A Shape Grammar with Feedback Generative Model for the Design of Compact Microstrip Antennas

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***Abstract*—A shape grammar based model that generates functional initial microstrip antenna designs is developed. The functionality of the designs is ensured with the inclusion of a feedback mechanism that influences the rule selection process. The grammar itself decomposes a complex shape into a chain of rectangles, whose parameters are used in a mathematical formula model that makes up the feedback mechanism. This approach to antenna design is demonstrated with the generation of compact microstrip antenna structures.**

***Keywords*-Shape Grammar; Compact Microstrip Antenna; Feedback; Intelligent Computer Aided Engineering;**

I. INTRODUCTION

The design of compact microstrip antennas is by far a manual task and in particular requires the antenna engineer to combine, articulate and evolve in a constrained volumetric space, a number of geometric shapes that yield the desired electrical characteristics. Fig.1 shows five examples of increasing design complexity. The first design is a standard full size single-band rectangular microstrip antenna that is not restricted in size and its design is a straightforward process that can be readily formalised, since the shape is always that of a parametric rectangle. The antenna in Fig.1(b) is a compact version of a larger single-band rectangular antenna. A narrower and longer rectangular patch is meandered into an S-shape. The aspect ratios of the various sections can be varied, but the shape remains the same, however distorted it may look. Therefore for example adding an extra twist in the shape cannot be simply done with a change in the value of one parameter. The antenna in Fig.1(c) is a dual-band single feed compact design which is then modified to accommodate within the same volume a separately fed third band, Fig.1(d). The design process for the latter prototypes is not obvious and difficult to formalize. Additionally the designer is rarely free to decide on the positions of the feed and shorting posts (shown as dots in fig.1) as these are generally dictated by other system design considerations. This means that the shape has to fit around the position of these components and not the other way round. The antenna in fig.1(e) is an example of a reconfigurable antenna that can be switched between wide-band and narrow-band operation. In such a case it is desired to use the minimum number of switches, while at the same time, satisfy the characteristics at a discrete number of frequency bands. The synthesis task for this case is even more difficult to formalise. Furthermore the design task is a complex one because the electrical characteristics are strongly coupled to the form and shape of the antenna structure and a small change in the topology or form can result not only in a significant change in performance, but can also render the design invalid. Because of these two reasons such designs are carried out by expert designers. Hence modifying a design to suit some additional specifications requires a significant time investment.

During the design process, the antenna engineer has at his disposal two types of Computer Aided Design (CAD) tools, empirically derived mathematical formula based models, and full-wave numerical models. The former type is limited to specific simple geometries, is very fast to compute, and to some extent relates dimensional attributes to the electrical properties. On the other hand the latter is applicable to any arbitrary geometrical shape and is very accurate, but is computationally intensive and does not give an explanation of how the device works and how it can be modified to fit the specifications. During the design of compact antennas the engineer makes use of the simple formula models to select the components and advance the process of shape evolution and manipulation. Once the initial form is developed numerical CAD and optimisation techniques are used to refine the design, or shape. It would therefore be useful to have a design tool that formalizes and mimics some of the informal processes that an antenna designer goes through. This will help speed up the synthesis task.

In [1] the use of a shape grammar with feedback based CAD system to assist the designer in the generation of valid and functional shapes is proposed and this system is demonstrated on narrow constant-width-meander-line microstrip designs. This paper extends this work to include microstrip antennas of variable line widths and therefore shapes similar to the ones in fig.1. The grammar makes use of feedback to yield an estimate of the electrical properties of arbitrary shaped meander-line microstrip antennas.

Shape grammars have been originally developed by Stiny[3] and have been used to generate architectural designs and art works that pertain to a particular style [3], [4]. They were later applied in engineering design for example in the representation of solid objects [5], the design of optimal truss structures [6], the design of coffee makers [7] and design of microresonators [8].

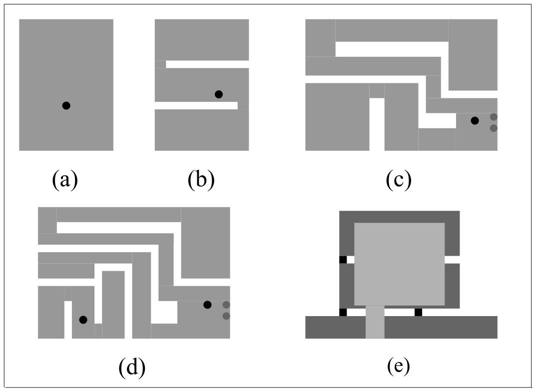


Figure 1. Microstrip and patch antenna designs (a) relatively straightforward to design and describe, (b) this design makes full use of the available space and is harder than (a) to design, (c & d) two harder designs where two modes are matched at the same point, and (e) an even harder problem where the antenna can be switched between wide-band and narrow-band operation [2].

Architectural designs have a well-defined function-form decomposition such that the generating engine first satisfies the functional requirements and then proceeds to generate the physical forms in a particular style. However most engineering artifacts do not have such a straightforward form-function decomposition and are characterised by a considerable function-form coupling. Agarwal and Cagan[9] propose shape grammars as a new framework for geometry-based engineering expert systems as an analysis and synthesis tool. It is further stated that it is the class of products characterised by a strong function-form coupling that stand to gain most from formal design tools because the lack of a pre-defined generation sequence limits human designs to a small set of valid configurations. In [9] a coupled form-function shape grammar is proposed for the design of micro-electromechanical resonators. This is used to generate designs that function as resonators. The functional requirement is enforced by making sure that the grammar includes all the necessary elements that are required for the device to function and validity is ensured in this respect. The form of the device is then altered by choosing the rules at random or through a search process. This however means that the design may not necessarily meet or be close to meeting all the required specifications for the intended applications. However, the goal in this paper is to generate designs that are close to satisfying all specifications without the use of numerical CAD. An approximate model is therefore developed and used within the generative or synthesis cycle. The system is therefore described as a shape grammar with feedback generative system.

The automation of the synthesis task in microstrip antenna design has been mainly approached through the application of the Genetic Algorithm (GA) in search of structures that yield the required specifications. Genetic algorithms are useful to explore poorly-understood search spaces. They are coded with the minimum of domain knowledge and assumptions, and use domain independent genetic operators to explore the search space. They can be very robust and persistent throughout the search but are very slow to converge to a result and are inefficient for local optimization. Systems that rely solely on the GA, therefore, require hundreds of iterations to reach their goal, if a solution exists in the defined search space. Several methods like pruning and structuring of the search space can be applied to improve the GA efficiency for CAD purposes. Some of these methods are domain specific and some are not. Johnson and Yahya Rahmat-Samii pioneered the use of the genetic algorithm (GA) for machine evolved planar microstrip antenna shapes that exhibit wide-band characteristics [10]. In this work a rectangular design space is defined in terms of a $M \times N$ matrix of pixels and the GA is allowed to search new and unknown shapes that exhibits the required specifications. Jones and Jones developed a wire antenna array language that defines the structure of Yagi antenna arrays and combines this with a genetic programming (GP) algorithm [11]. The setup is used to obtain a machine evolved classical Yagi structure as well non standard Yagi designs, where the GP is allowed to try new configurations. A similar experiment is reported by Koza [12] for loaded co-linear wire antennas and uses the concept of a turtle that drops elements along a linear track. In [13], a Knowledge-Based Genetic Algorithm (KBGA) is developed to reduce the time required to evolve novel microstrip antennas. The KBGA makes use of the abstract shape representation described in [10] and selects antenna design heuristics (rules of thumb) that influence the genetic operators. The KBGA is similar to a language of design techniques, but does not deal directly with shape. In [14] the same encoding and crossover techniques are used for the purpose of miniaturization. In [15] it is shown experimentally that this encoding method is not suitable to evolve compact structures of the types shown in Fig.1(c& d). However this does not mean that other encoding techniques do not yield results. The approaches described above are limited in three ways (a) These methods call a numerical model at each iteration and this slows down the process, (b) the quest of these papers is machine evolved designs rather than acting as a partner within the design team and therefore there is little need to link any shape modification to its properties, and (c) these methods do not deal with the geometric shape in a direct way.

In this paper the shape grammar is primarily used as an automated generative tool, where the system chooses rules to be applied based on immediate feedback on the electrical characteristics of the partial design and on the specifications to be met by the final design. The feedback is obtained through the use of the shape grammar as an analysis tool. The emergent shapes in the design are recognized and linked to an analytical equation that yields the electrical characteristics. Recognition of the emergent shapes is carried out implicitly during the shape evolution process. This is a similar process to the procedure adopted by the human designer where first order models are used to evolve the conceptual or initial shape or form. The usefulness of this methodology lies in the fact that a large number of designs valid for the intended application can be machine generated allowing a system to explore a wide variety of configurations.

The rest of the paper is organized as follows: The microstrip antenna shape grammar is first described followed by a discussion on how shape attributes and grammar labels are used to obtain an approximate electrical analysis of the structure; next, the whole process is demonstrated with an example; in the final section conclusions and future work are discussed.

II. THE COMPACT MICROSTRIP ANTENNA SHAPE GRAMMAR

A shape grammar consists of four components; the initial shape, a finite set of shapes, a finite set of symbols and a finite set of shape rules. A labeled shape is defined as a shape augmented by symbols. By successively applying the transformation rules to an evolving shape, a design is derived in the language specified by the shape grammar. Parametric shape grammars extend shape grammars to deal with shapes whose dimensions can be arbitrarily specified. Additionally *weights* can be specified for the various shapes and used to redefine shape boolean operations. *Weights* ensure that certain components are not written off by newly introduced components. The work described in this paper makes extensive use of symbols, while weights and parametric shapes are implicitly included in the rules.

The compact microstrip antenna shape grammar is to generate designs that fit in the 2D design space available and satisfy the electrical specifications to the point that the structures generated can be either efficiently optimized with the aid of a full-wave numerical model or combined further to evolve reconfigurable structures.

In order to fulfill these tasks this system should (1) ensure the inclusion of all the components necessary for the antenna to function, (2) explore the space available so as to be able to generate the shapes, and (3) trace the electrical current paths from the driving point to the radiating edges so as to be able to obtain approximate quantities for the electrical characteristics.

In this paper the grammar is limited to describe patch shapes that can be generated in 2D from an array of small square shapes or *pixels*. Fig.2 shows some example shapes that can be generated from an 12×10 array of square pixels. The dimensions of the square pixel and the size and boundaries of the array are variable and this arrangement can scale-up and accommodate many real-life cases. This system of shape representation is chosen to simplify the problem of reasoning and interpretation of shape as well as shape algebra. This representation has been often applied in studies that make use of a genetic algorithm to evolve a shape that yields the required electrical and physical characteristics, such as in device miniaturization [14] or for wide-band designs [10].

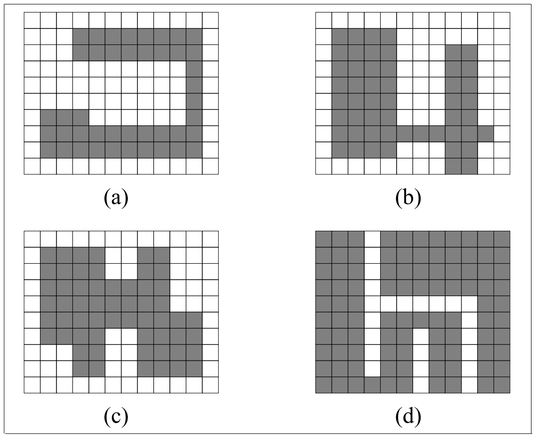


Figure 2. Shapes generated from a matrix of square pixels. The grammar described in this paper limits the number of shapes that can be generated. Shapes (a) and (d) conform with the grammar in this paper.

In this section the basic grammar that generates one pixel-wide meander line shapes and explores the design space is described first. The grammar is then extended to evolve these initial shapes to shapes characterised by wider sections, like the ones shown in fig.1(c & d) and fig.3(c). This is followed by an explanation on how feedback is coupled with the shape grammar to synthesize or generate valid shapes and results are used to demonstrate the system.

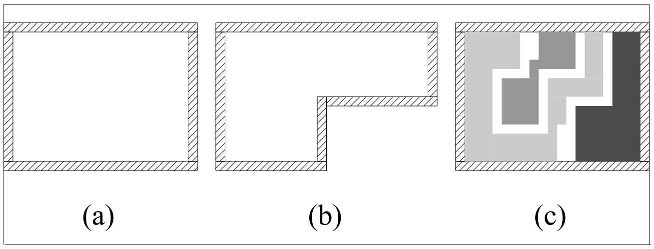


Figure 3. Examples of 2D design spaces (a) rectangular shape, (b) irregular space, and (c) Space leftover by other antenna elements - a 2D-space optimisation problem.

A. The Basic Shapes and Symbols

The proposed grammar is a parametric 2D grammar augmented by symbols. Fig.4 shows the basic shapes and symbols defined in this grammar. The basic shape is the square pixel and rectangles emerge from the union of adjacent pixels and are labeled as such. In this grammar rectangles are defined in terms of pixels rather than as a

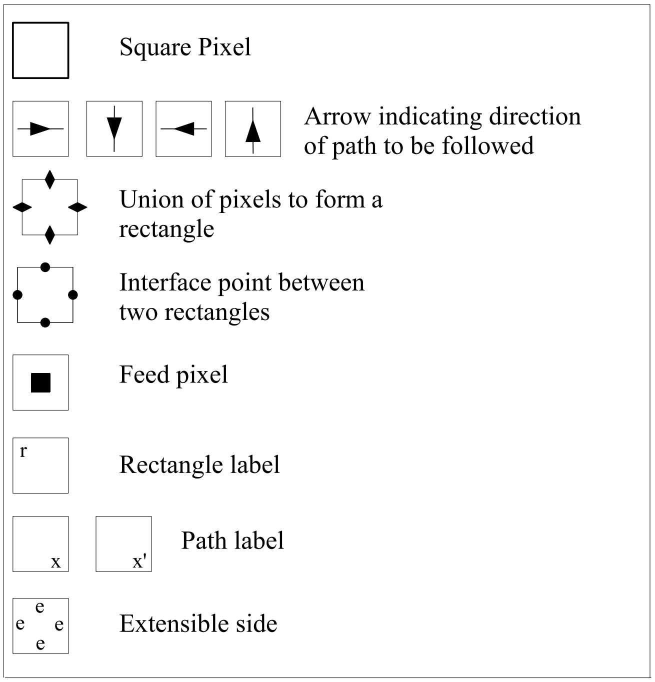


Figure 4. Grammar elements symbols and labels.

union of lines, as in [8]. The parametric term in this grammar relates to the width and length of the rectangle in pixels. Fig.5(a) shows an example of a rectangle. The symbol ’r’ is a unique integer number given to an emergent rectangle as defined by the shape rules. The diamond symbol together with the arrows define how the pixels are connected to form a rectangle. The arrows define the general direction to be followed and the diamonds define transverse paths. Fig.5(b) depicts two rectangles adjacent to each other and also defines the interface points, with the *dot* symbol. The rest of the labels and symbols in fig.4 are explained in the next section.

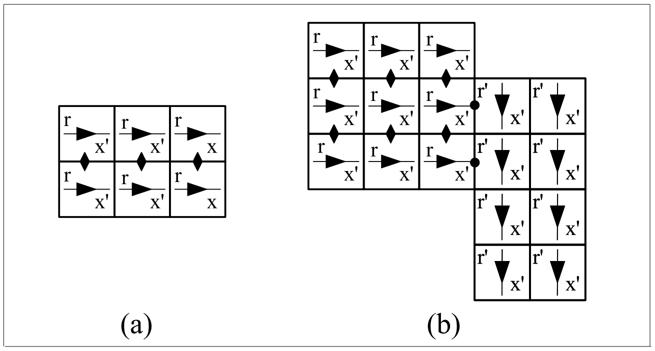


Figure 5. (a) An example of how a rectangle is defined, (b) an example of how adjacent rectangles are defined and connected.

B. *The Initial and Line Grammar Rules*

Physically the compact microstrip antenna is made up of a patch of metal supported on a substrate and attached to a probe feed. The substrate is further backed by a ground-plane. For the structure to perform as an antenna, these four elements must be present; the feed or driving point, the patch, the substrate and ground-plane. Optional components like shorting posts and capacitive patches can also be added. In this grammar the substrate and the ground-plane are assumed to be infinitely large and assumed to be implicitly present. The inclusion of the feed point and the shape are enforced by the grammar. The grammar evolves a shape as two branches coming out of the probe feed or a shorting post. The branches are labeled *a* and *b* and the interface point between them is located at the probe feed or shorting post. This setup is suitable to model structures resonating at the fundamental mode and can also be used to model shorted patch structures. These two types of structures are the most common building blocks used in the design of compact microstrip antennas.

Fig.6 shows the initial rules that define the initial patch shape consisting of a probe feed and two branches. *Rule 1* defines a pixel as a feed pixel. *Rule 2* labels the extensible edges with the label *e*. Extensible edges in this context means that adjacent pixels can be appended to the current pixel given that these do not touch other elements belonging

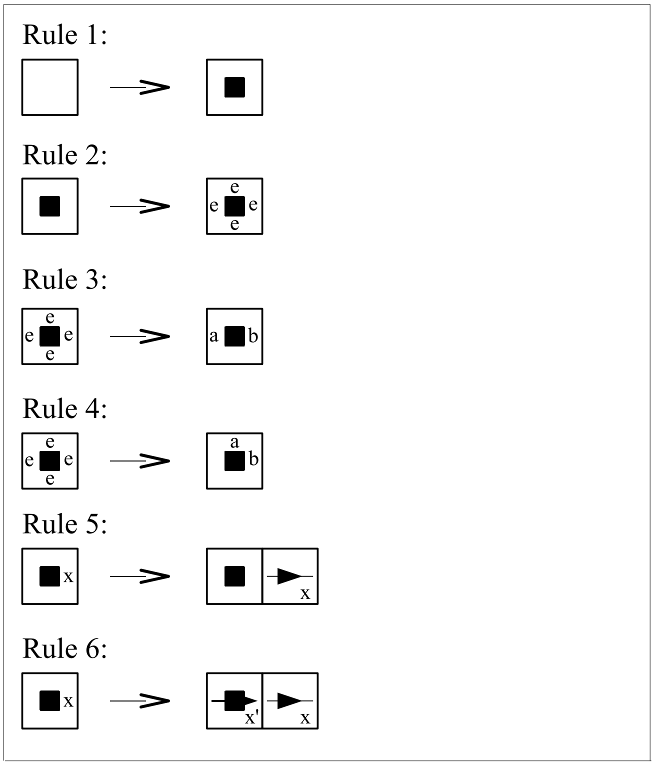


Figure 6. The initial rules.

to other branches or systems. *Rule 3* or *4* define the starting points for the two branches 'a' and 'b'. *Rule 5* attaches a pixel to one of the chosen sides and the first rectangle is defined. *Rule 6* attaches a pixel to the other chosen side and the second rectangle is so defined. This rectangle is composed of two pixels. The initial shape is thus defined and is composed of two branches labeled *a* and *b* and the location of the probe feed. The radiating edges or pixels are defined by labels *a* and *b*. Some possible starting shapes are shown in fig.7.

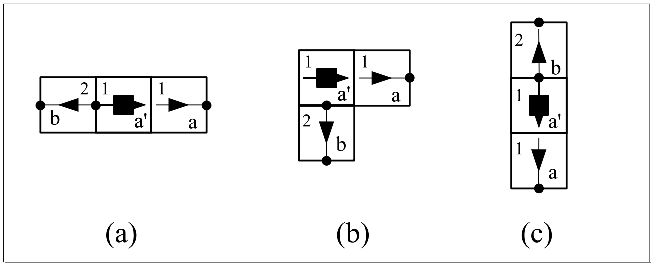


Figure 7. Some initial shapes using the initial shape grammar of fig.6.

The initial shape is further evolved or extended using the line generation rules given in fig.8. The rules are constructed using the shape and label algebra together. The rules as depicted here also assume that the side to be extended is indeed extensible. One way to implement this is to include the states of the neighbouring nodes in the rule, as depicted in fig.9. For the rest of the rules given in this paper the provision of a mechanism that is aware of the state of the neighbouring pixels is assumed and no further comments on this issue are given. Furthermore fig.10 defines three operators that transform rules to suit different orientations and an example of the *rotate* operator is given.

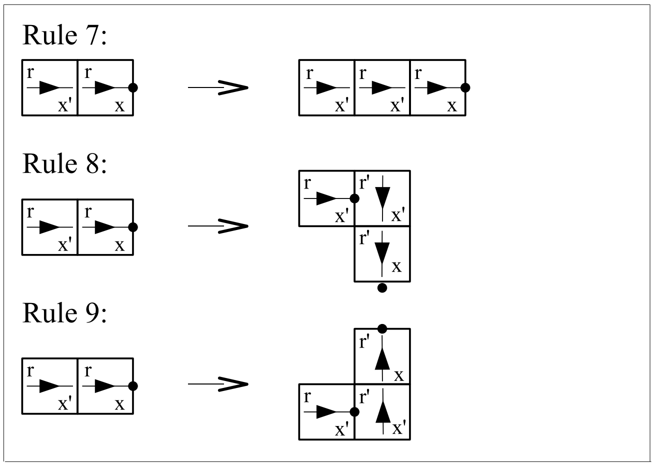


Figure 8. Grammar rules that evolve one pixel-wide meander lines.

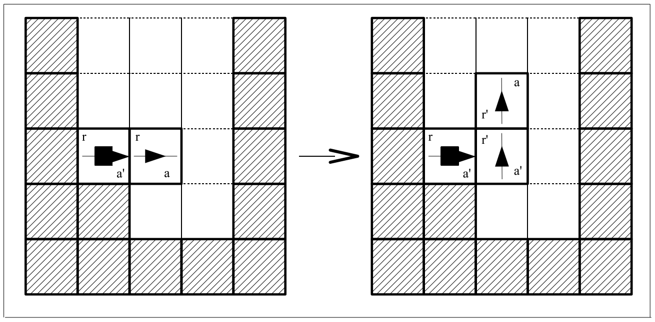
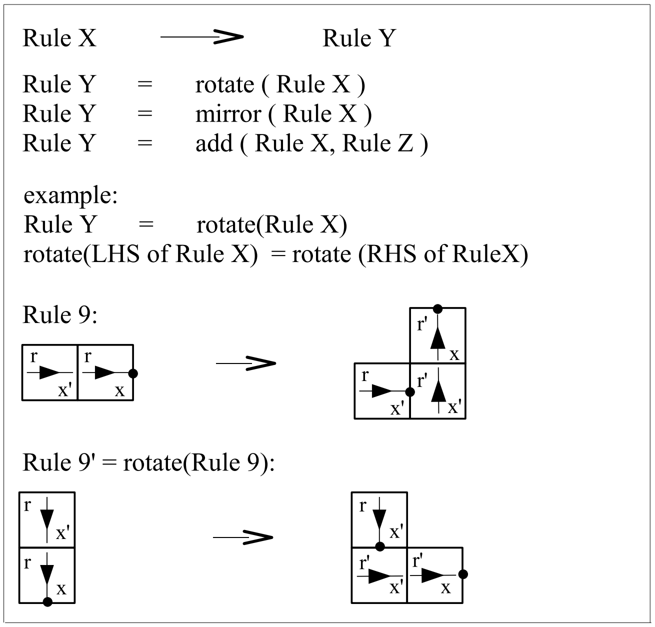


Figure 9. *Rule 9* defined with neighbouring pixels included.


Rule X ——> Rule Y

Rule Y = rotate (Rule X)
Rule Y = mirror (Rule X)
Rule Y = add (Rule X, Rule Z)

example:
Rule Y = rotate(Rule X)
rotate(LHS of Rule X) = rotate (RHS of RuleX)

Rule 9:

Rule 9' = rotate(Rule 9):

Figure 10. Rule tranformation operators and example for the *rotation* operator. This operator does not extend the shape but modifies a rule, such that it is applied in a different orientation.

Fig.11 illustrates the successive application of the *line generation rules* and how rectangles emerge and replace or modify previous ones. The arrow label indicates the direction of where next pixel is to be found. In this case rectangles are limited to a width of one pixel in one dimension. The basic shape grammar of fig.8 therefore generates the shapes discussed in [1]. The grammar ensures that two and only two branches *a*, *b* are always present; the shape evolves by considering the extension of a branch by one pixel; and a branch is extended if the new pixel to be added does not touch any other pixel that is *present* except for the one at the end of the branch. The shape grammar therefore ensures that (a) the overall shape of the patch is a meandered line of width one pixel, (b) there are no loops along the line, and (c) one feed is always present.

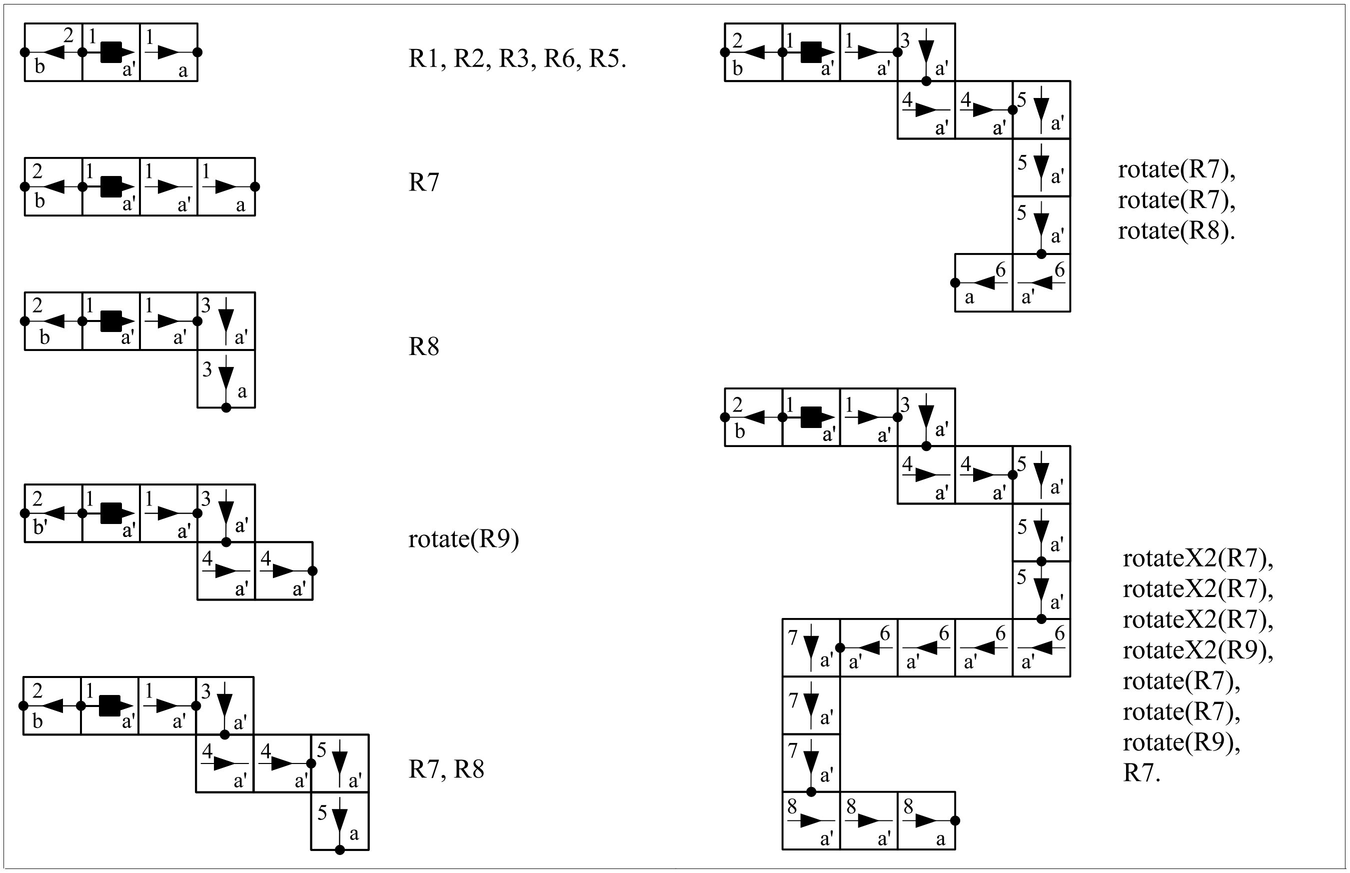


Figure 11. Application of the *line grammar* to evolve a shape.

C. The Extended grammar

The grammar described in the previous section is limited to shapes made of lines that are one pixel wide. It is desired that the grammar generates meander lines that are wider than one pixel and of non-uniform width, commonly used to optimise bandwidth and the driving point impedance. This section extends the grammar with a set of rules that further evolve a shape originally developed by the line grammar and is termed the *extended grammar*. Additionally the line grammar is implicitly used to explore the design space and therefore takes into consideration other concurrent shapes common in multi-band designs. The initial shape for the extended grammar is therefore the final shape evolved with the line grammar rules. The final shape for the extended grammar is a line defined by a chain of rectangles rather than pixels. Subsets of the extended grammar can also be defined to generate specific shapes, like for example rectangular shapes, L-shapes and C-shapes.

The extended grammar starts with a modification to the line grammar. *Rule 4* is removed from the initial rule set. Fig.12 shows *Rule 10* and *Rule 11* that define how the line in the vicinity and including the probe feed is extended.

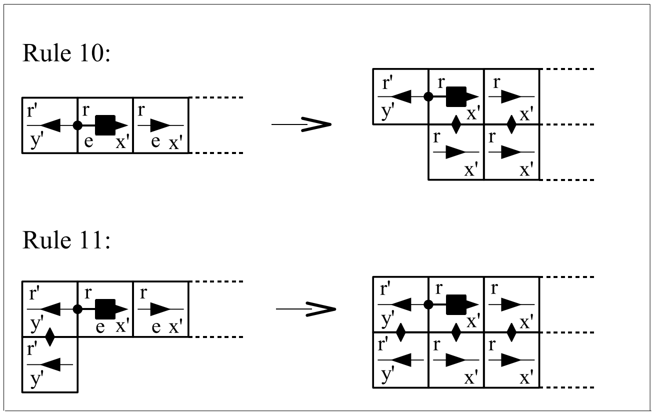


Figure 12. Shape rules in the vicinity of and including the probe feed.

Fig.13 and fig.14 give the rules for widening the line at corner sections, while fig.15 defines how branch ends are widened or extended. The rules given in figs.12, 13, 14 & 15 must be applied in pairs. Fig.16 combines rules to widen a straight run, a U shape and an S-shape. Likewise in fig.17 an S-shape is evolved into a straight run and the branch end

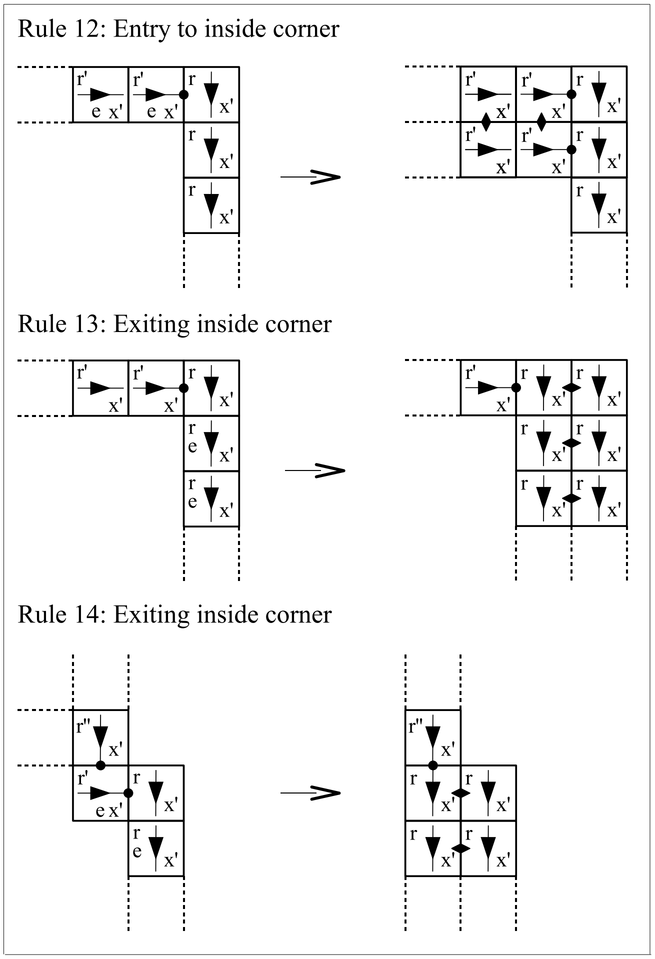


Figure 13. Shape evolution at the inside of corners.

is widened. The rules update the symbols accordingly and the chain of rectangles comes out in an implicit way as a product of the application of the rules. Fig.18 shows how the staircase line evolves into a straight *tapered* run consisting of rectangles of different widths. At this point no further rules can be applied on the right hand side other than *Rule 19* at the end of the branch. The end result may not be desired in practice but it demonstrates how rectangles emerge and replace other rectangles, while at the same time maintain the flow of arrows along the path.

In general an edge is extended as follows: an edge for extension is first chosen, based on extensibility and priority or a random list; this is followed by a search for a rule that matches the section and if the search is successful the change is carried out. Probabilities in the rule selection can be used to influence the path taken by the line grammar. Fig.19 gives some examples of shapes generated by the shape grammar. Subsets of the grammar can also be defined to describe particular classes of shapes, for example L-shape, C-Shape, G-shape. These shapes can be recognized from the number

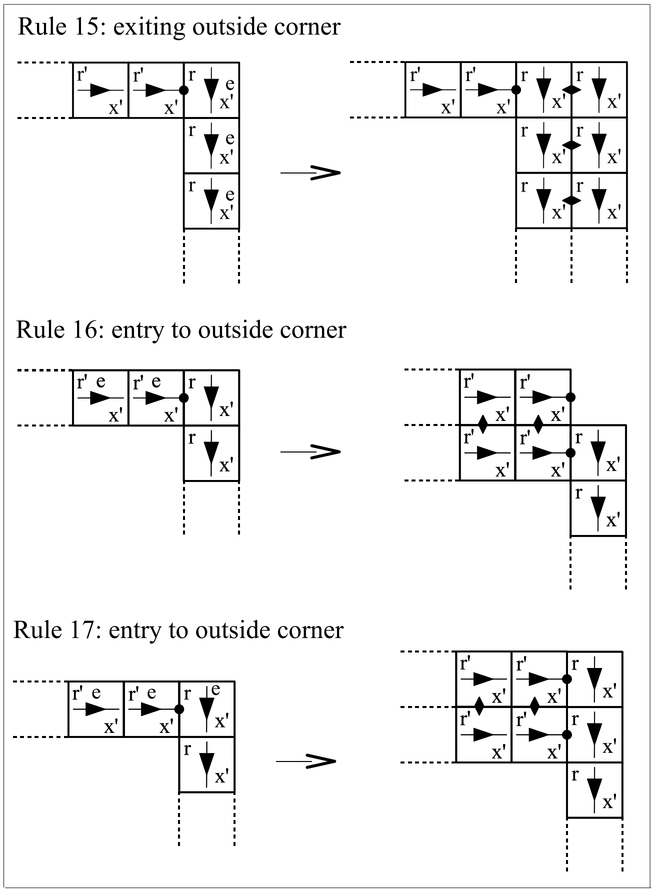


Figure 14. Shape evolution at the outside of corners.

of corners and the sense of arrow directions. These features can be defined as a subset of rules, some of which having a limit on the number of rule instances that can be applied. Other shapes having other features like for example, wider branch ends than the rest of the line can be defined as a sequence of rules, that when applied generate these specific shapes.

D. The Grammar in Analysis

The antenna electrical properties of major interest are the frequency of operation, input impedance, efficiency, radiation pattern and bandwidth. In compact antenna design, radiation patterns are usually not a design variable since the designer has very limited ability to control the patterns [16] and most often exhibit omni-directional properties characterised by considerable cross-polar levels. Efficiency is a function of many variables including materials used and bandwidth is mostly a function of physical configuration, including shape characteristic and is catered for by the sequence of the grammar rules applied as described in the previous section. Furthermore, in practice, the bandwidth is usually enhanced with a matching circuit[17]. The frequency

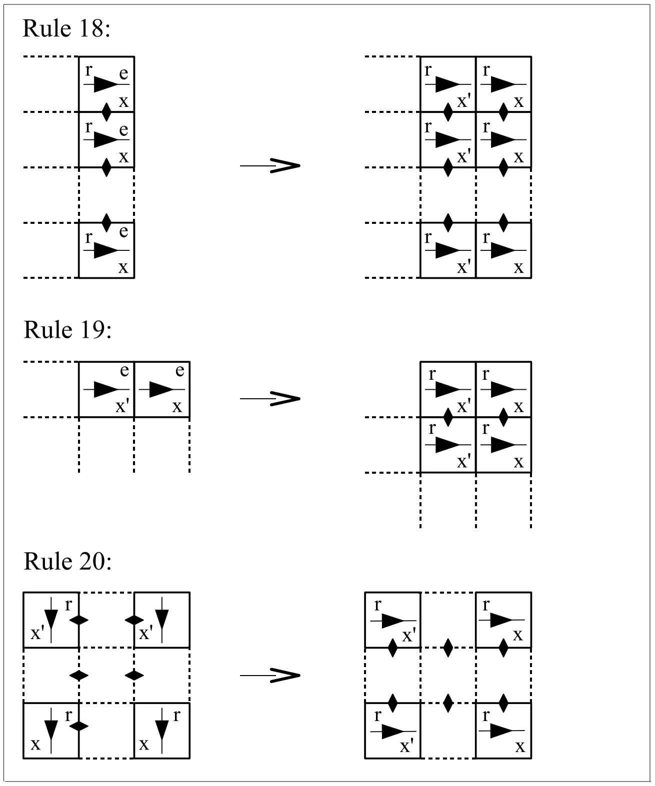


Figure 15. Shape evolution at branch end.

of operation and the input impedance are the two properties that need to be considered explicitly in the analysis task and this section describes how these are estimated.

In theory the microstrip antenna resonates at an infinitely number of modes. However it is mostly used in its fundamental mode which is excited when the size of the patch is roughly half a wavelength long in at least one dimension, or quarter of a wavelength long for shorted patches. This is the concept on which the simple and natural transmission line model for the microstrip antenna is built. The transmission line model treats the antenna as a transmission line with open or shorted ends. Additionally the height of the substrate and size of the ground plane have some minor effects on the value of the frequency of resonance. The input impedance is modeled on a standing wave analysis that gives the impedance as a function of position. The effect of the patch width has a considerable effect on the input impedance and the height of the substrate usually effects the inductive part because of the higher inductance of the probe feed. These models are used as *guides* by the compact antenna designer during the initial design stage where exactness and high accuracy are not the priority. The designer follows *rules of thumb* to relate the shape of the compact antenna to these models and obtains an approximation of the electrical characteristics prior to optimisation using a numerical model. The feedback mechanism in the generative grammar reported in this paper models this analysis process and therefore can be thought of as a formalization of the transmission line model applied to the analysis of the shapes generated by the shape grammar, i.e. shapes that range from straight lines to irregular meander lines.

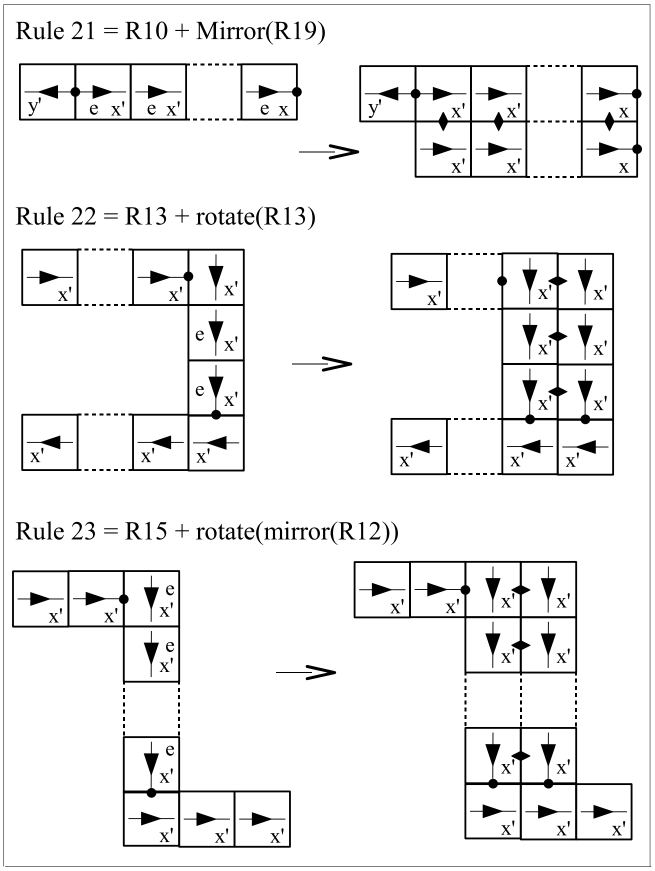


Figure 16. Set one of Modification rules in pairs.

The main task for the algorithm is to analyze the shape and link its *geometrical attributes and dimensions* to the transmission line model. This is carried out by decomposing the shape into rectangles and tracing the current paths that are required to estimate the resonant frequency and the input impedance. These two tasks are carried out implicity by the grammar rules. The next step is to pick up the shape attributes that are used as inputs in the formula models to yield the electrical properties. The previous discussion highlighted that the main attribute used in the calculation of the frequency of resonance as well as the input impedance, is the *effective length* of the current path. Other physical characteristics of the structure that have less effect on the electrical characteristics are the height of the substrate and the shape itself - how *compact* the shape is as well as the aspect ratios of the sub-shapes.

The mechanism of this process is first discussed and

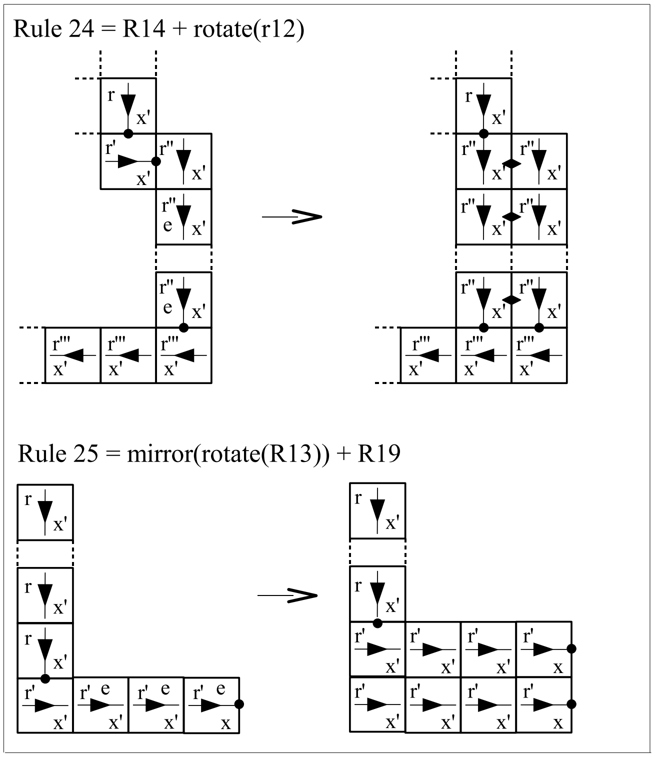


Figure 17. Set two of Modification rules in pairs.

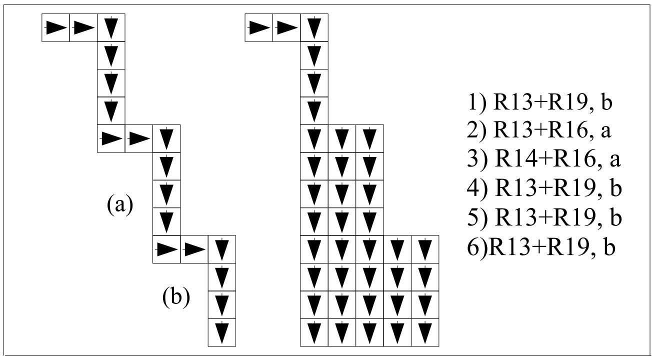


Figure 18. Stair-case line evolved to a quasi straight run by applying the extended grammar

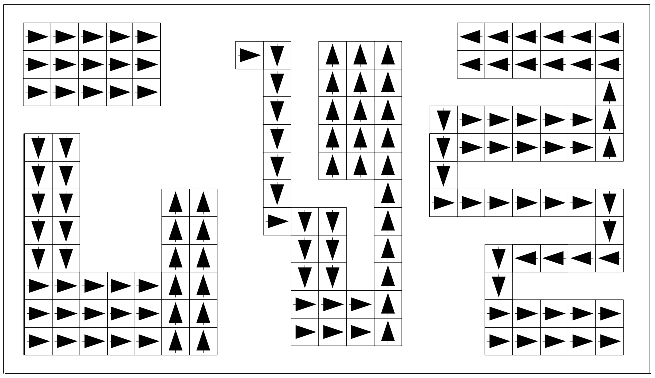


Figure 19. Examples of shapes generated by the shape grammar.

demonstrated for shapes derived from the grammar of fig.8, and later for shapes derived by the whole grammar, i.e including the extended grammar. A set of metrics is adopted and listed below, where reference is made to fig.20.

1) Number of corners along the meandered line. This is split into two. NC_a is equal to the number of corners along branch a, and NC_b is equal to the number of corners along branch b.
2) Length of meandered line in terms of pixels. This is split in two. L_a is equal to the number of pixels between A and B, and L_b is equal to the number of pixels between A and C.
3) Separation in space of the two end points, B and C. This is equal to distance BC.
4) Angle subtended by lines AB and AC. This angle is equal to θ.

Metrics 1 and 2 are used in the calculation of the *effective lengths* and metrics 3 and 4 give an indication of the compactness of the structure.

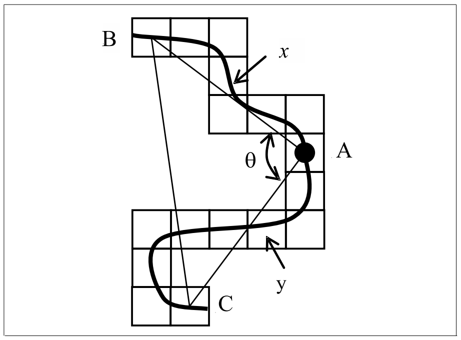


Figure 20. Metrics that are used to obtain properties of the shape.

The next step is to relate the metrics to the electrical properties, so as to be able to carry out the approximate quantitative analysis desired for the feedback mechanism. In general the relationship can be described by a weighted function of the shape properties as,

$$EP_n = \sum \textit{Weighted Shape Metrics}, \tag{1}$$

where EP_n is the electrical property under consideration. For this purpose a set of 20 random elements are generated from the grammar without feedback of fig.8. Fig.21 shows the algorithm used to generate these elements. Table I lists the data collected for the 20 elements. The shape is described by the metrics, and the electrical properties are obtained with a Finite-difference-Time-Domain (FDTD) numerical tool developed by the author [18]. The next step is to select the shape properties that have a strong influence on

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01  Generate initial shape and define branches
02  Choose a branch to extend. The choice is based on
    a random number and the probability of a branch
    being selected more often than the other is variable
03  Check if branch is extensible: If YES repeat step
    02. If no extensible branch can be found terminate
    element
04  Extend branch in a random extensible direction
05  Decide with a probability of $P_{continue}$ whether to
    continue extending the branches. If YES goto step
    02 else terminate element
```

Figure 21. Algorithm Generate Random Elements

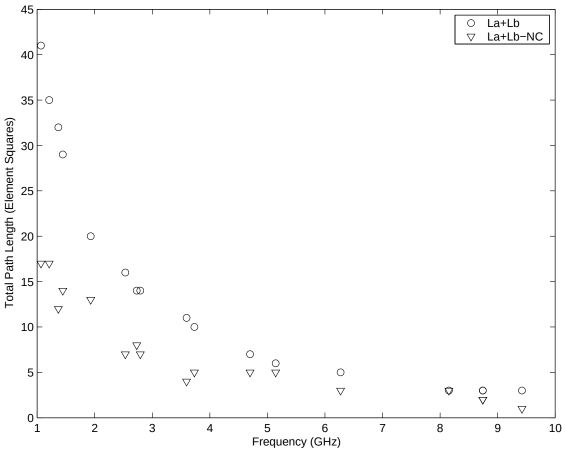

Figure 22. Scatter plot for the resonant frequency.

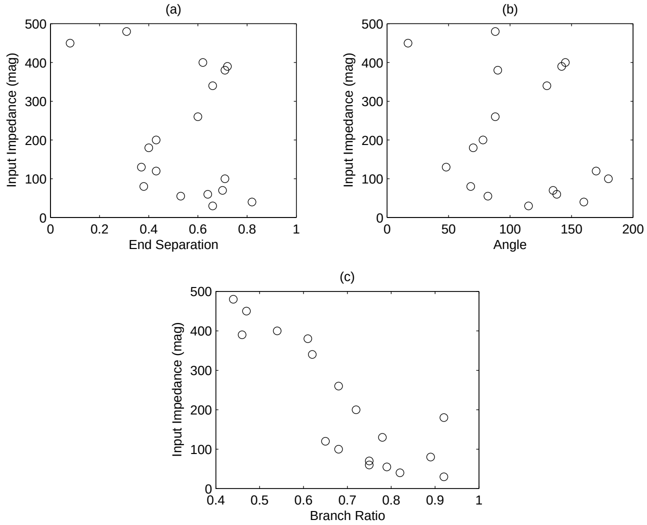

Figure 23. Scatter plots for the input impedance.

each electrical property. In general this could be posed as a regression problem. In this paper, microstrip antenna design experience discussed previously supported by scatter plots are used to guide the selection of the most significant metrics to approximate the resonant frequency and driving point impedance.

The resonant frequency, f_0 is dependent mostly on the effective length of the meandered line given by $(path_x + path_y)$ in fig.20, which in turn is a function of L_a, L_b and NC. Scatter plots for these variables against f_0 confirm a higher dependence on the branch lengths than on the number of corners present, fig.22. The relationship for f_0 derived by considering the simple transmission line model is,

$$\begin{aligned} f_0 &= 3 \times 10^{-4}/((L_a + L_b + 2a_0 \\ &\quad -(NC_a + NC_b) * a_1) * L_p * 2) \end{aligned} \tag{2}$$

The coefficients a_0 and a_1 are obtained by minimizing the error and L_p is the width of the square pixel in millimeters. The angle θ and distance BC are ignored in this equation because the frequency's sensitivity to these is minimal.

The driving point impedance depends mostly on the relative position of the feed point along the current path. Another set of scatter plots, fig.23 confirm that, Z_{in}, depends on the relative position of the probe feed to the ends of the line. The scatter plots for the angle θ and the normalized BC metrics show a weak and inconclusive relationship with Z_{in} and therefore are not included in the model. The model does not yield a numerical value for Z_{in} but gives an indication of how far away it is from 50Ω, the system impedance. The input impedance estimate is obtained by considering the $B0 : B1$ ratio from which it is derived that when the ratio is close to 0.85 the input impedance is in close proximity to 50Ω. These two relationships are used to obtain an estimate for the frequency of resonance and the input impedance level for the structures of I. The set of designs in table I group is termed the *Tuning Set* as it is used to obtain the coefficients a_0 and a_1 in eq.2 using a trial and error approach. The values for the optimised coefficients are $a_0 = 0.6$ and $a_1 = 0.31$. The last two columns in table I give the estimated frequency, and the error. The average error is 2.96% with a standard deviation of 2.03.

A second set elements was generated using the same algorithm as that for the *tuning set*. This second set is used as a *Validation Set*. The average error in the estimated frequency is 3.15% with a standard deviation of 1.85.

The randomly generated elements for the tuning set span the frequency range from approximately 1 to $7GHz$ and a histogram of these shows that their distribution is approximately uniform especially in the 1 to $4GHz$ range. The model is therefore fitted to this wide frequency range. A plot of the averaged error against frequency shows a gentle downward slope favoring the high frequencies. The same observations are noted in the validation set. If a smaller error

Table I
TUNING SET

#	L_a	L_b	NC	NC_a	NC_b	BC	Θ	$B0:B1$	$B0/(B0+B1)$	F_0	Zin	f_0	$Error$
0	10	5	9	6	3	33.4	88	0.56	0.63	2.67	hi	2.72	1.70
1	4	2	1	0	0	17.6	90	0.61	0.66	5.14	me	5.14	0.08
2	4	1	3	2	0	14.5	132	0.47	0.71	6.4	me	6.61	3.34
3	10	3	7	6	1	33.4	129	0.41	0.71	2.85	hi	3.02	5.86
4	12	11	13	7	5	25.2	52	0.97	0.32	1.82	lo	1.82	0.17
5	11	6	6	4	2	32.1	72	0.60	0.50	2.3	hi	2.24	2.60
6	5	4	5	1	3	17.0	81	0.72	0.50	4.2	me	4.14	1.34
7	2	9	6	1	4	32.1	159	0.31	0.79	3.4	me/hi	3.49	2.69
8	4	2	3	2	0	17.6	111	0.69	0.73	5.5	me	5.62	2.22
9	9	3	5	2	2	12.6	64	0.37	0.28	3.05	hi	3.11	2.03
10	12	15	14	6	8	12.0	16	0.83	0.13	1.55	lo	1.55	0.27
11	8	2	7	6	0	28.3	122	0.43	0.80	3.83	hi	3.98	3.83
12	11	4	7	4	2	37.8	137	0.41	0.68	2.64	hi	2.60	1.56
13	15	2	7	6	1	44.1	180	0.20	0.70	2.2	hi	2.28	3.75
14	12	31	18	2	15	20.2	29	0.45	0.13	1.02	hi	0.96	5.78
15	12	9	7	3	3	29.0	57	0.75	0.37	1.92	me	1.84	4.40
16	27	8	23	16	6	36.5	78	0.31	0.32	1.29	hi	1.27	1.36
17	13	10	9	7	1	32.1	55	0.90	0.38	1.8	me	1.72	4.48
18	7	4	2	2	0	22.0	15	0.68	0.48	3.27	me/hi	3.13	4.27
19	18	16	18	11	7	28.3	71	0.95	0.24	1.35	lo	1.25	7.47

is desired the model can be fitted over a smaller frequency range. In this case the grammar will have associated with it a set of coefficient vectors from which the appropriate one has to be selected. In the modelling mode this has to be carried out at least over a two step iteration. However for the initial design phase a smaller error is generally not required.

The model is tested in the synthesis task. The simple synthesis algorithm in fig.24 was developed for this purpose. The task of the algorithm was to develop a structure that resonates at $2.0GHz$ and provides a close match to 50Ω. The algorithm terminates either when the specifications are satisfied or when the shape cannot be changed any more. Further more, since the feed is fixed at a given position, it may be the case that the desired input impedance is unattainable. In this case the algorithm does not attempt to shift the feed. Table II lists ten structures that have been developed by this algorithm. All ten cases were simulated with the FDTD numerical tool and the average error in the resonant frequency is 2.85% with standard deviation of 1.5, whereas the average deviation from the $2.0GHz$ desired is 1.72% with a standard deviation of 1.44. The average error for the input impedances is 31.8% with a standard deviation of 26.2. This figure seems high. However, the input impedance is a very sensitive quantity and for this type of model it should be considered sufficiently accurate. Furthermore the accuracy required depends on where and how the model is utilized in practice. Certainly it is an area for further study.

```
01  Set frequency of resonance and desired input impedance
02  Start Synthesis
03  Generate initial shape at a random position
04  Obtain an estimate for the input impedance
05  If the input impedance estimate is greater than the
    target randomly pick one branch that can be extended,
    else pick the shortest branch that can be extended
06  If neither branch is extensible then terminate synthesis
07  Extend branch in a random extensible direction
08  Obtain an estimate of the resonant frequency: If
    estimate is less than the desired value terminate
    synthesis else repeat as from step 04
```

Figure 24. Algorithm Synthesize Elements

Table II
SYNTHESIS SET

#	Freq Model	Freq FDTD	Prediction Error	Target Error	Branch Ratio	Zin FDTD	Target Error
0	1.954	2.03	3.744	2.300	0.881	53.0	6
1	1.947	2.01	3.132	2.648	0.912	13.0	74
2	1.954	2.02	3.260	2.293	0.963	15.0	70
3	1.986	2.01	1.183	0.689	0.993	35.0	30
4	1.994	2.07	3.690	0.300	0.939	44.0	12
5	1.923	1.91	0.685	3.846	0.851	45.0	10
6	1.994	2.10	5.048	0.300	0.869	57.0	14
7	1.986	2.00	0.689	0.689	0.902	80.0	60
8	1.994	2.08	4.153	0.319	0.869	60.0	20
9	1.923	1.98	2.875	3.846	0.879	40.0	20

The method described above cannot be applied directly to shapes evolved with the extended grammar and a general method that can be applied to any shape is desired. The way the *effective length* is calculated is modified. Fig.25(a) demonstrates how this is done. The *dot* symbols that define the interface points for the rectangles are joined together by straight lines and the summation of the length of these lines gives the initial effective length, which is equivalent to $(L_a + L_b)$ in eqn.2. The final effective length is corrected in the same way as in the previous section, i.e. by considering the number of corners as well as the coefficients a_0 and a_1. To test this method the tuning set is used and the updated coefficients are $a_0 = 0.55$ and $a_1 = 0.02$. The average error is 2.82% with standard deviation of 1.95. The average error for the validation set is 3.32% with a standard deviation of 2.02%.

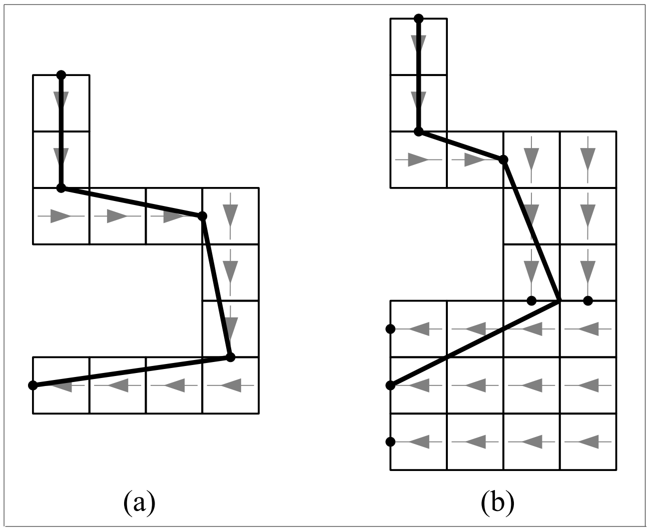

Figure 25. Evaluation of the *effective length*.

The accuracy of the formula model (eqn.2) when applied for shapes evolved with the *extended grammar* is discussed next. Three slightly different approaches are considered in calculating the *effective length* for the branches; (1) $(L_a + L_b)$ is equivalent to the minimum length end to end path passing through the sub-shapes interface points and the feed point, (2) $(L_a + L_b)$ is equivalent to the length of the path passing through the midpoint of each sub-shape interface zone (fig.25(b)), and (3) $(L_a + L_b)$ is equal to the sum of the lengths of all the rectangles that make up the shape. A set of fifty prototypes that operate over a frequency range of 1 to 4 GHz and various shapes ranging from rectangular to L-shapes and meander lines are simulated with the FDTD model and used as a second tuning set. This set is used to optimise the a_0 and a_1 coefficients. For approach (1) $a_0 = 0.60$ and $a_1 = 0.30$ and the average error is 6.73% with a standard deviation of 6.90. For approach (2) $a_0 = 0.55$ and $a_1 = 0.20$ and the average error is 4.72% with a standard deviation of 3.19, and for approach (3) $a_0 = 0.60$ and $a_1 = -0.28$ and the average error is 5.75% with a standard deviation of 8.13. For (1) and (2) the path length is underestimated and therefore the a_1 coefficients are negative. The results show that approach (2) performed best. Intuitively this approach is the closet to approximating the current path along the antenna structure. However approach (3) has the advantage of being less computationally intensive. It is also possible to obtain higher accuracies if the set is restricted to a particular type. The errors are greatest for the most irregular shapes and when the interface width between rectangles is greater than 1 pixel. The simple formula model does not take into account the various widths of the rectangles and how they scale relative to each other. The accuracy is however adequate for its intended application, i.e for the conceptual design stage. This does not mean that a better model is not possible and this issue is further discussed in the last section.

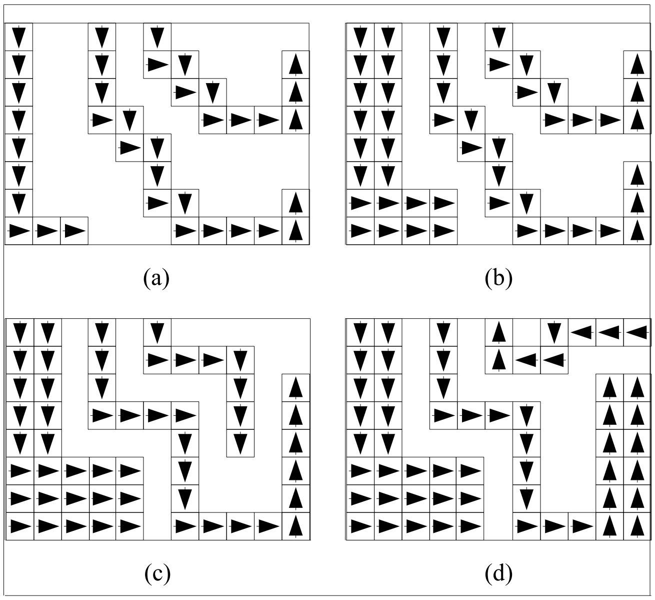

Figure 26. Stages during the design process of a tri-band separately fed structure.

This section ends with a multi-band structure. The design of multi-band structures can be either evolved separately or simultaneously. Fig.26 depicts an extracted sequence of interim designs during the evolution of a three element antenna with separate feed structures. Fig.26(a) is the result of applying the line grammar rules to the three elements simultaneously. In fig.26(b) the structure on the left-hand-side is widened but is short from satisfying the specifications and can't be further evolved. Therefore in fig.26(c) the other designs are removed and the first structure is allowed to evolve, at which point the other structures are evolved again. Fig.26(d) shows the final design, that can be further evolved if necessary. The intention of this design is to operate as a single feed dual-band antenna at $0.925GHz$ and $1.8GHz$

and a separately fed antenna for $2.45GHz$. These frequencies correspond to cellular licensed mobile communications bands and the unlicensed Industry, Scientific and Medical (ISM) band. The frequencies of operation as predicted by the shape grammar in analysis are $1.86GHz$, $1.05GHz$, and $2.47GHz$, while the respective deviations from the target values are 3.5%, 13.2% and 2.1%. These deviations are not due to errors in the analysis model, but are function of the stopping criteria in the application process of grammar rules. The two major shapes derived by the grammar are joined together to create a single feed dual-band structure and shorting planes are added. The third shape is fed separately. The height of the substrate is $4mm$ and $\varepsilon_r = 1.0$. The resulting structure, fig.27, is analyzed numerically (FDTD). The frequencies obtained from the FDTD model for the three fundamental modes are $0.85GHz$, $1.69GHz$, and $2.52GHz$ and the deviations from the grammar model for the grammar derived shapes are 18%, 9% and 2%. The high deviation for the dual-band structure is due to the increase in length when the two shapes are joined together and due to the shorting plane which is shorter than the width of the line. The input impedance is also close to the system impedance of 50Ω for both feeds. The next step for the antenna engineer is to proceed to the second phase of the design and optimise the structure so as to deliver the final design prior to measurements. Potential variables for the optimisation stage are indicated in fig.27 with lines ended in arrows. The optimisation process does not change the character of the shape itself, but varies the dimensions of the sub-shapes or rectangles.

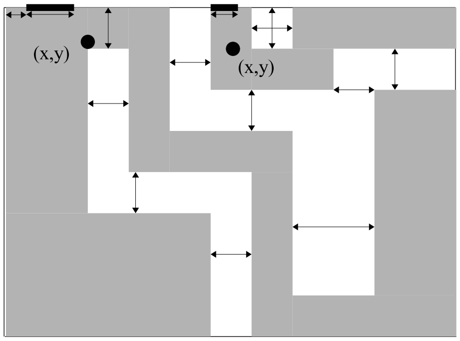

Figure 27. The numerical model ready for optimisation derived from fig.26(d). The arrows and positions for the probe feeds are suggested variables for the optimisation process. Possible variables for optimisation are indicated by the arrow-ended-lines.

III. CONCLUSIONS

This paper described an original microstrip antenna shape grammar that generates compact designs similar to meander line structures, as well as some standard shapes. The shape derived is represented as a chain of rectangles identified by labels and symbols. The decomposition into sub-shapes or rectangles is carried out by the grammar itself. The radiating edges are labeled and the nature of the shape can be derived from the orientation and sequence of the arrow symbols. A set of sub-shape and full-shape parameters are used as variables in a mathematical formula model that yields approximate values for the electrical characteristics. The result from the formula is used as feedback to guide the selection of rules and therefore shape evolution. The shape grammar is useful to synthesize and analyze compact microstrip antennas.

The shape grammar is a tool for the conceptual or initial design stage prior to the optimisation of the details. This tool formalizes and mimics some of the informal processes that an antenna designer goes through. The rule selection can be done by the machine to generate a wide variety of potentially useful designs and can form the basis of an Intelligent Computer Aided Engineering (ICAE) software. Forbus describes an ICAE system as a junior partner in the design team and the goal for the system is to capture a significant fraction of an engineer's knowledge that is later used to generate potential designs as well as modifications[19]. The shape grammar described in this paper provides such a tool and can be used to record design knowledge that can be used later. Alternatively the shape grammar can be integrated as part of a CAD software and the rules applied manually by the designer. The feedback mechanism automatically provides an estimate of the electrical properties of the antenna.

In this paper feedback is demonstrated with a simple weighted function. This function performs very well for the narrow line designs but its accuracy degrades when more variables are introduced. Not surprisingly the results show that the accuracy of the quantitative results improve if the formula is fitted over a narrow range of shapes and frequencies. Nevertheless, the accuracy of the model is still good enough for the initial design phase. On the other hand it is always desired to have a single model applicable to a wide range of devices. The inclusion of more variables and non-linear terms may improve the results. Of special interest is the use of Neural Network architectures (NN) as black boxes that replace the CAD formula. This model is suitably applied to well known and understood configurations. Geometrical dimensions are selected as inputs to a NN which in turn yields the electrical properties of the device. This approach has been shown to work for planar and microstrip antennas [20],[21],[22] and [23]. It would also be very useful to the designer if the model gives an estimate of how accurate the result is [24]. The work presented in this paper may be augmented with NN to automate the analytical derivation of the approximate model used in the generative system.

The work reported in this paper demonstrates the feasibility of a shape grammar based model that can act as an assistant to the designer. It would therefore be useful to extend this grammar to include other configurations and components such that it has a wider range of applicability.

Equally interesting should be the application of boolean shape operations to shapes generated by the grammar to extract sub-shapes that can be re-interconnected by switches. Such a tool would be very useful during the design of reconfigurable antennas.

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