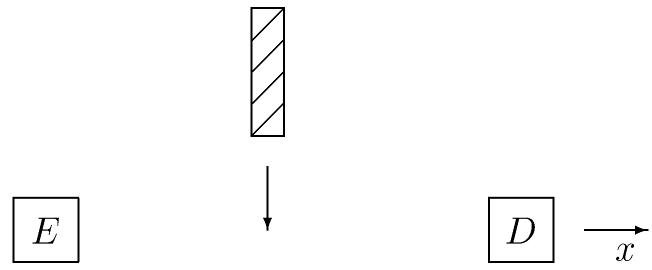

Fig. 2. Shielding material prevents D from clicking

At this stage everything happens as if E emitted some kind of particles and D detected them, and the particles behaved according to the rules of quantum mechanics. Nevertheless, we shall nor commit ourselves to the existence or nonexistence of these particles, except on one count. Such particles, if they exist, are not in any way related to hypothetical constituents of the material making up D, E, or the shield, or of any macroscopic object whatsoever. Whatever the microscopic structure of macroscopic objects is, it has nothing to do with what is responsible for the correlations between D and E.

In a similar way, we can perform in the hypothetical world experiments with Stern-Gerlach devices, Mach-Zehnder interferometers, or other setups used in the typical quantum-mechanical investigations carried out in the real world. Correlations are observed between initial states of "emittors" and final states of "detectors" which are unexplainable by classical mechanics but follow the rules of quantum mechanics. We assume again that, if these correlations have something to do with the emission and absorption of particles, these are in no way related to eventual microscopic constituents of the macroscopic devices.

In the experiments just described that relate to the hypothetical world, quantum mechanics correctly predicts the correlations between D and E (or other "emittors" and "absorbers") when suitable experimental configurations are set up. In these situations, the theory can be interpreted in (at least) two broadly different ways. In the first one, the theory is understood as applying to genuine microscopic objects, emitted by E and detected by D. Perhaps these objects follow Bohmian-like trajectories (see Sec. VI), or behave between E and D in some other way compatible with quantum mechanics. In the other interpretation, there are no microscopic objects whatsoever going from E to D. There may be something like an action at a distance. At any rate the theory is in that case interpreted instrumentally, for the purpose of quantitatively accounting for correlations in the stochastic behavior of E and D.

In the hypothetical world we are considering, I believe that both interpretations are logically consistent and adequate. Of course, each investigator can find more satisfaction in one interpretation than in the other. The epistemic view of quantum mechanics corresponds to the instrumentalist interpretation. It simply rejects the existence of microscopic objects that have no other use than the one of predicting observed correlations between macroscopic objects.

In the world in which we live, however, the situation is crucially different. The electrons, neutrons, photons, and other particles that diffract or interfere are the same that one appeals to in order to explain the structure of macroscopic objects. Denying their existence, as is done in the approach of genuine fortuitousness, dissolves such explanatory power. Denying that they have states, as is done in the epistemic view, leaves one to explain the state of a macroscopic object on the basis of entities that have no state.

V. Interpreting quantum mechanics

The epistemic and related views therefore fail to account for the second explanatory role of quantum mechanics. To reinforce this conclusion, it is instructive to investigate what it means to interpret a theory.

With most physical theories, interpretation is rather straightforward. But this should not blind us to the fact that even very familiar theories can in general be interpreted in more than one way. A simple example is classical mechanics.

Classical mechanics is based on a well-defined mathematical structure. This consists of constants m_i, functions $\mathbf{x}_i(t)$, and vector fields $\mathbf{F}_i$ (understood as masses, positions, and forces), together with the system of second-order differential equations $\mathbf{F}_i = m_i\mathbf{a}_i$. A specific realization of this structure consists in a system of ten point masses interacting through the $1/r^2$ gravitational force. A hypothesis may then assert that the solar system corresponds to this realization, if the sun and nine planets are considered pointlike and all other objects neglected. Predictions made on the basis of this model correspond rather well with reality. But obviously the model can be made much more sophisticated, taking into account for instance the shape of the sun and planets, the planets' satellites, interplanetary matter, and so on.

Now what does the theory have to say about how a world of interacting masses is really like? It turns out that such a world can be viewed in (at least) two empirically equivalent but conceptually very different ways. The first one consists in asserting that the world is made only of small (or extended) masses that interact by instantaneous action at a distance. The second way asserts that the masses produce everywhere in space a gravitational field, which then locally exerts forces on the masses. These two ways constitute two different interpretations of the theory. Each one expresses a possible way of making the theory true (assuming empirical adequacy). Whether the world is such that masses instantaneously interact at a distance in a vacuum, or a genuine gravitational field is produced throughout space, the theory can be held as truly realized.

Similar remarks apply to classical electromagnetism. The mathematical equations can be interpreted as referring to charges and currents interacting locally through the mediation of electric and magnetic fields. Alternatively, they can be viewed as referring to charges and currents only, interacting by means of (delayed) action at a distance [35].

In this respect, quantum mechanics seems different from all other physical theories. There appears to be no straightforward way to visualize, so to speak, the behavior of microscopic objects. This was vividly pointed out by Feynman [36, p. 129] who, after a discussion of Young's two-slit experiment with electrons, concluded that "it is safe to say that no one understands quantum mechanics. [...] Nobody knows how it can be like that." But the process of interpreting quantum mechanics lies precisely in taking up Feynman's challenge. It is to answer the question, "How can the world be for quantum mechanics to be true?"

If we adopt this point of view (known as the semantic view of theories [37], [38]), we can understand the motivation to look for interpretative schemes of quantum mechanics. Each such scheme provides one clear way that the microscopic objects can behave so as to reproduce the quantum-mechanical rules and, therefore, the observable behavior of macroscopic objects. In the following sections, we shall look at three such approaches, and see how each ones deals with the three problems outlined before.

VI. Bohmian mechanics

A. One particle

Bohmian mechanics [15], [39], also known as the de Broglie-Bohm theory owing to de Broglie's early work [40], is a realistic causal theory that (in its standard form) exactly reproduces the statistical results of quantum mechanics. To see how it works, consider a particle of mass m whose Hamiltonian is given by

$$H = \frac{1}{2m}\mathbf{P}\cdot\mathbf{P} + V(\mathbf{X}, t). \tag{6}$$

The Schrödinger equation can be written as

$$i\hbar\frac{\partial}{\partial t}\psi(\mathbf{x}, t) = -\frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{x}, t) + V(\mathbf{x}, t)\psi(\mathbf{x}, t), \tag{7}$$

where the wave function $\psi(\mathbf{x}, t)$ is assumed to be normalized.

The complex function ψ can be written in polar form as

$$\psi(\mathbf{x}, t) = R(\mathbf{x}, t)\exp\left\{\frac{i}{\hbar}S(\mathbf{x}, t)\right\}, \tag{8}$$

where R and S are two real functions. Substituting (8) in (7) and manipulating, one easily finds that

$$\frac{\partial}{\partial t}(R^2) + \boldsymbol{\nabla}\cdot\left\{R^2\frac{1}{m}\boldsymbol{\nabla}S\right\} = 0, \tag{9}$$

$$\frac{\partial S}{\partial t} + \frac{1}{2m}\,(\boldsymbol{\nabla}S)\cdot(\boldsymbol{\nabla}S) + V(\mathbf{x}, t) - \frac{\hbar^2}{2m}\frac{1}{R}\nabla^2 R = 0. \tag{10}$$

Suppose that, for an initial value $\psi(\mathbf{x}, t_0)$ of the wave function, the solution of (7) has been found. Define

$$V_{\mathrm{Q}}(\mathbf{x}, t) = -\frac{\hbar^2}{2m}\frac{1}{R}\nabla^2 R \tag{11}$$

and

$$V_{\mathrm{tot}}(\mathbf{x}, t) = V(\mathbf{x}, t) + V_{\mathrm{Q}}(\mathbf{x}, t). \tag{12}$$

Equation (10) then becomes

$$\frac{\partial S}{\partial t} + \frac{1}{2m}(\boldsymbol{\nabla} S)\cdot(\boldsymbol{\nabla} S) + V_{\mathrm{tot}}(\mathbf{x}, t) = 0. \tag{13}$$

Formally, (13) coincides with the Hamilton-Jacobi equation associated with a classical particle with momentum

$$\mathbf{p} = \boldsymbol{\nabla} S \tag{14}$$

and Hamiltonian

$$H(\mathbf{x}, \mathbf{p}, t) = \frac{1}{2m}\mathbf{p}\cdot\mathbf{p} + V_{\mathrm{tot}}(\mathbf{x}, t). \tag{15}$$

That observation is at the root of the de Broglie-Bohm theory, which rests on the following hypotheses:

1) At every instant t, a quantum particle has a well-defined position $\mathbf{x}$ and momentum $\mathbf{p}$.
2) The particle's trajectory is governed by the Hamilton-Jacobi equation (13) or, equivalently, by Newton's equation

$$m\frac{d^2\mathbf{x}}{dt^2} = -\boldsymbol{\nabla} V_{\mathrm{tot}}(\mathbf{x}, t). \tag{16}$$

   The potential V_{tot} is the sum of an external potential V and of the *quantum potential* V_{Q}, determined by the solution of Schrödinger's equation (7).
3) The particle's position and momentum, although well-defined, cannot be known exactly. One can only know the probability density that at time t, the particle is at point $\mathbf{x}$. This probability density is equal to $|\psi(\mathbf{x}, t)|^2 = R^2(\mathbf{x}, t)$, where $R(\mathbf{x}, t)$ satisfies (9).[5]

Since it is usually not known, and is unpredictable on the basis of the wave function alone, the well-defined particle's position, in Bohmian mechanics, is often called a *hidden variable*.

We should note that hypothesis (3) on the probability density is consistent with the trajectories of the particles that make up the statistical ensemble. Indeed let a large number of identical particles be distributed according to a density $R^2(\mathbf{x}, t)$, with velocities equal to $\frac{1}{m}\mathbf{p}(\mathbf{x}, t)$. The particle number conservation law can then be written as

$$\frac{\partial}{\partial t}(R^2) + \boldsymbol{\nabla}\cdot\left\{R^2\frac{1}{m}\mathbf{p}\right\} = 0 \tag{17}$$

which, owing to (14), coincides with (9).

[5]More general forms of Bohmian mechanics relax the identification of the probability density with $|\psi|^2$ [39].

One can also check the consistency of (14) and (16). Indeed taking the total time derivative of the former and making use of (10), we get

$$\begin{aligned}\frac{d\mathbf{p}}{dt} &= \frac{\partial}{\partial t}\boldsymbol{\nabla} S + (\mathbf{v}\cdot\boldsymbol{\nabla})(\boldsymbol{\nabla} S)\\ &= \frac{\partial}{\partial t}\boldsymbol{\nabla} S + \frac{1}{m}(\boldsymbol{\nabla} S\cdot\boldsymbol{\nabla})(\boldsymbol{\nabla} S)\\ &= \boldsymbol{\nabla}\left\{\frac{\partial}{\partial t}S + \frac{1}{2m}(\boldsymbol{\nabla} S)^2\right\}\\ &= -\boldsymbol{\nabla}\left\{V(\mathbf{x}, t) - \frac{\hbar^2}{2m}\frac{1}{R}\nabla^2 R\right\}.\end{aligned} \tag{18}$$

Hypothesis (3) provides the answer that Bohmian mechanics gives to the first problem raised in Sec. II, the one of the interpretation of the wave function. Here the absolute square of the wave function quantifies the best knowledge one can have of the particle's precise position. But note the difference with the epistemic view. In Bohmian mechanics, the particle always has a precise position, which we do not know exactly. In the epistemic view, no precise position is attached to the particle, and in fact there may even be no particle at all. The absolute square of the wave function, in the epistemic view, only represents the probability that, after a suitable macroscopic preparation procedure, a position-measuring macroscopic apparatus will yield such and such values. As we will see, Bohmian mechanics also correctly predicts probabilities of measurement results. But it does so, in position measurements, because the measurement result corresponds to a position the particle really has.

With hypothesis (3), Bohmian mechanics always reproduces the statistical results of quantum mechanics. It is instructive to see how this works in the paradigmatic example of the two-slit experiment. Here we look for the probability of detection at various points on a screen behind the slits. It is well known that in quantum mechanics, the detection probability when both slits are open is not the sum of the detection probability when the first slit is open and the detection probability when the second slit is open. This is often interpreted by saying that when both slits are open, one cannot affirm that the particle went through one specific slit at the exclusion of the other.

In Bohmian mechanics, however, whether one or two slits are open, any given particle goes through only one slit. How can this reproduce the interference pattern at the screen? It turns out that, for a given initial value of the wave function (say, when a particle is emitted), the solution of the Schrödinger equation behind the slits, when only one slit is open, is different from the corresponding solution when both slits are open. The quantum potential, given in (11), is therefore also different. Thus for a given value of the particle's position, its trajectory when one slit is open is different from its trajectory when both slits are open, even if in both cases the particle goes through the same slit. Bohmian trajectories in the two-slit experiment were numerically calculated by Philippidis *et al* [41]. The statistical results they obtained precisely reproduce Young's interference pattern, thereby providing an illuminating answer to Feynman's challenge.

B. Two particles

Bohmian mechanics can be generalized to any number of particles but, for our purposes, it will be enough to consider only two. To be explicit, we will consider in this subsection the case of two spinless particles interacting through a potential $V(\mathbf{x}_1, \mathbf{x}_2, t)$. The system's configuration space has six dimensions.

The Schrödinger equation can be written as

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m_1}\nabla_1^2\Psi - \frac{\hbar^2}{2m_2}\nabla_2^2\Psi + V\Psi, \tag{19}$$

where ∇_i^2 $(i = 1, 2)$ stands for the Laplacian with respect to coordinates $\mathbf{x}_i$. Letting

$$\Psi(\mathbf{x}_1, \mathbf{x}_2, t) = R(\mathbf{x}_1, \mathbf{x}_2, t)\exp\left\{\frac{i}{\hbar}S(\mathbf{x}_1, \mathbf{x}_2, t)\right\}, \tag{20}$$

one finds two coupled equations for the real functions R and S. The equation that generalizes (10) is the Hamilton-Jacobi equation for two particles with momenta

$$\mathbf{p}_i = \boldsymbol{\nabla}_i S. \tag{21}$$

The particles interact through the potential $V_{\text{tot}} = V + V_{\text{Q}}$, where now

$$V_{\text{Q}}(\mathbf{x}_1, \mathbf{x}_2, t) = -\frac{\hbar^2}{2m_1 R}\nabla_1^2 R - \frac{\hbar^2}{2m_2 R}\nabla_2^2 R. \tag{22}$$

They follow well-defined trajectories governed by (21) or, equivalently,

$$m_i\frac{d^2\mathbf{x}_i}{dt^2} = -\boldsymbol{\nabla}_i V_{\text{tot}}(\mathbf{x}_1, \mathbf{x}_2, t). \tag{23}$$

The probability density that, at time t, the first particle is at $\mathbf{x}_1$ and the second particle is at $\mathbf{x}_2$ is given by $R^2(\mathbf{x}_1, \mathbf{x}_2, t)$. The equation that generalizes (9) represents the conservation of probability, and it ensures that the evolution of the probability density due to the particles' motion is consistent with the evolution of the wave function.

The most general solution of the two-particle Schrödinger equation (19) can be written as a sum of products of functions of the form

$$\Psi(\mathbf{x}_1, \mathbf{x}_2, t) = \sum_i \phi_i(\mathbf{x}_1, t)\alpha_i(\mathbf{x}_2, t). \tag{24}$$

Let us assume that, for some interval of time, the potential V is the sum of two terms that each involve the coordinates of one particle only, that is,

$$V(\mathbf{x}_1, \mathbf{x}_2, t) = V_1(\mathbf{x}_1, t) + V_2(\mathbf{x}_2, t). \tag{25}$$

Let us further assume that, at some time t_0 in that interval, the wave function Ψ is a product state, which means that there is only one term in the right-hand side of (24). In other words,

$$\Psi(\mathbf{x}_1, \mathbf{x}_2, t_0) = \phi(\mathbf{x}_1, t_0)\alpha(\mathbf{x}_2, t_0). \tag{26}$$

It is then easy to show that the wave function remains a product state for as long as V satisfies (25), with ϕ and α satisfying one-particle Schrödinger equations associated with potentials V_1 and V_2, respectively.

If we write

$$\phi = R_\phi\exp(iS_\phi/\hbar) \quad \text{and} \quad \alpha = R_\alpha\exp(iS_\alpha/\hbar), \tag{27}$$

one immediately sees that

$$R(\mathbf{x}_1, \mathbf{x}_2, t) = R_\phi(\mathbf{x}_1, t)R_\alpha(\mathbf{x}_2, t) \tag{28}$$

and

$$S(\mathbf{x}_1, \mathbf{x}_2, t) = S_\phi(\mathbf{x}_1, t) + S_\alpha(\mathbf{x}_2, t). \tag{29}$$

Equation (28) implies that the quantum potential V_{Q} in (22), and therefore the total potentiel V_{tot}, are sums of one-particle terms. The first particle's Bohmian trajectory is therefore independent of the second particle's coordinates, and vice versa. This conclusion also follows from (21) and (29).

In the general case where V is not the sum of one-particle terms, however, or even when it is such a sum but the initial wave function is not a product state, then the wave function at time t is given by (24) with the right-hand side having more than one term. In this case, (28) and (29) do not hold and the first particle's Bohmian trajectory will in general depend on the second particle's coordinates. This, we will see, is what accounts for the long-distance correlations.

C. The measurement problem

In a measurement interaction, the initial state of the quantum system and apparatus is a product state, which transforms into an entangled state according to (4). The problem consists in understanding why is the joint system described by only one term of the superposition.

In Bohmian mechanics, the particle whose observable is to be measured and the apparatus pointer both have well-defined positions at $t = 0$, before the measurement interaction begins. As the interaction unfolds, they follow trajectories governed by (21), to end up again at well-defined positions at $t = T$. The position of the pointer at that time is directly interpreted as the apparatus reading, which is entirely well-defined.

We can calculate the probability $P(i)$ that, at time T, the pointer shows the value α_i. This is obtained through the marginal distribution of the pointer observable, equal to the average over the particle's coordinates of the absolute square of the total wave function at T. Making use of the orthogonality of the eigenfunctions $\phi_j(\mathbf{x})$, we find that

$$\begin{aligned}\int d\mathbf{x}\,|\Psi(\mathbf{x}, \xi)|^2 &= \sum_{i,j} c_i^* c_j \alpha_i^*(\xi)\alpha_j(\xi)\int d\mathbf{x}\phi_i^*(\mathbf{x})\phi_j(\mathbf{x}) \\ &= \sum_j |c_j|^2|\alpha_j(\xi)|^2. \end{aligned} \tag{30}$$

Since the pointer's wave functions $\alpha_j(\xi, T)$ are essentially non-overlapping, the probability $P(i)$ that the pointer's position is within the support of α_i is equal to $|c_i|^2$, which is the statement of Born's rule.[6]

[6]If the measurement interaction does not yield orthogonal particle states, a similar argument can be made using the orthogonality of the final states of the environment.

When the measurement interaction is over, the pointer's wave functions α_j remain orthogonal for as long as one may care. This, in fact, is due to the myriad of degrees of freedom of the pointer other than ξ, whose evolution is different corresponding to different pointer positions. If the pointer has entered wave packet α_i, therefore, it will never end up in a different α_j. To do so, it would have to go through a region of configuration space associated with zero probability. The Bohmian trajectories of both the particle and apparatus henceforth develop as though only the i^{th} term was present. The other ones still are, but they have no effect whatsoever on subsequent trajectories. Although the wave function has never collapsed, the system evolves as if it had.

D. Long-distance correlations

As I summarized in [42], one can incorporate spin in Bohmian mechanics by adding spinor indices to the wave function, in such a way that $\Psi \to \Psi_{i_1 i_2}$. There can be several ways to associate particle spin vectors with the wave function [39], but one way or other they involve the expressions

$$\mathbf{s}_1 = \frac{\hbar}{2\Psi^\dagger\Psi}\Psi^\dagger\boldsymbol{\sigma}_1\Psi, \qquad \mathbf{s}_2 = \frac{\hbar}{2\Psi^\dagger\Psi}\Psi^\dagger\boldsymbol{\sigma}_2\Psi. \tag{31}$$

Here $\boldsymbol{\sigma}_1$ and $\boldsymbol{\sigma}_2$ are Pauli spin matrices for the two particles.

In the singlet state, the initial wave function typically has the form

$$\Psi = \phi_1(\mathbf{x}_1)\phi_2(\mathbf{x}_2)|\chi\rangle, \tag{32}$$

where $|\chi\rangle$ is given in (5). With such a wave function, it is easy to show that $\mathbf{s}_1 = 0$ and $\mathbf{s}_2 = 0$. That is, both particles initially have spin zero. This underscores the fact that in Bohmian mechanics, values of observables outside a measurement context do not in general coincide with eigenvalues of associated operators.

Spin measurement was analyzed in detail by Dewdney *et al.* [43], [44]. In the EPR context, in particular, these investigators first wrote down the two-particle Pauli equation adapted to the situation shown in Fig. 1. With Gaussian initial wave packets ϕ_1 and ϕ_2, the equation can be solved under suitable approximations. Bohmian trajectories can then be obtained by solving (21). These equations of motion involve the various components of the two-particle wave function in a rather complicated way, and must be treated numerically.

Suppose that the magnetic field in the spin-measuring apparatus on the left of Fig. 1 is oriented in the $\mathbf{n}$ direction. Consider the case where particle 1 enters that apparatus much before particle 2 enters the one on the right-hand side. What was shown was the following. When particle 1 enters the apparatus along a specific Bohmian trajectory, the various forces implicit in (21) affect both the trajectory and the spin vector, the latter building up through interaction with the magnetic field. The beam in which particle 1 eventually ends up depends on its initial position. If particle 1 ends up in the upper beam of the spin-measuring apparatus, its spin becomes aligned with $\mathbf{n}$. Meanwhile there is an instantaneous action on particle 2, simultaneously aligning its spin in the $-\mathbf{n}$ direction. Similarly, if particle 1's initial position is such that it ends up in the lower beam, its spin becomes aligned with $-\mathbf{n}$, and the spin of particle 2 simultaneously aligns in the $\mathbf{n}$ direction.

Thus the nonlocal forces, present in Bohmian mechanics as a consequence of the nonfactorizability of the wave function, have, once the measurement of the spin of particle 1 has been completed, resulted in particle 2 having a spin exactly opposed. A subsequent measurement of the spin component of particle 2 along $\mathbf{n}$ then reveals the perfect correlation predicted by quantum mechanics.

VII. Everett's relative states

Everett's relative states, or many-worlds, interpretation is an attempt to meet the challenge of interpretation while eschewing the introduction of the collapse of the wave function or of hidden variables. The wave function is taken to apply to individual systems and is meant to represent the true state of the quantum system at all times. Everett also claimed to be able to deduce Born's rule from the other postulates of quantum mechanics. That claim has been the subject of much controversy [45], but its analysis falls outside the scope of the problems raised here.

Everett considers the wave function of a compound system after a quantum measurement, represented for instance by the right-hand side of (4). Confronted with the fact that all pointer readings appear, Everett takes the bull by the horns and claims that they indeed all exist. They all exist, but each reading (say α_j) is associated with only one value of the quantum observable (in this case q_j). Everett calls $\phi_j(\mathbf{x})$ and $\alpha_j(\xi)$ *relative states*. In other words, the value q_j exists relative to α_j, and vice versa.

Since all pointer readings exist at once, understanding that multiplicity is a crucial question, to which many answers have been given. For simplicity, I shall focus on the one usually attributed to DeWitt [46], called the many-worlds view. Although that answer was frequently criticized as extravagant, it has the merit of being perhaps the clearest one.

In the many-worlds view, whenever there is a quantum measurement, the world in which the measurement is initiated splits into a large number of worlds.[7] There is at least one such world corresponding to each term in the right-hand side of (4). In world j, for instance, the quantum system ends up in state $\phi_j(\mathbf{x})$ and the apparatus in state $\alpha_j(\xi)$. That world henceforth continues to evolve according to the Schrödinger equation, in a way completely independent of the other ones.

This is Everett's solution to the measurement problem. There is no need for collapse because different readings occur in different worlds. Everett also shows that if the same quantum observable is repeatedly measured, every observer in every world will record that the results are repeated identically, just as quantum mechanics with collapse predicts.

Long-distance correlations are also explained quite straightforwardly in the many-worlds view. Suppose that two particles

[7]Some believe that splitting occurs whenever there is a quantum interaction, not necessarily one involving a macroscopic object.

have been prepared in the singlet state (5), and that Alice and Bob each have spin-measuring apparatus in initial states $|\alpha_0\rangle$ and $|\beta_0\rangle$, respectively. The compound system's initial state is thus given by

$$\frac{1}{\sqrt{2}}\left\{|+;\mathbf{n}\rangle|-;\mathbf{n}\rangle - |-;\mathbf{n}\rangle|+;\mathbf{n}\rangle\right\}|\alpha_0\rangle|\beta_0\rangle. \qquad (33)$$

After each particle has interacted with its measurement apparatus, the final state of the compound system is, in obvious notation, given by

$$\frac{1}{\sqrt{2}}\{|+;\mathbf{n}\rangle|-;\mathbf{n}\rangle|\alpha_+\rangle|\beta_-\rangle - |-;\mathbf{n}\rangle|+;\mathbf{n}\rangle\}|\alpha_-\rangle|\beta_+\rangle. \qquad (34)$$

The splitting into many worlds yields worlds where Alice's pointer shows $+$ and Bob's pointer shows $-$, and worlds where Alice's pointer shows $-$ and Bob's pointer shows $+$. There are no worlds where Alice's and Bob's pointers both show $+$, nor are there worlds where they both show $-$. Hence in all worlds, correlations predicted by standard quantum mechanics are perfectly satisfied.

VIII. Cramer's transactional interpretation

Cramer's transactional interpretation [17], [47] postulates that quantum processes (e.g., the emission of an alpha particle, followed by its absorption by one of several detectors) involve the exchange of offer waves (solutions of the Schrödinger equation) and confirmation waves (complex conjugates of the former). The confirmation waves propagate backward in time. Cramer's approach is inspired by the Wheeler-Feynman electromagnetic theory [35], [48], in which advanced electromagnetic waves are as important as retarded waves. The wave function and its complex conjugate are thus real fields, very much like the classical electric and magnetic fields.

Suppose that D, at point $\mathbf{x}$, is one of a number of detectors that can absorb the particle. The offer wave, emitted at t_0 from the alpha particle source, will arrive at D with an amplitude proportional to $\psi(\mathbf{x},t)$, the Schrödinger wave function. The confirmation wave produced by D is stimulated by the offer wave, and Cramer argues that it arrives back at the source with an amplitude proportional to $\psi(\mathbf{x},t)\psi^*(\mathbf{x},t) = |\psi(\mathbf{x},t)|^2$. Similar offer and confirmation waves are exchanged between the source and all potential detectors, and all confirmation waves reach the source exactly at t_0, the time of emission. Eventually, what Cramer calls a *transaction* is established between the source and one of the detectors, with a probability proportional to the amplitude of the associated confirmation wave at the source. The quantum process is then completed.

The transaction is, in Cramer's approach, what corresponds to the collapse of the wave function in standard quantum mechanics. Like collapse, the transaction picks just one of the pointer positions (which corresponds, in our example, to the detector that has fired). But unlike collapse, the transaction does not occur at a specific time. It occurs on the whole space-time region that links the source and the detector, in what Cramer calls *pseudotime*.

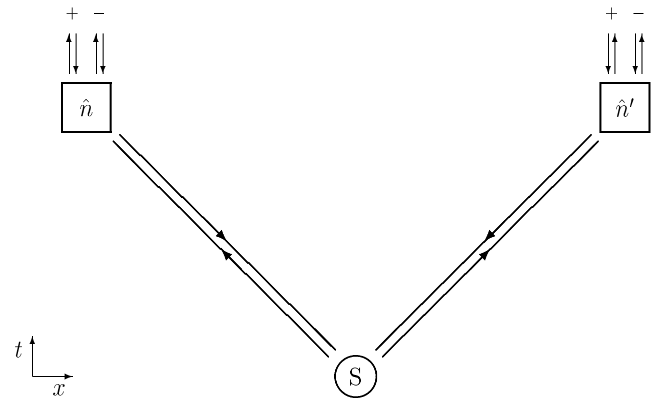

Fig. 3. Offer waves (upward arrows) and confirmation waves (downward arrows) in the EPR setup.

The transactional interpretation provides a rather vivid representation of the mechanism of long-distance correlations. Fig. 3 is a space-time representation of an EPR setup, viewed in the transactional interpretation [42]. Arrows pointing in the positive time direction label offer waves, and those pointing in the negative direction label confirmation waves. Two particles are emitted by the source, and in Cramer's sense both Alice's and Bob's particles can be absorbed by two detectors. They correspond to the two beams in which each particle can emerge upon leaving its spin-measuring device.

Let us focus on what happens on the left-hand side. An offer wave is emitted by the source, and in going through the spin-measuring device it splits into two parts. One part goes into the detector labelled $+$, and the other goes into detector $-$. Each detector sends back a confirmation wave, propagating backward in time through the apparatus and reaching the source at the time of emission. A transaction is eventually established, resulting in one of the detectors registering the particle. A similar process occurs on the right-hand side, with one of the two detectors on that side eventually registering the associated particle.

If offer and confirmation waves represent a special kind of causal influences, one can see that these influences can be transmitted between the spacelike-separated detectors on different sides along paths that are entirely timelike or lightlike. The EPR correlations are thus explained without introducing any kind of superluminal motion, which is one more way to meet Feynman's challenge.

IX. Discussion

Bohmian mechanics, the many-worlds view, and the transactional interpretation are three possible answers to the question of how can the world be for quantum mechanics to be true. Bohmian mechanics tells us that microscopic particles follow deterministic trajectories influenced by the quantum potential. The many-worlds view asserts that all results of a quantum measurement simultaneously exist, but in different worlds that cannot communicate with each others. The transactional interpretation tells us that backward-in-time connections are effected through the complex conjugate Schrödinger field, and that transactions are established between emittors and specific detectors.

Of course these interpretations of quantum mechanics, just like others that I have not considered explicitly, also have problems, since none of them has gained universal acceptance. Bohmian mechanics, for instance, is not easy to reconcile with the theory of special relativity. The many-worlds view is often deemed extravagant, while more benign implementation of Everett's approach may not be so well-defined. And the notion of transaction needs to be spelled out more precisely.

Apart from specific criticisms, the whole program of interpreting quantum mechanics has been questioned by adherents of the epistemic view. Why bring forward interpretations that add no empirical content to the theory? If, for instance, Bohmian mechanics exactly reproduces the statistical results of quantum mechanics, aren't the trajectories superfluous, and shouldn't they be discarded? The analogy has been made between such trajectories and the concept of the ether prevalent at the turn of the twentieth century [49], [50]. H. A. Lorentz and his contemporaries viewed electromagnetic phenomena as taking place in a hypothetical medium called the ether. From this, Lorentz developed a description of electromagnetism in moving reference frames, and he found that the motion is undetectable. Following Einstein's formulation of the electrodynamics of moving bodies, the ether was recognized as playing no role, and was henceforth discarded. So should it be, according to most proponents of the epistemic view of quantum states, with interpretations of quantum mechanics that posit observer-independent elements of reality like Bohmian trajectories. They predict no empirical differences with the Hilbert space formalism, and should therefore be discarded.

It is true that, just like the ether in special relativity, constructs like Bohmian trajectories don't lead to specific empirical consequences. I have argued, however, that although they could be dispensed with in the hypothetical world of Sec. IV, they cannot in the real world unless, just like the ether was eventually replaced by the free-standing electromagnetic field, they are replaced by something that can account for the structure of macroscopic objects.

In all physical theories other than quantum mechanics, there are straightforward and credible answers to the question raised above, of "How can the world be for the theory to be true?" In quantum mechanics there are a number of answers, like the ones we have reviewed in this paper. None is straightforward, and none gains universal credibility. Should we then adopt the attitude of the epistemic or related views, which decide not to answer the question? I believe that, from a foundational point of view, this is not tenable. For how can we believe in a theory, if we are not prepared to believe in any of the ways it can be true, or worse, if we do not know any way that it can be true?

The epistemic view of quantum mechanics is an attempt to solve or attenuate the foundational problems of the theory. It would succeed if quantum mechanics were used only to explain nonclassical correlations between macroscopic objects. But it is also used to explain the microscopic structure of such objects. Interpreting the theory means finding ways that it can be intelligible. I believe that each clear and well-defined way to do so adds to the understanding of the theory. In many instances, however, much work remains to be done to achieve that clarity and precision.

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