

Empty Container Management in Multi-port System with Inventory-based Control

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Abstract – One of challenges that shipping companies face today is to effectively manage empty containers in order to meet customer demands and to reduce operation cost. This paper considers the joint empty container repositioning and container fleet sizing problem in a multi-port system with inventory-based control mechanism. A single-level threshold policy with repositioning rule in terms of minimizing the repositioning cost is proposed to manage empty containers. The objective is to optimize the fleet size and the parameters of the policy to minimize the expected total cost per period. Taking advantage of an interesting property of the problem, this paper proposes two approaches, a non-linear programming and a gradient search approach to solve the problem. From a methodological perspective, this paper deploys infinitesimal perturbation analysis method to improve computational efficiency. A numerical study demonstrates the effectiveness of the proposed policy and provides some managerial insights for shipping companies in managing empty containers.

Keywords - empty container management; repositioning; inventory control; simulation; infinitesimal perturbation analysis

I. INTRODUCTION

Growth in maritime transportation industries has been stimulated by the increase in international merchandise trade as a result of globalization in the last few decades. In particular, the containerization of cargo transportation has been the fastest growing sector of the maritime industries. Containerized cargos have grown at an annual average rate of 9.5% over the period 1987 through to 2006, which is exceeding the average maritime trade growth rate 4.1% over the same period [2]. In 2010, it estimated that container trade volumes reached 140 million twenty-foot equivalent units (TEUs) [3]. The growth of containerized shipping has presented challenges inevitably, in particular to the management of empty containers (ECs) arising from the highly imbalanced trade between continents. For example, in 2010, the annual container flow from Asia to Europe was 13.5 million TEUs and 5.6 million TEUs in the reverse direction, resulting in container flow imbalance of 7.9 million TEUs [3]. To manage the imbalance of container flows, repositioning ECs is an essential approach used by shipping companies. It is reported that empty container movements have accounted for at least 20% of the global port handling activity ever since 1998 [2]. Song et al. [4] estimated that the cost of repositioning ECs around the globe exceeded US\$15 billions, which was 27% of the total world fleet running cost based on the data in 2002. If the cost of

repositioning ECs can be reduced, the shipping companies can increase profit and improve competitiveness. Therefore, how to effectively and efficiently operate ECs is a very important issue for shipping companies and it is known as the problem of empty container repositioning (ECR).

Research on ECR has increased quite substantially in recent years. Much of work has adopted deterministic programming approach, which uses dynamic network programming formulations (see, e.g., [5][6][7][8]). The stochastic factors of the problem, such as future customer demands and returned containers, have attracted much attention since 1990s. In [9], Cheung and Chen proposed a two-stage stochastic network model to determine the ECR decisions. In [10], Lam et al. formulated the ECR problem as a dynamic stochastic programming. In [11], Long et al. applied a two-stage stochastic model to incorporate uncertainties in ECR problem with random demand, supply and ship space capacity. The mathematical models often successfully capture the dynamic and stochastic nature of the problem, while give rise to some concerns, such as requirement of a pre-specified planning horizon, sensitivity of the decisions to data accuracy and variability, and implementation of the decisions in the stochastic systems [12][13].

Another interesting development is to explore inventory-based control policies for managing ECs. Characterized by a set of rules and a set of parameters, such policies utilize the feedback of inventory information to manage ECs. Once the rules and parameters are designed in advance, ECs can be repositioned by following these simple rules. One of the advantages of using inventory-based control policies is that it is easy to operate and easy to understand, while producing near-optimal or even optimal solutions [14].

In this study, we consider the joint ECR and container fleet sizing problem in a multi-port system with inventory-based control mechanism. The system comprises a set of ports connected to each other and a fleet of own containers are used to meet the stochastic customer demands. A single-level threshold policy with repositioning rule in terms of minimizing the repositioning cost is proposed to manage ECs with periodical review. Although in general such policy does not guarantee optimality, its simplicity in implementation makes them attractive in practice and this fact motivates us to study its application in the ECR problem. The objective of this study is to optimize the fleet size and the parameters of the policy in terms of minimizing the expected total cost per period, including repositioning cost and holding and leasing cost.

The remainder of the paper is organized as follows: related work is presented in Section II. Then, Section III presents the mathematical formulation of the ECR problem and discusses the methods to solve the problem, followed by the description of the infinitesimal perturbation analysis (IPA) based gradient algorithm in Section IV. Section V illustrates the numerical studies. Finally, the work in this study is concluded and several issues for future research are discussed.

II. RELATED WORK

Inventory-based control policies for ECR problem have recently received increasing attentions. Studies in [15][16][17][18][19] focused on the examination of the structural properties of the optimal inventory-based repositioning policies and demonstrated that the optimal repositioning policies were of threshold type in some situations such as one-port and two-port systems. Once the parameters and rules of such policies are designed in advance, they are easy to operate and easy to understand from a managerial perspective. Several researchers further considered the implementation of the threshold-type control policies in more general systems. Song and Carter [14] addressed the ECR problem in a hub-and-spoke system with the assumption that ECs can be only repositioned between hub and spokes. Song and Dong [20] considered the implementation of a two-level threshold policy in a cycle shipping route system, and extended to optimize the fleet size and the parameters of the policy by using a simulation-based method with genetic algorithm in a typical liner shipping system [13]. Song and Dong [21] studied the repositioning policy for ECs with flexible destination ports in a liner shipping system. Yun et al. [22] considered the ECR problem in the inland system and optimized the parameters of the policy by applying a simulation optimization tool.

From the literature we find that the current studies are inadequate in addressing the implementation of the inventory-based control policies in general maritime systems. To the best of our knowledge, few of the studies have considered the implementation of such policies in multi-port system with direct empty container flows between each pair of ports, to which this study attempts to contribute. Li et al. [23] considered the ECR problem in a multi-port system, and proposed a heuristic algorithm based on a two-level threshold policy to allocate ECs among multiple ports. However, it could be computationally expensive when the numbers of ports and fleet size are very large. Besides, the fleet sizing problem is also not fully studied in such system. Moreover, most policies in literature apply simple rule, such as linear rationing rule to allocate ECs. The repositioning rule in terms of minimizing the repositioning cost has not been considered yet.

This paper is an extended version of [1]. It provides a detailed description of the IPA-based gradient technique, as well as a previously omitted mathematical proof on the important property in [1]. Moreover, the gradient technique is extended to optimize the fleet size.

III. PROBLEM FORMULATION

A multi-port system, which consists of ports connected to each other, is considered in this study. A fleet of own ECs meets exogenous customer demands, which are defined as the requirements for transforming ECs to laden containers and then transporting these laden containers from original ports to destination ports. A single-level threshold policy with periodical review is adopted to manage ECs. At the beginning of a period, the ECR decisions are made for each port, involving whether to reposition ECs, to or from which ports, and in what quantity. Then, when the customer demands occur in the period, those ECs that are currently stored at the port and those ECs that are repositioned to the port in this period can be used to satisfy customer demands. If it is not enough, additional ECs will be leased from vendors.

Several assumptions are made as follows:

- Only one type of container, i.e., TEU is considered.
- The customer demands must be satisfied in each period; and customer demands for each pair of ports in each period follow independent normal distributions.
- Short-term lasing is considered and the quantity of the leased ECs is always available in each port at any time.
- The leased ECs are not distinguished from owned ECs, i.e., the shipping company can return own ECs to vendors when it has sufficient ECs available.
- The travel time for each pair of ports is less than one period length.
- When the repositioned ECs arrive at destination ports, they will become available immediately; and when laden containers arrive at destination ports, they will become empty and be available at the beginning of next period.
- The cost of repositioning an EC from p to port m is the summation of the handling cost of an EC at port p , the handling cost of an EC at port m , and the transportation cost of an EC from p to port m .

The notations used in this paper are presented in Table I. In every period t , the ECR decisions are firstly made at the beginning of this period. Then, the inventory position can be obtain by

$$x_{p,t} = x_{p,t} + a_{p,t}, \quad \forall p \in P \quad (1)$$

After customer demands are realized and the laden containers become available, the beginning on-hand inventory of the next period can be updated by

$$x_{p,t+1} = y_{p,t} + \varphi_{p,t}, \quad \forall p \in P \quad (2)$$

It should be noted that $x_{p,t}$ can be negative. This is due to the fact that customer demands are random and beyond control. If $x_{p,t}$ is negative, it represents the number of containers that are leased from port p and stored at other ports. In this

TABLE I. LIST OF NOTATIONS

Sym bol	Description
N	the fleet size, which is the number of owned ECs
P	the set of ports
t	the discrete time decision period
P_t^S	the surplus port subset in period t
P_t^D	the deficit port subset in period t
$x_{p,t}$	the beginning on-hand inventory of port p in period t
$y_{p,t}$	the inventory position of port p in period t after making the ECR decisions
$z_{p,m,t}$	the number of ECs repositioned from port p to port m in period t
$\varepsilon_{p,m,t}$	the random customer demand from port p to port m in period t
$u_{p,t}^S$	the number of estimated EC supply of surplus port p in period t
$u_{p,t}^D$	the number of estimated EC demand of deficit port p in period t
\mathbf{x}_t	$= [x_{p,t}]_{p \in P}$ the vector of the beginning on-hand inventory in period t
\mathbf{y}_t	$= [y_{p,t}]_{p \in P}$ the vector of the inventory position in period t
\mathbf{Z}_t	$= (z_{p,m,t})_{p \in P, m \in P, p \neq m}$ the array of repositioned quantities for all ports
ω_t	$= (\varepsilon_{p,m,t})_{p \in P, m \in P, p \neq m}$ the stochastic customer demands in period t
$C_{p,m}^R$	the cost of repositioning an EC from port p to port m , $\forall p \neq m$
C_p^H	the cost of holding an EC at port p per period
C_p^L	the cost of leasing an EC at port p per period
γ_p	the threshold of port p
$\boldsymbol{\gamma}$	$= [\gamma_p]_{p \in P}$ vector of the thresholds
$\eta_{p,t}^O$	$= \sum_{m \in P, m \neq p} \varepsilon_{p,m,t}$ the number of total exported laden containers of port p in period t
$a_{p,t}$	$= \sum_{l \in P, l \neq p} z_{l,p,t} - \sum_{m \in P, m \neq p} z_{p,m,t}$ the net actual imported EC of port p in period t
$F_p(\cdot)$	the cumulative distribution function for $\eta_{p,t}^O$
$\varphi_{p,t}$	$= \sum_{l \in P, l \neq p} \varepsilon_{l,p,t} - \eta_{p,t}^O$ the net customer demand of port p in period t
$E_{p,t}$	the set of ports whose net actual imported ECs is changed by perturbing the estimated supply or demand of port p in period t , $e_{p,t} \in E_{p,t}$ and $e_{p,t} \neq p$
$Q_{p,t}$	the set of ports whose beginning on-hand inventory in period t is affected by perturbing threshold of port p , $q_{p,t} \in Q_{p,t}$ and $q_{p,t} \neq p$
$\pi_{p,t}$	the corresponding dual variable for port p constraint in the transportation model in period t
q_t^N	the port whose beginning on-hand inventory in period t is affected by perturbing the fleet size
$I\{\cdot\}$	a indicator function, which takes 1 if the condition in the brace is true and otherwise 0

situation there are no ECs stored at port p ; otherwise, they will be returned to the vendor to reduce the number of leased containers according to the assumption. If $x_{p,t}$ is positive, it represents the number of ECs that are available at port p , which implies that there are no container leased out from vendor at this port. Note that there are N owned ECs in the system, we have

$$N = \sum_{p \in P} x_{p,t} \quad \forall t \quad (3)$$

Let $J(\mathbf{x}_t, \boldsymbol{\gamma}, \omega_t)$ be the total cost in period t . It can be defined as:

$$J(\mathbf{x}_t, \boldsymbol{\gamma}, \omega_t) = H(\mathbf{Z}_t) + G(\mathbf{y}_t, \omega_t) \quad (4)$$

where the value of \mathbf{Z}_t is determined by the beginning on-hand inventory \mathbf{x}_t and the policy $\boldsymbol{\gamma}$; $H(\mathbf{Z}_t)$ and $G(\mathbf{y}_t, \omega_t)$ are the EC repositioning cost and the EC holding and leasing cost in period t , respectively. They are defined as follows

$$H(\mathbf{Z}_t) = \sum_{p \in P} \sum_{m \in P, m \neq p} C_{p,m}^R \cdot z_{p,m,t} \quad (5)$$

$$\begin{aligned} G(\mathbf{y}_t, \omega_t) &= \sum_{p \in P} g(y_{p,t}, \eta_{p,t}^O) \\ &= \sum_{p \in P} (C_p^H (y_{p,t} - \eta_{p,t}^O)^+ + C_p^L (\eta_{p,t}^O - y_{p,t})^+) \end{aligned} \quad (6)$$

where $g(y_{p,t}, \eta_{p,t}^O)$ represents the EC holding and leasing cost of port p in period t ; $x^+ = \max(0, x)$. More specifically, the EC repositioning cost $H(\mathbf{Z}_t)$ refers to the cost of repositioning ECs between multiple ports. The EC holding and leasing cost $G(\mathbf{y}_t, \omega_t)$ is the cost incurred when ECs are stored at some ports and additional ECs are leased from vendors at the other ports.

Next, a single-level threshold policy is developed to make the ECR decisions \mathbf{Z}_t at the beginning of period t .

A. A Single-level Threshold Policy

Note that when the ECR decisions are made in a period, the customer demands in this period have not been realized yet. Hence, we try to maintain the inventory position at a target threshold value $\boldsymbol{\gamma}$. More specifically, γ_p is the target threshold of port p . In period t , if $x_{p,t} > \gamma_p$, then port p is a surplus port and the quantity excess of γ_p should be repositioned out to other ports that may need ECs to try to bring the inventory position down to γ_p ; if $x_{p,t} < \gamma_p$, then it is a deficit port and ECs should be repositioned in from other ports that may supply ECs to try to bring the inventory position up to γ_p ; if $x_{p,t} = \gamma_p$, then it is a balanced port and nothing is done.

From the policy, therefore, if there are no surplus or deficit ports in period t , no ECs should be repositioned. Otherwise, ECs should be repositioned from surplus ports to deficit ports in the right quantity at the least movement. Without loss of generality, the ECR decisions in period t are considered. The two subsets, i.e., surplus port subset and deficit port subset can be obtained as $P_t^S = \{i: x_{i,t} > \gamma_i\}$ and $P_t^D = \{j: x_{j,t} < \gamma_j\}$, respectively. For a surplus port, its number of excess ECs, namely the number of estimated EC supply is calculated by (7); and for a deficit port, its number of estimated EC demand by (8).

$$u_{i,t}^S = x_{i,t} - \gamma_i, \quad \forall i \in P_t^S \quad (7)$$

$$u_{j,t}^D = \gamma_j - x_{j,t}, \quad \forall j \in P_t^D \quad (8)$$

If either P_t^S or P_t^D are empty, then $\mathbf{Z}_t = 0$. Otherwise, the value of $z_{l,m,t}$, $\forall l \in (P - P_t^S)$, $m \in (P - P_t^D)$, $l \neq m$ should be equal to zero, and the value of $z_{i,j,t}$, $\forall i \in P_t^S$, $j \in P_t^D$ are determined by solving a transportation model.

Note that when P_t^S and P_t^D are nonempty, the total number of estimated EC supplies $\sum_{i \in P_t^S} u_{i,t}^S$ could be not equal to the total number of estimated EC demands $\sum_{j \in P_t^D} u_{j,t}^D$. Thus, we propose to move as many excess ECs of surplus ports as possible to the deficit ports to satisfy their demands. Hence, when $\sum_{i \in P_t^S} u_{i,t}^S \geq \sum_{j \in P_t^D} u_{j,t}^D$, a transportation model is formulated as follows to determine the repositioned quantities in each period:

$$\min_{z_{i,j,t}} \sum_{i \in P_t^S} \sum_{j \in P_t^D} C_{i,j}^R \cdot z_{i,j,t} \quad (9)$$

$$\text{s.t. } \sum_{j \in P_t^D} z_{i,j,t} \leq u_{i,t}^S, \quad \forall i \in P_t^S \quad (10)$$

$$\sum_{i \in P_t^S} z_{i,j,t} \geq u_{j,t}^D, \quad \forall j \in P_t^D \quad (11)$$

$$z_{i,j,t} \geq 0, \quad \forall i \in P_t^S, j \in P_t^D \quad (12)$$

Constraints (10) ensure that the repositioned out ECs of a surplus port should not exceed its estimated EC supply; constraints (11) ensure that the EC demand of a deficit port can be fully satisfied; constraints (12) are the non-negative quantity constraints. When $\sum_{i \in P_t^S} u_{i,t}^S < \sum_{j \in P_t^D} u_{j,t}^D$, similar model is built by substituting $\sum_{j \in P_t^D} z_{i,j,t} \geq u_{i,t}^S, \forall i \in P_t^S$ and $\sum_{i \in P_t^S} z_{i,j,t} \leq u_{j,t}^D, \forall j \in P_t^D$ for (10) and (11), respectively.

B. The Optimization Problem

Let $J(N, \boldsymbol{\gamma})$ be the expected total cost per period with the fleet size N and policy $\boldsymbol{\gamma}$. The problem, which is to optimize the fleet size and the parameters of the policy in terms of minimizing the expected total cost per period, can be formulated as

$$\min_{N, \boldsymbol{\gamma}} J(N, \boldsymbol{\gamma}) \quad (13)$$

subject to the inventory dynamic equations (1)~(3) and the single-level threshold policy. As in many papers on empty container movement (see, e.g., [9] [11]), the variables that relate to the flow of ECs are considered as continuous variables in this study. That is, the fleet size and the parameters of the policy are considered as real numbers in this study. It is fine when the values of these variables are large.

In general, it is difficult to solve problem (13), since there is no closed-form formulation for the computation of $J(N, \boldsymbol{\gamma})$ involving the repositioned empty container quantities determined by transportation models. However, when the transportation model is balanced in period t , i.e., the total number of estimated EC supplies, namely $\sum_{i \in P_t^S} u_{i,t}^S$ is equal to the total number of estimated EC demands, namely $\sum_{j \in P_t^D} u_{j,t}^D$, the excess ECs in all surplus ports can be fully repositioned out to satisfy the demands of all deficit ports. Hence, the inventory position of each port can be kept at its target threshold level in this period, i.e., $y_{p,t} = \gamma_p$.

Further, the EC repositioning cost in next period will be only related to the customer demands in this period. From (3), (7) and (8), we obtain that $\sum_{i \in P_t^S} u_{i,t}^S = \sum_{j \in P_t^D} u_{j,t}^D \quad \forall t$ if and only if $N = \sum_{p \in P} \gamma_p$. Hence, an important property of the problem is presented as follows:

Property 1: When the fleet size is equal to the sum of thresholds, the inventory position of a port can be always maintained at its target threshold value and then the EC repositioning cost in a period is only dependent on the customer demands.

Let Scenario-I (Scenario-II) be the scenario in which $N = \sum_{p \in P} \gamma_p$ ($\neq \sum_{p \in P} \gamma_p$). Taking advantage of this property, we propose two approaches to solve problem (13) under both scenarios, respectively.

1) Scenario-I

Consider the problem under Scenario-I. From Property 1, it implies that $y_{p,t} \equiv \gamma_p \quad \forall p \in P, t$, and only the EC holding and leasing cost in a period is related to values of N and $\boldsymbol{\gamma}$. Hence, the optimal solution which minimizes $J(N, \boldsymbol{\gamma})$ should be equivalent to the optimal solution which minimizes the expected EC holding and leasing cost per period. From (6), since that holding and leasing cost function for each port is independent, the problem (13) under Scenario-I can be simplified to an non-linear programming (NLP) problem as follows:

$$\min_{N, \boldsymbol{\gamma}} \sum_{p \in P} E\left(\left(C_p^H \cdot (\gamma_p - \eta_p^O)^+ + C_p^L \cdot (\eta_p^O - \gamma_p)^+\right)\right)$$

$$\text{s.t. } N = \sum_{p \in P} \gamma_p$$

where the subscript t in the notation of $\eta_{p,t}^O$ is dropped since customer demands in each period are independent and identically distributed. The NLP problem can be further decomposed into P independent newsvendor problems with the optimal solutions as follows:

$$\gamma_p^* = F_p^{-1}\left(C_p^L / (C_p^L + C_p^H)\right) \quad (14)$$

$$N^* = \sum_{p \in P} \gamma_p^* \quad (15)$$

where $F_p^{-1}(\cdot)$ is the inverse function of $F_p(\cdot)$.

2) Scenario-II

The problem under Scenario-II is more complex than that under Scenario-I, since the EC repositioning cost is also affected by the values of N and $\boldsymbol{\gamma}$. Consider that the minimum expected holding and leasing cost of problem (13) can only be achieved under Scenario-I due to the convexity of the holding and leasing cost function. The problem under Scenario-II is worth to study if and only if the minimum expected EC repositioning cost under this scenario could be less than that under Scenario-I.

Consider there is no closed-form formulation for the expected EC repositioning cost. Without loss of generality, the EC repositioning costs in a period under both scenarios are mathematically compared with same customer demands. More specifically, we have Scenario-I with parameters $(N, \boldsymbol{\gamma}^I)$ and Scenario-II with parameters $(N, \boldsymbol{\gamma}^{II})$ where $N = \sum_{p \in P} \gamma_p^I \neq \sum_{p \in P} \gamma_p^{II}$. Let other various quantities in both scenarios be distinguished by displaying the arguments I and II , respectively. We have next proposition.

Proposition 3.1: For all t , we have

$$H(\mathbf{x}_{t+1}^{II}, \boldsymbol{\gamma}^{II}) \Big|_{\omega_t} \leq H(\mathbf{x}_{t+1}^I, \boldsymbol{\gamma}^I) \Big|_{\omega_t}$$

with same customer demand ω_t .

Proof: The proof is given in Appendix A. ■

From this proposition, it implies that the expected EC repositioning cost per period under Scenario-II could be less than that under Scenario-I. Therefore, it is worth to study the problem under Scenario-II, and the simulation technique is adopted to estimate $J(N, \boldsymbol{\gamma})$ with given values of N and $\boldsymbol{\gamma}$ as follows:

$$J(N, \boldsymbol{\gamma}) \approx \frac{1}{T} \sum_{t=1}^T J(\mathbf{x}_t, \boldsymbol{\gamma}, \omega_t) = \frac{1}{T} \sum_{t=1}^T (H(\mathbf{x}_t, \boldsymbol{\gamma}) + G(\mathbf{y}_t, \omega_t)) \quad (16)$$

where T is the amount of the simulation periods. A gradient search method is developed to solve the problem in next section.

IV. IPA-BASED GRADIENT TECHNIQUE

IPA [24] is able to estimate the gradient of the objective function from one single simulation run, thus reducing the computational time. Moreover, it has been shown that

variance of IPA estimator is lower, compared with many other gradient estimators [25]. The main idea behind IPA is that: if a decision parameter of a system is perturbed by an infinitesimal amount, the derivation of the system performance to that parameter can be estimated by tracing its pattern of propagation through the system. This will be a function of the fraction of the propagations that die before having a significant effect on the response of interest [26]. Once the performance derivatives are obtained, they can be imbedded in optimization algorithms to optimize the interest parameters. Successful applications of IPA have been reported in inventory modeling [27], stochastic flow models [28][29], persistent monitoring problems [30], budget allocation for effective data collection [31], multi-location transshipment problem with capacitated production [32]. In this study, an IPA-based gradient technique is proposed to search the optimal solution under Scenario-II. The overall gradient technique is briefly described in Figure 1.

For a given $(N, \boldsymbol{\gamma})$ at iteration n , simulation is run to estimate the expected cost $J(N, \boldsymbol{\gamma})$ in (16). At the same time, the gradient vector of the expected cost $\nabla J(N, \boldsymbol{\gamma})_{(N, \boldsymbol{\gamma})=(N, \boldsymbol{\gamma})_n}$ can be estimated. Briefly, the simulation is run by firstly making the ECR decision in a given period by solving transportation models and then the customer demands are realized. At the same time, the cost and the gradient for this period can also be computed. After all periods are run, the overall cost and gradient can be computed from the individual values obtained in each period. Based on the gradient information, the parameters can be updated by using the steepest descent algorithm, i.e., $(N, \boldsymbol{\gamma})_{n+1} = (N, \boldsymbol{\gamma})_n - \alpha_n (\nabla J(N, \boldsymbol{\gamma})_{(N, \boldsymbol{\gamma})=(N, \boldsymbol{\gamma})_n}) \cdot \alpha_n$ is the step size at the n th iteration. After $(N, \boldsymbol{\gamma})$ is updated, the process for the new iteration is repeated until the stopping criterion is met.

To estimate the gradient of expected cost, we take a partial derivation of (16) with respect to the fleet size and the threshold of port i , respectively and have

$$\begin{aligned} \partial J(N, \boldsymbol{\gamma}) / \partial N &\approx 1/T \cdot \sum_{t=1}^T (\partial J(\mathbf{x}_t, \boldsymbol{\gamma}, \omega_t) / \partial N) \\ &\approx \frac{1}{T} \sum_{t=1}^T \left(\frac{\partial H(\mathbf{Z}_t)}{\partial N} + \sum_{p \in P} \frac{\partial E(g(y_{p,t}, \eta_{p,t}^o))}{\partial y_{p,t}} \cdot \frac{\partial y_{p,t}}{\partial N} \right) \end{aligned} \quad (17)$$

$$\begin{aligned} \partial J(N, \boldsymbol{\gamma}) / \partial \gamma_i &\approx 1/T \cdot \sum_{t=1}^T (\partial J(\mathbf{x}_t, \boldsymbol{\gamma}, \omega_t) / \partial \gamma_i) \\ &\approx \frac{1}{T} \sum_{t=1}^T \left(\frac{\partial H(\mathbf{Z}_t)}{\partial \gamma_i} + \sum_{p \in P} \frac{\partial E(g(y_{p,t}, \eta_{p,t}^o))}{\partial y_{p,t}} \cdot \frac{\partial y_{p,t}}{\partial \gamma_i} \right) \end{aligned} \quad (18)$$

Here, for the EC holding and leasing cost function, the expected cost function is utilized to estimate the gradient instead of sample cost function since we are able to get the explicit function to evaluate the average gradient. $\partial H(\mathbf{Z}_t) / \partial N$ or $\partial H(\mathbf{Z}_t) / \partial \gamma_i$, which measures the impact of the transportation cost in period t when the fleet size or threshold of port i is changed, can be found using the dual

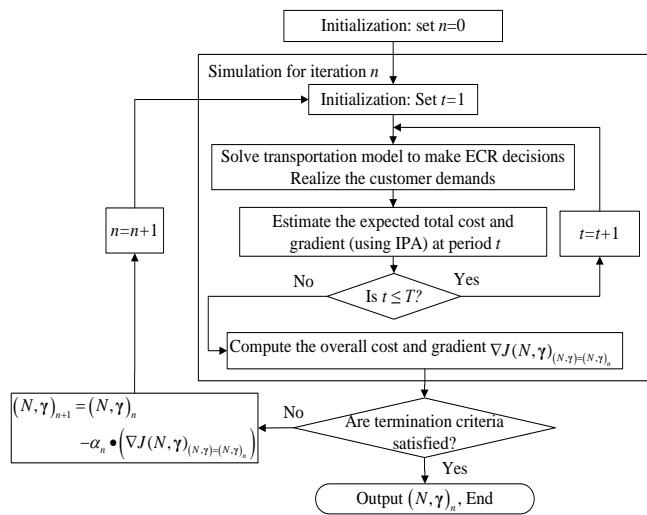


Figure 1. The flow of the IPA-based gradient technique

model information from the transportation model. $\partial g(y_{p,t}, \eta_{p,t}^o) / \partial y_{p,t}$, which measures the impact of the holding and leasing cost function of port p in period t when the inventory position is changed, can be easily found taking the derivation of (6) with respect to the inventory position level. $\partial y_{p,t} / \partial N$ or $\partial y_{p,t} / \partial \gamma_i$, which measures the impact of the inventory position level of port p in period t when the fleet size or threshold of port i change, can be estimated using the IPA technique.

Next, the gradient of expected total cost with respect to the threshold is analyzed.

A. Gradient with Respect to the Threshold

In this section, the gradient of expected total cost with respect to the threshold is studied with given fleet size N .

The nominal path is defined as the sample path generated by the simulation model with parameter γ and the perturbed path as the sample path generated using the same model and same random seeds, but with parameter $(\gamma)'$, where $(\gamma)' = \gamma + \Delta\gamma$. Without loss of generality, only the threshold of port i is perturbed and the thresholds of the other ports are unchanged, i.e., $(\gamma_i)' = \gamma_i + \Delta\gamma_i$ and $(\gamma_p)' = \gamma_p$ for other $p \in (P - \{i\})$, where the value of $\Delta\gamma_i$ is sufficiently small. By “sufficiently” small, we mean such that the surplus port and deficit port subsets are same in the both paths in every period. Oftentimes, the changes in various quantities will be presented by displaying with argument Δ . For example, $\Delta\mathbf{x}_t$ represents the change in the beginning on-hand inventory in period t .

The threshold of port i is perturbed with same quantities in all periods and the representative perturbation flow in period t is shown in Figure 2. The perturbation of $\Delta\mathbf{x}_t$ and the perturbation of $\Delta\gamma_i$ will affect the estimated EC supply or demand, i.e., $u_{p,t}^S$ or $u_{p,t}^D$ of some ports, which are the right-hand side (RHS) of the constraints in the transportation model. Hence, the perturbation of $\Delta u_{p,t}^S$ or $\Delta u_{p,t}^D$ could change the transportation cost and the net actual imported ECs, i.e., $a_{p,t}$ of some ports. The perturbations of $\Delta a_{p,t}$ and $\Delta\mathbf{x}_t$ will affect the perturbation on the EC inventory position $y_{p,t}$ of some ports. Further, perturbation of $\Delta y_{p,t}$ will affect the total holding and leasing cost in this period and the beginning on-hand inventory of next period.

From the definition of $Q_{i,t}$, we have $Q_{i,t} = \{p: \Delta x_{p,t} \neq 0\}$. Thus, depending on the status of $Q_{i,t}$, only two types of scenarios are possible – one with $Q_{i,t} = \emptyset$, the other with

$Q_{i,t} \neq \emptyset$. Since the value of \mathbf{x}_1 , i.e., the initial on-hand inventory, is given, it implies that $\Delta\mathbf{x}_1 = 0$ and $Q_{i,1} = \emptyset$. If the perturbation in the first period is propagated to the second period, it could lead $Q_{i,2} \neq \emptyset$. Hence, the perturbations in the first two periods are investigated in order to conclude the general formulations for the perturbation terms in (18).

1) Perturbation with Respect to Threshold in the First Period

The perturbations in period $t = 1$ are traced following the flow in Figure 2.

Since $\Delta\mathbf{x}_t = 0$, we have $\Delta u_{i,t}^S = -\Delta\gamma_i$ or $\Delta u_{i,t}^D = \Delta\gamma_i$, and $\Delta u_{p,t}^S$ or $\Delta u_{p,t}^D = 0$ for other $p \in P$ from (7) and (8). It implies that only the RHS of port i constraint is changed.

From the transportation model, the perturbation of $\Delta u_{i,t}^S$ or $\Delta u_{i,t}^D$ will affect the perturbation of $\Delta a_{p,t}$, depending on the status of port i constraint at the optimal solution. Only two scenarios are possible:

Scenario 1: The port i constraint is not binding. The perturbation of $\Delta u_{i,t}^S$ or $\Delta u_{i,t}^D$ will not affect the optimal repositioned quantities. Hence, it implies that $\Delta a_{p,t} = 0$ for all $p \in P$.

Scenario 2: The port i constraint is binding. The perturbation could affect the optimal repositioned quantities of some ports. However, only for a pair of ports, i.e., i and k , their net actual imported ECs will be changed (explained in Appendix B); while for the other port, no changes. It implies that $\Delta a_{p,t} \neq 0$ for any $p = i, k$, and $\Delta a_{p,t} = 0$ for other $p \in (P - \{i, k\})$.

From the definition of $E_{i,t}$, we get that $E_{i,t} = \{p: \Delta a_{p,t} \neq 0\}$. Thus, if the port i constraint is not binding, we have $E_{i,t} = \emptyset$. Otherwise, we have $E_{i,t} = \{i, e_{i,t}\}$ with $e_{i,t} = k$. It implies that $e_{i,t}$ is unique in period t . To find $E_{i,t}$, a modified stepping stone (MSS) approach (elaborated in Appendix B) is proposed.

Without investigating the details on the perturbation of $\Delta a_{p,t}$, we next consider the perturbation on $y_{p,t}$. From (1), we have $\Delta y_{p,t} = \Delta a_{p,t}, \forall p \in P$ since $\Delta x_{p,t} = 0, \forall p \in P$. Hence, we can get that:

- If $E_{i,t} = \emptyset, \Delta y_{p,t} = 0$ for all $p \in P$.
- If $E_{i,t} \neq \emptyset, \Delta y_{p,t} \neq 0$ for any $p \in E_{i,t}$, and $\Delta y_{p,t} = 0$ for other $p \in (P - E_{i,t})$.

From the transportation model, for the port with binding constraint, its inventory position will be equal to its threshold. It implies that $\Delta y_{i,t} = \Delta\gamma_i$. Further, we have $\Delta y_{e_{i,t},t} = -\Delta\gamma_i$ since $\sum_{p \in P} \Delta y_{p,t} = \Delta N = 0$ from (1) and (3).

We continue to study how the perturbations are carried forward to next period. From (2), we have $\Delta x_{p,t+1} = \Delta y_{p,t}, \forall p \in P$. It implies that the perturbation on $x_{p,t+1}$ will be fully from the perturbation on $y_{p,t}$, and then we have $Q_{i,t+1} = \{p: \Delta y_{p,t} \neq 0\}$. From the analysis on the perturbation of $\Delta y_{p,t}$ in the case of $Q_{i,t} = \emptyset$, we have

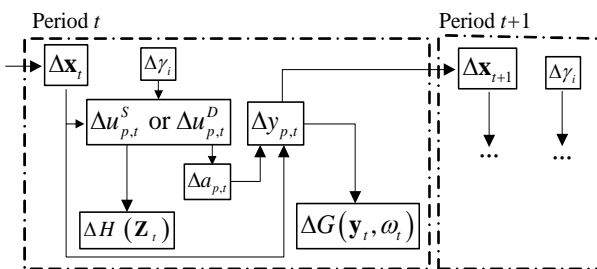


Figure 2. The Perturbation Flow

$$Q_{i,t+1} = \begin{cases} \emptyset, & \text{if } E_{i,t} = \emptyset \\ \{i, q_{i,t+1}\}, & \text{otherwise} \end{cases} \quad (19)$$

where $q_{i,t+1} = e_{i,t}$ when $E_{i,t} \neq \emptyset$. And when $Q_{i,t+1} \neq \emptyset$, we have $\Delta x_{i,t+1} = \Delta \gamma_i$, $\Delta x_{q_{i,t+1},t+1} = -\Delta \gamma_i$, and $\Delta x_{p,t+1} = 0$ for other $p \in (P - Q_{i,t+1})$. It indicates that if there are perturbations propagated to the next period, only for a pair of ports, i and $q_{i,t+1}$, their beginning on-hand inventories will be changed by $\Delta \gamma_i$ and $-\Delta \gamma_i$, respectively; while for the other ports, no changes.

From the analysis on the perturbations of the relevant variables, we have next lemma.

Lemma 4.1: In the period $t=1$, we have

$$\begin{aligned} 1) \partial H(\mathbf{Z}_t) / \partial \gamma_i &= (-1)^{I(i \in P_t^S)} \cdot \pi_{i,t} \\ 2) U_{p,t} &= \begin{cases} -C_p^L, & \text{if } y_{p,t} < 0 \\ ((C_p^H + C_p^L) \cdot F_p(y_{p,t}) - C_p^L), & \text{if } y_{p,t} \geq 0 \end{cases}, \forall p \in P \\ 3) \frac{\partial y_{p,t}}{\partial \gamma_i} &= \begin{cases} 1, & \text{if } I(Q_{i,t+1} \neq \emptyset, p = i) = 1 \\ -1, & \text{if } I(Q_{i,t+1} \neq \emptyset, p = q_{i,t+1}) = 1, \forall p \in P \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

where $U_{p,t} = \partial E(g(y_{p,t}, \eta_{p,t}^0)) / \partial y_{p,t}$.

Proof: Since only the RHS of port i 's constraint is changed in the first period with $\Delta u_{i,t}^S = -\Delta \gamma_i$ or $\Delta u_{i,t}^D = \Delta \gamma_i$, the perturbation on the optimal transportation cost value in the first period is estimated by $\Delta H(\mathbf{Z}_t) = (-1)^{I(i \in P_t^S)} \cdot \Delta \gamma_i \cdot \pi_{i,t}$ from the sensitivity analysis about RHS of linear programming. By hypothesis the value of $\Delta \gamma_i$ is sufficiently small. The change in the RHS value is within its allowable range and the status of all constraints remains unchanged in the perturbed path. In our problem with real variables, the probability of having degenerate optimal solutions or multiple optimal solutions is close to 0. Hence, $\frac{\partial H(\mathbf{Z}_t)}{\partial \gamma_i} =$

$$\lim_{\Delta \gamma_i \rightarrow 0} \frac{\Delta H(\mathbf{Z}_t)}{\Delta \gamma_i} = (-1)^{I(i \in P_t^S)} \cdot \pi_{i,t}.$$

For assertion 2), by taking the first derivation of the holding and leasing cost function of port p in (6) with respect to the inventory position, it is easy to have the equation.

For assertion 3), recall that the perturbation on $y_{p,t}$ is equal to the perturbation on $x_{p,t+1}$ for all $p \in P$, i.e., $\Delta y_{p,t} = \Delta x_{p,t+1}$. We have $\frac{\partial y_{p,t}}{\partial \gamma_i} = \lim_{\Delta \gamma_i \rightarrow 0} \frac{\Delta y_{p,t}}{\Delta \gamma_i} = \lim_{\Delta \gamma_i \rightarrow 0} \frac{\Delta x_{p,t+1}}{\Delta \gamma_i}, \forall p \in P$. Hence, if $Q_{i,t+1} = \emptyset$, we have $\partial y_{p,t} / \partial \gamma_i = 0$ for all $p \in P$. Otherwise, we have $\partial y_{i,t} / \partial \gamma_i = 1$, $\partial y_{q_{i,t+1},t} / \partial \gamma_i = -1$, and $\partial y_{p,t} / \partial \gamma_i = 0$ for other $p \in (P - Q_{i,t+1})$. ■

2) Perturbation with Respect to Threshold in the Second Period

If $Q_{i,2} = \emptyset$, the analysis about the perturbation in the second period will be similar with that in the first period.

Otherwise, the other case with $Q_{i,t} = \{i, q_{i,t}\}$ for $t=2$ will be studied.

Similarly, the perturbations on relevant variables are analyzed. We can have $\Delta u_{q_{i,t},t}^S = -\Delta \gamma_i$ or $\Delta u_{q_{i,t},t}^D = \Delta \gamma_i$ and $\Delta u_{p,t}^S$ or $\Delta u_{p,t}^D = 0$ for other $p \in (P - \{q_{i,t}\})$. It implies that only the RHS of port $q_{i,t}$ constraint is changed.

Hence, if the port $q_{i,t}$ constraint is not binding, we have $E_{q_{i,t},t} = \emptyset$ with $\Delta a_{p,t} = 0$ for all $p \in P$. Otherwise, we have $E_{q_{i,t},t} = \{q_{i,t}, e_{q_{i,t},t}\}$ with $\Delta a_{p,t} \neq 0$ for any $p \in E_{q_{i,t},t}$, and $\Delta a_{p,t} = 0$ for other $p \in (P - E_{q_{i,t},t})$.

Differently, the perturbation on $y_{p,t}$ could be from the both perturbations of $\Delta \mathbf{x}_t$ and $\Delta a_{p,t}$ in this period since $\Delta \mathbf{x}_t \neq 0$. Hence, from (1), i.e., $y_{p,t} = x_{p,t} + a_{p,t}$, we obtain:

- If $E_{q_{i,t},t} = \emptyset$, we have $\Delta y_{i,t} = \Delta \gamma_i$, $\Delta y_{q_{i,t},t} = -\Delta \gamma_i$, and $\Delta y_{p,t} = 0$ for other $p \in (P - \{i, q_{i,t}\})$.

- If $E_{q_{i,t},t} \neq \emptyset$, we have $\Delta y_{p,t}$ could be $\neq 0$ for any $p = i, e_{q_{i,t},t}$, and $\Delta y_{p,t} = 0$ for other $p \in (P - \{i, e_{q_{i,t},t}\})$.

Depending on whether $e_{q_{i,t},t} = i$, we get that:

- ◊ If $e_{q_{i,t},t} \neq i$, then it implies that $\Delta y_{i,t} = \Delta x_{i,t} = \Delta \gamma_i$, and $\Delta y_{e_{q_{i,t},t},t} = -\Delta \gamma_i$.

- ◊ If $e_{q_{i,t},t} = i$, then it implies that $\Delta y_{i,t} = 0$.

From the analysis about the perturbation of $\Delta y_{p,t}$ in the case of $Q_{i,t} \neq \emptyset$, we have:

$$Q_{i,t+1} = \begin{cases} \emptyset, & \text{if } E_{q_{i,t},t} \neq \emptyset \text{ and } e_{q_{i,t},t} = i \\ \{i, q_{i,t+1}\}, & \text{otherwise} \end{cases} \quad (20)$$

where $q_{i,t+1}$ is $= q_{i,t}$ when $E_{q_{i,t},t} = \emptyset$, or $= e_{q_{i,t},t}$ when $E_{q_{i,t},t} \neq \emptyset, e_{q_{i,t},t} \neq i$. When $Q_{i,t+1} \neq \emptyset$, the values of the perturbations of $\Delta x_{i,t+1}$ and $\Delta x_{q_{i,t+1},t+1}$ are same with that in the first period.

3) Total Perturbation with Respect to Threshold

Based on the analysis about $Q_{i,t}$ for $t = 1, 2$, it can be concluded that in any period t , $Q_{i,t}$ will be either empty or consists of a pair of ports, i.e., $\{i, q_{i,t}\}$. It implies that the analysis about the perturbations in period $t > 2$ by the perturbation of $\Delta \gamma_i$ will be similar with that in either of the first two periods.

From lemma 4.1 and (18), in a general form, we have

$$\frac{\partial J(N, \gamma)}{\partial \gamma_i} \approx \frac{1}{T} \sum_{t=1}^T \left(\begin{aligned} & I(Q_{i,t} = \emptyset) \bullet (-1)^{I(i \in P_t^S)} \bullet \pi_{i,t} \\ & + I(Q_{i,t} \neq \emptyset) \bullet (-1)^{I(q_{i,t} \in P_t^S)} \bullet \pi_{q_{i,t},t} \\ & + I(Q_{i,t+1} \neq \emptyset) \bullet (U_{i,t} - U_{q_{i,t+1},t}) \end{aligned} \right) \quad \forall i \in P \quad (21)$$

where the first two terms on the RHS of (21) present the perturbations on the EC repositioning cost under the two

conditions, i.e., $Q_{i,t} = \emptyset$ and $Q_{i,t} \neq \emptyset$, respectively; the third term presents the perturbation on the EC holding and leasing cost when $Q_{i,t+1} \neq \emptyset$; For $t = 1$, $Q_{i,t} = \emptyset$, and for $t > 1$, $Q_{i,t}$ can be obtained from either (19) and (20), depending on the status of $Q_{i,t-1}$; the values of $U_{i,t}$ and $U_{q_{i,t+1},t}$ can be calculated from assertion 2) of lemma 4.1.

B. Gradient with Respect to the Fleet Size

In this section, the gradient of expected total cost with respect to the fleet size is studied with given policy γ .

Similarly, the nominal path is defined as the sample path generated by the simulation model with parameter N and the perturbed path as the sample path generated using the same model and same random seeds, but with parameter $(N)'$, where $(N)' = N + \Delta N$ and ΔN is sufficiently small. Since $\sum_{p \in P} \Delta x_{p,t} = \Delta N$ from (3), we can perturb $x_{i,1}$ by ΔN to investigate the perturbations on relevant variables. That is, we set $q_t^N = i$ with $\Delta x_{q_t^N,t} = \Delta N$ and $\Delta x_{p,t} = 0$ for other $p \in (P - \{q_t^N\})$ in the period $t = 1$.

The perturbation flow for this problem is similar with that shown in Figure 2, but the unique difference is that there is no perturbation from the threshold. In other word, the perturbations on various variables are only from the perturbation on the beginning on-hand inventory.

1) Perturbation with Respect to the Fleet Size in the First Period

In the period $t = 1$, we have $\Delta u_{q_t^N,t}^S = \Delta N$ or $\Delta u_{q_t^N,t}^D = -\Delta N$ and $\Delta u_{p,t}^S$ or $\Delta u_{p,t}^D = 0$ for other $p \in (P - \{q_t^N\})$. It implies that only the RHS of port q_t^N constraint is changed. Hence, we have

- If the port q_t^N constraint is not binding, we have $E_{q_t^N,t} = \emptyset$. And then $\Delta y_{q_t^N,t} = \Delta N$ and $\Delta y_{p,t} = 0$ for other $p \in (P - \{q_t^N\})$.

- Otherwise, we have $E_{q_t^N,t} = \{e_{q_t^N,t}\}$. And then $\Delta y_{e_{q_t^N,t},t} = \Delta N$ and $\Delta y_{p,t} = 0$ for other $p \in (P - E_{q_t^N,t})$.

Similarly, we have

$$q_{t+1}^N = \begin{cases} q_i^N, & \text{if } E_{q_t^N,t} = \emptyset \\ e_{q_t^N,t}, & \text{otherwise} \end{cases} \quad (22)$$

where $\Delta x_{q_{t+1}^N,t+1} = \Delta N$ and $\Delta x_{p,t+1} = 0$ for other $p \in (P - E_{q_t^N,t})$. It implies that in each period, there is a unique port whose beginning on-hand inventory will be affected by ΔN . And we can have next lemma.

Lemma 4.2: In any period t , we have

$$\frac{\partial y_{p,t}}{\partial N} = \begin{cases} 1, & \text{if } p = q_{t+1}^N, \forall p \in P \\ 0, & \text{otherwise} \end{cases}$$

Proof: The proof is similar with that in the assertion 3) of the lemma 4.1. ■

2) Total Perturbation with Respect to the Fleet Size

From lemma 4.2 and (17), in a general form, we have:

$$\partial J(N, \gamma) / \partial N \approx 1/T \sum_{t=1}^T \left((-1)^{I(q_t^N \in P^S)+1} \bullet \pi_{q_t^N,t} + U_{q_{t+1},t}^N \right) \quad (23)$$

where the first term on the right hand side of (23) presents the perturbation on the EC repositioning cost; the second term presents the perturbation on the holding and leasing cost; for $t = 1$, $q_t^N = i$, and for $t > 1$, q_t^N can be obtained from (22); the value of $U_{q_t^N,t}^N$ can be calculated from the assertion 2) of lemma 4.1.

V. NUMERICAL RESULTS

In this section, we aim to evaluate the performance of the proposed single-level threshold policy (STP) and provide some insights about the EC management for shipping companies.

While the proposed policy can be applied in a multi-port system with an arbitrary number of ports, three problems that differ in the number of ports are considered: problem 1 has 6 ports, problem 2 has 9 ports and problem 3 has 12 ports, which represent small, moderate and large systems, respectively. Besides, since the trade imbalance could be the most important factor affecting the performance of the proposed policy, three kinds of trade imbalance patterns are designed, which include balanced trade pattern, moderately imbalanced trade pattern and severely imbalanced trade pattern. Therefore, a total of 9 cases are studied.

For each problem, the cost parameters and the average customer demands from port p to port m in a period, denoted by $\mu_{p,m}$, are randomly generated. In the balanced trade pattern, we set $\mu_{p,m} = \mu_{m,p} \forall p \neq m$ to balance the customer demands between any pair of ports. In the moderately (severely) imbalanced trade pattern, we double (treble) the values of one port's or several ports' exported laden containers and keep other values remain the same as those in the balanced trade pattern. The customer demands, i.e., $\varepsilon_{p,m,t} \forall p \neq m$ are assumed to follow normal distribution with mean $\mu_{p,m}$ and standard variance $0.2 \mu_{p,m}$, and be left-truncated at zero. For example, for problem 1, the holding cost parameter is uniformly generated from the interval (0, 5), the repositioning cost parameter is uniformly taken from the interval (5, 10), and the leasing cost parameter is uniformly generated from the interval (10, 30). This reflects the general view that the repositioning cost parameter is greater than the holding cost parameter, while much less than the leasing cost parameter. The parameter $\mu_{p,m}$ in the balanced trade pattern is uniformly generated from the interval (0, 200). In Appendix C, the cost parameters and the parameters of average customer demands in different trade imbalance patterns of problem 1 are presented.

For a multi-port system with STP, the optimal values of the fleet size and the thresholds can be obtained through sequentially solving the problem (13) under Scenario-I and Scenario-II. Under Scenario-I, the NLP is solved by Matlab (version 7.0.1). Under Scenario-II, the IPA-gradient based

algorithm is coded in Visual C++ 5.0. All the numerical studies are tested on and Intel Duo Processor E6750 2.67GHz CPU with 4.00 GB RAM under the Microsoft Vista Operation System. Based on preliminary experiments, we set the simulation period $T = 10100$ with 100 warm-up periods. The termination criteria for searching are that the maximum iteration, namely n_{max} is achieved or the total cost per period in current iteration is significantly greater than that in last iteration. We set $n_{max} = 1000$.

A. Policy Performance Evaluation

To evaluate the performance of the proposed STP, a match back policy (MBP) is introduced for comparison. Such policy is widely accepted and applied in practice and its principle is to balance the container flow in each pair of ports. In other words, ECs to be repositioned from port p to port m in period $t + 1$ should try to match the difference between the total number of laden containers exported from port m to port p and the total number of laden containers exported from port p to port m in period t . Mathematically, we have

$$(z_{p,m,t+1})_{MBP} = (\varepsilon_{m,p,t} - \varepsilon_{p,m,t})^+ \quad \forall t \quad (24)$$

When MBP is adopted, the repositioning cost is independent from the fleet size. Hence, the fleet size which minimizes the expected holding and leasing cost will be the optimal fleet size minimizing the expected total cost. We have $(N^*)_{MBP} = \sum_{p \in P} F_p^{-1}(C_p^L / (C_p^L + C_p^H))$.

To facilitate the comparison of STP and MBP, the percentage of expected total cost reduction achieved by STP from MBP is given in Figure 3. We use a doublet, i.e., (number of ports, trade pattern), to present a particular case. For example, (6, B), (6, M-IB), and (6, S-IB) mean the cases for problem 1 with 6 ports in the balanced, moderately imbalanced and severely imbalanced trade patterns, respectively.

There are three main observations from the results. First, STP outperforms MBP in all cases. As expected, the reduction of total cost by STP is major from the reduction of the repositioning cost. Second, as the system becomes larger, STP can reduce more cost. Hence, it is important for

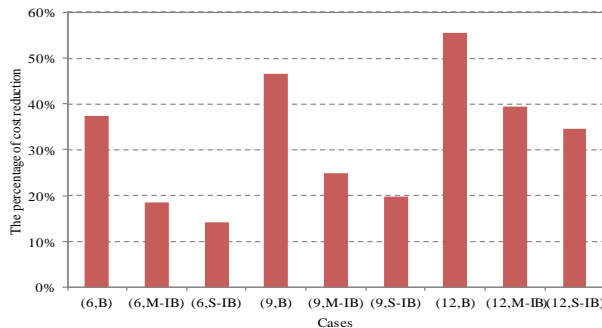


Figure 3. Percentages of expected total cost per period reduction achieved by STP from MBP

shipping companies to use better method in repositioning ECs, instead of resorting to simple way such as the MBP, especially in the complex system. Another observation is that for a problem, it seems that in the imbalanced trade patterns, the advantage of using STP over MBP seems not as great as that in the balanced trade pattern. However, STP still can reduce cost from MBP in the imbalanced trade pattern, such as over 30% cost reduction for the problem with 12 ports.

B. Policy Performance Sensitivity to the Fleet Size

Considering the fact that container fleet is often fixed by shipping companies in practice, we further investigate the policy performance under both policies with given fleet size. If the fleet size is given, the optimal thresholds under STP can be found by the two proposed approaches with little adjustment. Hence, let N^* be the optimal fleet size under STP. The fleet size is varied from $0.7N^*$ to $1.3N^*$ in all 7 cases to investigate the affect of the fleet size on the expected total cost.

Figure 4 shows the expected total cost per period under both policies in case (6, B) with different fleet sizes. First, it is also observed that STP outperforms MBP in all different fleet size cases. It reveals that the expected total cost per period savings achieved by STP over MBP are of the order of 13.18%~37.72%. One possible explanation is that the proposed policy makes the ECR decisions in terms of minimizing the repositioning cost. Besides, the trend of the diamond line shows that, the cost saving achieved by STP from MBP increases gradually when the fleet size increases, and as the fleet size increases further the saving decreases. It is also possible to have a small cost saving percentage when the fleet size is too little. The reason is that when the fleet size is severely insufficient to satisfy the customer demands, a large number of ECs are leased and few requirements for repositioning ECs. Third, the results also show that the minimum expected total cost per period appears to be convex with respect to the fleet size under both policies. It reflects the intuition that the optimal fleet size is the trade-off between the repositioning cost and the holding and leasing cost.

Similarly observations can be also found in other cases.

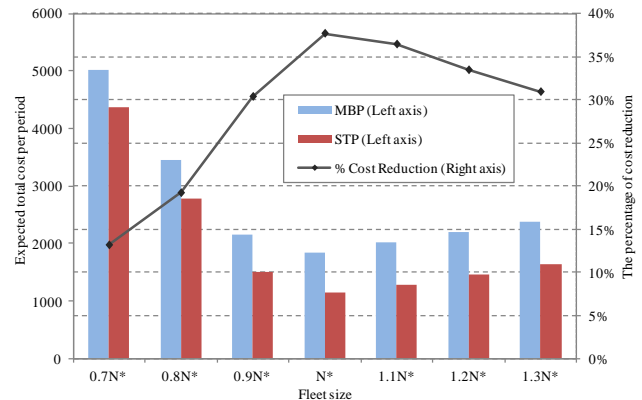


Figure 4. Comparison of STP and MBP in case (6, B)

C. Sensitivity Analysis of the Thresholds

Since the single-level threshold policy is applied to manage ECs in the multi-port system, the thresholds of the policy will significantly affect the performance of the proposed policy. Many factors may impact the thresholds of the policy. The most significant factors are the fleet size and the parameters of leasing cost and holding cost. Next, based on the case (6, B), we explore the sensitivity of the thresholds.

As shown in Figure 5, the optimal threshold values generally increase as the fleet size increases. More interesting, as the fleet size increases from $1.1N^*$ to $1.3N^*$, ports 1~5 have very small increments on their thresholds, while port 6 has large increment on its threshold. This reflects the fact that as the fleet size is much greater than its optimal value, the low holding cost at port 6 works in favor of keeping a large number of ECs at this port. While as the fleet size decrease from $0.9N^*$ to $0.7N^*$, port 3 has the largest decrement on its threshold. A possible explanation is that as owned ECs are insufficient at all ports, many leasing containers have to be used to satisfy the customer demands. And the low leasing cost at port 3 supports it to keep low inventory level of ECs and lease a large number of ECs.

Next, the sensitivity of the thresholds to the holding and leasing cost parameters is considered. From (21), we obtain that $\partial \left(\frac{\partial(J(N,\gamma))}{\partial \gamma_i} \right) / \partial C_i^L \leq 0$ and $\partial \left(\frac{\partial(J(N,\gamma))}{\partial \gamma_i} \right) / \partial C_{q,i,t}^L \geq 0$. It implies that $\frac{\partial(J(N,\gamma))}{\partial \gamma_i}$ will decrease (increase) as C_i^L ($C_{q,i,t}^L$) increases. Thus, when the leasing cost of port i increases, the threshold for this port will increase while the thresholds for some other ports decrease. The similar property for the holding cost parameter can be derived. That is, as the holding cost of port i increases, the threshold of this port will decrease while the thresholds for some other ports increase.

Focusing on the case (6, B) with optimal fleet size under STP, we consider two more cases, i.e., cases A and B, in which the holding and leasing cost parameters of port 6 are increased by 2 times, respectively. Figure 6 shows the results about thresholds changes by cases A and B from the original case. It can be observed that when the holding (leasing) cost

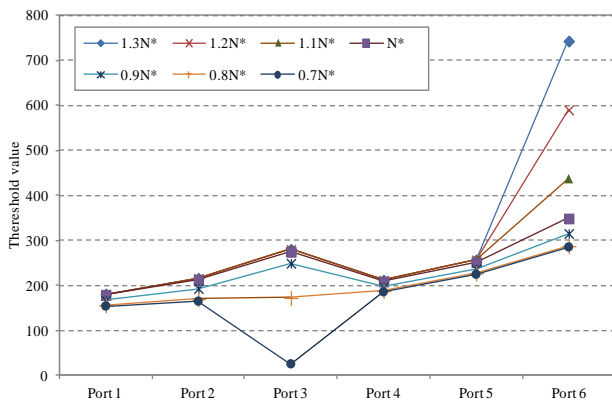


Figure 5. The optimal threshold value for case (6, B) under STP

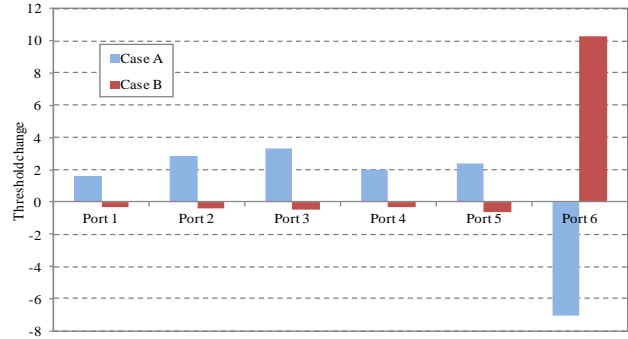


Figure 6. Optimal thresholds changes in cases A and B

parameter of port 6 increases by 2 times, the threshold of port 6 decreases (increases), but the thresholds of other ports increase (decrease). This testifies above phenomenon. Hence, the results reflect the fact that a higher leasing cost at a port works in favor of keeping a large number of ECs while a higher holding cost encourages repositioning out more ECs in a surplus port or repositioning in less ECs in a deficit port.

VI. CONCLUSION AND FUTURE WORK

In this paper, the joint ECR and container fleet sizing problem in a multi-port system is addressed. A single-level threshold policy is developed to reposition ECs periodically by taking into account customer demand uncertainty and dynamic operations. Two approaches, a NLP and IPA-based gradient technique are developed to solve the problem optimizing the fleet size and the parameters of thresholds under Scenario-I and Scenario-II, respectively. The numerical results provide insight that by repositioning the ECs intelligently, the total cost can be significantly reduced.

The main contributions of the study are as follows: (a) a single-level threshold policy with a repositioning rule in terms of minimizing repositioning cost was developed for repositioning ECs in a multi-port system. To the best of our knowledge, few works considered the repositioning rule related to the repositioning cost; (b) the optimal values of the fleet size and the thresholds of the policy can be obtained. These values could be used as reference points for shipping companies to make strategic decisions; (c) by developing the method to solve the difficult EC management problem, i.e., using IPA to estimate gradient, it is innovative and provides a potential methodology contribution in this field.

There are several limitations in the paper. Firstly, ECs were assumed to be dispatched in the right one period. Further research is needed to relax the one-period assumption and consider the problem with different dimensions for the repositioning time. The main challenge is to track the perturbations along the sample path. The second limitation is that the simulation time of the proposed algorithm could be further reduced. Although the proposed algorithm can be utilized to solve the problem with an arbitrary number of ports, as the number of ports increase, more simulation time is needed to search the optimal solution. For example, given a starting point, the proposed

algorithm under Scenario-II takes about 1 hour to get the minimum expected total cost for the small size system, and more time for a larger system. Thus, improving the effectiveness of the proposed algorithm should be an interesting area for future research. We could consider combining the gradient search approach with the evolutionary approach to take advantage of both approaches. In addition, a comparison of the proposed algorithms with other meta-heuristics could be another possible research direction.

APPENDIX A

Note that when ECs are repositioned in a period, the repositioning cost will be the minimum objective values of the transportation models. Hence, the constraints of ports under both scenarios are investigated and some lemmas will be presented, which will be used in proving proposition 2.1.

From property I, we can get that under Scenario-I, $y_t^I = \mathbf{Y}^I, \forall t$. Similarly, under Scenario-II, we can have $y_t^{II} \geq \mathbf{Y}^{II} (\leq \mathbf{Y}^{II})$ if N is $> \sum_{p \in P} \gamma_p^{II} (< \sum_{p \in P} \gamma_p^{II})$. It can be explained as follows. When N is $> \sum_{p \in P} \gamma_p^{II} (< \sum_{p \in P} \gamma_p^{II})$, the transportation model in each period should be unbalanced, i.e., the total number of estimated EC supplies, namely $\sum_{i \in P_t^S} u_{i,t}^S$ is greater (less) than the total number of estimated EC demands, namely $\sum_{j \in P_t^D} u_{j,t}^D$. This unbalanced situation, such as $\sum_{i \in P_t^S} u_{i,t}^S > \sum_{j \in P_t^D} u_{j,t}^D$, means that all EC demands at the deficit ports can be satisfied and some excess ECs still stay at some surplus ports. Hence, the inventory position of each port in each period could be not less (greater) than its target threshold level when N is $> \sum_{p \in P} \gamma_p^{II} (< \sum_{p \in P} \gamma_p^{II})$. The surplus port and deficit port subsets under both scenarios are investigated firstly.

Lemma A.1: For all period t , we have $P_{t+1}^{S,I} \subseteq P_{t+1}^{S,II}$ and $P_{t+1}^{D,II} \subseteq P_{t+1}^{D,I}$ if $N > \sum_{p \in P} \gamma_p^{II}$; $P_{t+1}^{S,II} \subseteq P_{t+1}^{S,I}$ and $P_{t+1}^{D,I} \subseteq P_{t+1}^{D,II}$, if $N < \sum_{p \in P} \gamma_p^{II}$ with same customer demand ω_t .

Proof: With the help of (2), we have $x_{p,t+1}^I = \gamma_p^I + \varphi_{p,t}, \forall p \in P$. It implies that $\varphi_{i,t} > 0, \forall i \in P_{t+1}^{S,I}$ and $\varphi_{j,t} < 0, \forall j \in P_{t+1}^{D,I}$.

Note that both scenarios are investigated with same ω_t . We have $x_{i,t+1}^{II} > y_{i,t}^{II} \geq \gamma_i^{II}, \forall i \in P_{t+1}^{S,I}$ if $N > \sum_{p \in P} \gamma_p^{II}$, and $x_{j,t+1}^{II} < y_{j,t}^{II} \leq \gamma_j^{II}, \forall j \in P_{t+1}^{D,I}$ if $N < \sum_{p \in P} \gamma_p^{II}$. It implies that $P_{t+1}^{S,I} \subseteq P_{t+1}^{S,II}$ if $N > \sum_{p \in P} \gamma_p^{II}$, and $P_{t+1}^{D,I} \subseteq P_{t+1}^{D,II}$ if $N < \sum_{p \in P} \gamma_p^{II}$.

Note that $P_t^S \cup P_t^D = P$ for any scenario. We have $P_{t+1}^{D,II} \subseteq P_{t+1}^{D,I}$ if $N > \sum_{p \in P} \gamma_p^{II}$ and $P_{t+1}^{S,I} \subseteq P_{t+1}^{S,II}$ when $N < \sum_{p \in P} \gamma_p^{II}$. ■

From lemma A.1, there are only two possible cases about the port subsets under both scenarios. One is that there are same port subsets. The other is that there are greater (smaller) surplus port subset and smaller (greater) deficit port subset under Scenario-II than that under Scenario-I when the fleet size is greater (less) than the sum of thresholds.

Furthermore, for the port that is common to surplus or deficit port subsets under both scenarios, its quantity of excess (deficit) ECs is investigated.

Lemma A.2: For all period t , we have

1) If $N > \sum_{p \in P} \gamma_p^{II}$, for $i \in P_{t+1}^{S,I}$ and $j \in P_{t+1}^{D,II} (P_{t+1}^{D,II} \neq \emptyset)$, we have $u_{i,t+1}^{S,II} \geq u_{i,t+1}^{S,I}$ and $u_{j,t+1}^{D,II} \leq u_{j,t+1}^{D,I}$.

2) If $N < \sum_{p \in P} \gamma_p^{II}$, for $i \in P_{t+1}^{S,II} (P_{t+1}^{S,II} \neq \emptyset)$ and $j \in P_{t+1}^{D,I}$, we have $u_{i,t+1}^{S,II} \leq u_{i,t+1}^{S,I}$ and $u_{j,t+1}^{D,II} \geq u_{j,t+1}^{D,I}$, with same customer demand ω_t .

Proof: From (2), (7) and (8), we get that $u_{i,t+1}^S = y_{i,t} + \varphi_{i,t} - \gamma_i$ and $u_{j,t+1}^D = \gamma_j - y_{j,t} - \varphi_{j,t}, \forall i \in P_{t+1}^S, j \in P_{t+1}^D$.

Hence, with help of lemmas A.1, when $N > \sum_{p \in P} \gamma_p^{II}$, we have $u_{i,t+1}^{S,II} \geq \varphi_{i,t} = u_{i,t+1}^{S,I}, \forall i \in P_{t+1}^{S,I}$, and if $P_{t+1}^{D,II}$ is non-empty, $u_{j,t+1}^{D,II} \leq -\varphi_{j,t} = u_{j,t+1}^{D,I}, \forall j \in P_{t+1}^{D,I}$.

Similarly, when $N < \sum_{p \in P} \gamma_p^{II}$, we have $u_{i,t+1}^{S,II} \leq u_{i,t+1}^{S,I}, \forall i \in P_{t+1}^{S,II}$ if $P_{t+1}^{S,II} \neq \emptyset$, and $u_{j,t+1}^{D,II} \geq u_{j,t+1}^{D,I}, \forall j \in P_{t+1}^{D,I}$. ■

From lemma A.2, it implies that for example, when the fleet size is greater than the sum of thresholds, for the common surplus port under both scenarios, its amount of excess ECs under Scenario-II is not less than that under Scenario-I.

We return now to the proof of proposition 2.1.

Proof: Since $P_t^{S,I}$ and $P_t^{D,I}$ are both nonempty, from the transportation model, we have $H(\mathbf{x}_{t+1}^I, \mathbf{Y}^I)|_{\omega_t} > 0$. Hence, when either $P_{t+1}^{D,II}$ or $P_{t+1}^{S,II}$ is empty, we can get that $H(\mathbf{x}_{t+1}^{II}, \mathbf{Y}^{II})|_{\omega_t} = 0 < H(\mathbf{x}_{t+1}^I, \mathbf{Y}^I)|_{\omega_t}$.

When $P_{t+1}^{D,II}$ is nonempty, in the case of $N > \sum_{p \in P} \gamma_p^{II}$, the constraints of the transportation models in period $t+1$ under both scenarios should be (10) and (11), which are as follows:

$$\sum_{j \in P_{t+1}^{D,I}} z_{i,j,t+1} \leq u_{i,t+1}^S, \forall i \in P_{t+1}^S$$

$$-\sum_{i \in P_{t+1}^S} z_{i,j,t+1} \leq -u_{j,t+1}^D, \forall j \in P_{t+1}^D$$

Based on the sensitivity analysis about RHS of linear programming, we get that increasing the RHS coefficient value of above constraint could reduce the minimum objective value. Depending on the port subsets under both scenarios, from lemmas A.1 and A.2, there are only two possible cases:

- Case 1: $P_{t+1}^{S,II} = P_{t+1}^{S,I}$ and $P_{t+1}^{D,II} = P_{t+1}^{D,I}$. It implies that the transportation models under both scenarios have similar constraints for all ports $i \in P_{t+1}^{S,I}, j \in P_{t+1}^{D,I}$. Hence, based on the model under the scenario-I, we increase the RHS values from $u_{i,t+1}^{S,I}$ and $-u_{j,t+1}^{D,I}$ up to $u_{i,t+1}^{S,II}$ and $-u_{j,t+1}^{D,II}$, respectively. Then, the new model will be equivalent to the model under scenario-II, and we can have $H(\mathbf{x}_{t+1}^{II}, \mathbf{Y}^{II})|_{\omega_t} \leq H(\mathbf{x}_{t+1}^I, \mathbf{Y}^I)|_{\omega_t}$.

• Case 2: $P_{t+1}^{S,II} \supset P_{t+1}^{S,I}$ and $P_{t+1}^{D,II} \subset P_{t+1}^{D,I}$. It implies that the transportation models under both scenarios have similar constraints for some ports $i \in P_{t+1}^{S,I}, j \in P_{t+1}^{D,I}$. Similarly, based on the model under the scenario-I (scenario-II), we increase (decrease) the RHS values of ports that are $\in (P_{t+1}^{D,I} - P_{t+1}^{D,II}) ((P_{t+1}^{S,II} - P_{t+1}^{S,I}))$ up (down) to zero. Then, the minimum objective cost of the new model-I (model-II), denoted by C' (C''), should be not greater (less) than that of the original model, i.e., $C' \leq H(\mathbf{x}_{t+1}^I, \boldsymbol{\gamma}^I)|_{\omega_t}$ ($C'' \geq H(\mathbf{x}_{t+1}^{II}, \boldsymbol{\gamma}^{II})|_{\omega_t}$). For the new model-I and model-II, they have similar constraints for ports $i \in P_{t+1}^{S,I}, j \in P_{t+1}^{D,II}$. Hence, as that in case 1, we have $C'' \leq C'$. Finally, we get that $H(\mathbf{x}_{t+1}^{II}, \boldsymbol{\gamma}^{II})|_{\omega_t} \leq C'' \leq C' \leq H(\mathbf{x}_{t+1}^I, \boldsymbol{\gamma}^I)|_{\omega_t}$.

Similarly, when $P_{t+1}^{S,II} \neq \emptyset$ in the case of $\sum_{p \in P} \gamma_p^{II} > N$, we can get that $H(\mathbf{x}_{t+1}^{II}, \boldsymbol{\gamma}^{II})|_{\omega_t} \leq H(\mathbf{x}_{t+1}^I, \boldsymbol{\gamma}^I)|_{\omega_t}$. ■

APPENDIX B

A numerical example is firstly presented to show that how the perturbation in the supply or demand of one binding port, such as i , will change the net actual imported ECs of a pair of ports. Then, a modified stepping stone (MSS) method is proposed to find $E_{i,t}$.

Consider a numerical example with three surplus ports and two deficit ports. ECs are repositioned from the surplus ports to the deficit ports. Assume the total number of EC supplies is greater than the total number of EC demands. And the transportation tableau is presented in Figure 7. Note that one dummy node is created in the transportation tableau to ensure that the total number of demands is equal to the total number of supplies. For illustration, the value of cell (m, l) represents the number of ECs repositioned from the m^{th} surplus port to the l^{th} deficit port. Two sub-examples are investigated.

Surplus port	Deficit port			Change of supply
	1	2	3(dummy)	
1	•			
2	•		•	
3		•	•	
Change of demand				

Figure 7. The transportation tableau
 • Basic cell

Sub-example 1: Perturb the number of EC supply of the first surplus port by $\Delta\gamma$

Consider the balance of the transportation tableau. Perturbing the number of EC supply of the first surplus port by $\Delta\gamma$ implies that the number of EC demand of the dummy deficit port will be perturbed by $\Delta\gamma$. However, since cell $(1, 3)$ is a non-basic cell (see Figure 7), its value should be kept as zero and cannot be changed by the perturbation. To track the changes on the basic variables, a loop is developed, which begins at cell $(1, 3)$ and is back to this cell (see Figure 8).

Surplus port	Deficit port			Change of supply
	1	2	3(dummy)	
1	$\Delta\gamma$ •		⊙	$\Delta\gamma$
2	$-\Delta\gamma$ •		•	$\Delta\gamma$
3		•	•	
Change of demand			$\Delta\gamma$	

Figure 8. Perturb the number of EC supply of the first surplus port by $\Delta\gamma$
 ⊙ Beginning cell

The loop in Figure 8 consists of successive horizontal and vertical segments whose end nodes must be basic variables, except for the two segments starting and ending at the non-basic variable.

More specifically, the changes on the basic variables can be obtained as follows: first increase the value of cell $(1, 1)$ by $\Delta\gamma$; then go around the loop, alternately decrease and then increase basic variables in the loop by $\Delta\gamma$, i.e. decrease the value of cell $(2, 1)$ and increase the value of cell $(2, 3)$ by $\Delta\gamma$. It is observed that the total numbers of repositioning out ECs at the first and the second surplus ports will be increased and decreased by $\Delta\gamma$, respectively.

Sub-example 2: Perturb the number of EC demand of the first deficit port by $\Delta\gamma$

Perturbing the number of EC demand of a deficit port (not the dummy deficit port) by $\Delta\gamma$ will result in the total number of demands from deficit ports (excluding the dummy deficit port) increasing by $\Delta\gamma$. To maintain the balance of the transportation tableau, a hypothetic surplus port is introduced and its supply is assumed to increase by $\Delta\gamma$. Besides, to make the transportation tableau non-degeneracy, the cell (the hypothetic surplus port, the dummy deficit port), i.e., cell $(4, 3)$ in Figure 9, is set as a basic cell.

To track the changes on the basic variables by the perturbation of $\Delta\gamma$ on the number of EC demand of the first deficit port, similar loop is developed in Figure 9. It is observed that the total number of repositioning in ECs at the first deficit port and the total number of repositioning out ECs at the second surplus port will be both increased by $\Delta\gamma$.

Just as the loop using in the typical stepping stone method, the loop in either Figure 8 or Figure 9 is unique. The uniqueness of the loop guarantees that there will be only a pair of ports, whose net actual imported ECs will be affected by perturbing the supply or demand of one binding port.

Surplus port	Deficit port			Change of supply
	1	2	3(dummy)	
1	•			
2	$\Delta\gamma$ •		$-\Delta\gamma$ •	
3		•	•	
4 (hypothetic)	⊙		•	$\Delta\gamma$
Change of demand	$\Delta\gamma$			

Figure 9. Perturb the number of EC demand of the first deficit port by $\Delta\gamma$

TABLE II. LIST OF ADDITIONAL NOTATIONS

	Description
M_t	the number of surplus ports in period t
L_t	the number of deficit ports in period t
$I_{i,t}^S$	the index of the surplus port i in the transportation tableau of period t
$I_{j,t}^D$	the index of the deficit port j in the transportation tableau of period t

Similar results can be obtained under the other case in which the total number of EC supplies is less than the total number of EC demands.

Some additional notations are defined Table II.

When the number of EC supply or demand of port i in period t is perturbed, a MSS method is proposed to find $E_{i,t}$ with the rules as follows.

MSS method to find $E_{i,t}$

- Step 1. Build the transportation tableau based on the transportation model solutions in period t . Create a dummy deficit node, $L_t + 1$ and a dummy surplus node, $M_t + 1$. And arbitrarily insert the value ε into cell $(M_t + 1, L_t + 1)$.
- Step 2. If port i is a surplus port in period t , i.e. $i \in P_t^S$, select cell $(I_{i,t}^S, L_t + 1)$; if port i is a deficit port in period t , i.e. $i \in P_t^D$, select cell $(M_t + 1, I_{i,t}^D)$.
- Step 3. If the selected cell is a basic cell, no port's total repositioned empties quantity will be affected, set $E_{i,t} = \emptyset$ and stop. Otherwise, go to Step 4.
- Step 4. Beginning at the selected cell, trace a loop back to the cell, turning corners only on basic cells. Only successive horizontal & vertical moves are allowed.
- Step 5. If the total EC supply is more than the total EC demand, record the basic cell $(v, L_t + 1)$, $v \neq (M_t + 1)$ in this loop and find port k with $I_{k,t}^S = v$. Otherwise, record the basic cell $(M_t + 1, v)$, $v \neq (L_t + 1)$ in this loop and find port k with $I_{k,t}^D = v$. Then, $E_{i,t} = \{i, e_{i,t}\}$ with $e_{i,t} = k$.

APPENDIX C

The parameters of problem 1 are presented Table III and Table IV.

TABLE III. THE COST PARAMETERS FOR PROBLEM 1

Original port p	Destination port m	$C_{p,m}^R$	C_p^H	C_p^L
1	2	8.421	4.396	16.537
	3	9.917		
2	1	6.761	2.374	12.395
	3	5.848		
3	1	9.027	1.900	24.599
	2	7.053		

TABLE IV. THE PARAMETERS OF AVERAGE CUSTOMER LADEN DEMAND IN DIFFERENT TRADE IMBALANCE PATTERNS FOR PROBLEM 1

Original port p	Destination port m	$\mu_{p,m}$		
		Balanced	Moderately imbalanced	Severely imbalanced
1	2	95.950	191.900	287.850
	3	126.385	252.771	379.155
2	1	95.950	95.950	95.950
	3	176.833	176.833	176.833
3	1	126.385	126.385	126.385
	2	176.833	176.833	176.833

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