A Regenerative Detect & Forward Relay Transmission in Linearly Precoded MU-MIMO Downlink

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Abstract—**Recently, Multi-User MIMO (MU-MIMO) downlink system which uses multiple antennas at Base Station (BS) and accommodates multiple users with multiple reception antennas attracts much attention. In MU-MIMO downlink system, by knowing the Channel State Information (CSI) at BS, Inter User Interferences (IUI's) among users are pre-excluded at BS. By increasing the number of transmission antennas assigned to each user, the transmission quality of each user is effectively improved. In MU-MIMO, there exists linear precoding or nonlinear precoding method, but linear precoding is considered more easily implemented and adjusted than nonlinear precoding. In this paper, we aim the coverage extension and the transmission quality improvement by using a regenerative Detect & Forward (DF) relay in MU-MIMO downlink system. We use Block Diagonalization (BD) + Eigen mode transmission (E-SDM) for linear MU-MIMO downlink scheme. By sharing the BD matrix at both BS and DF relay, the DF relay can demodulate the receive signal with the receive CSI only and can forward the detected signal to each user. At each user, the received signals from BS and DF relay are combined through bit LLR (Log Likelihood Ratio) addition to minimize the BER. With this system configuration and by employing large number of transmission antennas, we have shown the effectiveness of regenerative DF relay through simulations.**

Keywords-MU-MIMO; Block Diagonalization; Eigen mode transmission; Regenerative relay; Detect & Forward.

I. INTRODUCTION

Recently, Multi-User MIMO down link communication systems in which Base Station (BS) can transmit spatially multiplexed signals to multiple users without Inter-User Interference (IUI) are well investigated [1]-[9]. In MU-MIMO downlink system, in order to remove the IUI at BS, the Channel State Information (CSI) of downlink has to be known at BS. By increasing the number of transmission antennas at BS, the channel quality to each user can arbitrary be improved. As representative methods, there exist BD (Block Diagonalization) [4] and Channel Inversion (CI) [5] categorized as linear methods, and DPC (Dirty Paper Coding) [6], THP (Tomlinson-Harashima Precoding) [7] and Vector Perturbation (VP) [8],[9] as nonlinear methods. Although nonlinear methods can achieve greater channel capacity than linear methods, its complexity is higher and the design method is more difficult than linear methods. As for the linear methods, the CI method has the problems of increasing transmission power and limited sum-rate. The BD method consumes a lot of degree of freedom to make the nulls, but it can remove the IUI completely. Also, the BD method matches the Eigen mode transmission (E-SDM; Eigen beam-Space Division Multiplexing) [10] well and is considered as a practical design method. On the other hand, concerning the use of relay in MU-MIMO downlink, increasing the channel capacity by using the relay has been discussed [11],[12]. On the relay transmission in MU-MIMO downlink system, the BD methods are used at BS [13]-[18]. In [13], during the 1st time slot transmission from BS to relay, MU-MIMO is not employed, but during the 2nd time slot from relay to each user, it is used. In [14]-[18], the transmission from BS to relay is done during the 1st time slot, but the direct link from BS to each user during the 1st time slot is not considered.

In this paper, we employed the BD+E-SDM method for the transmission from BS to each user during the 1st time slot. We assume that the DF relay which locates between BS and each user already knows the precoding matrix of BS. By knowing the precoding matrix, the DF relay can demodulate the receive signal with only receive CSI. Accordingly, the BS does not need to assign the transmission antennas to the DF relay and the transmission from BS to DF relay becomes SU (Single-User)-MIMO. During the 2nd time slot, the DF relay transmits the detected signals to each user also with the BD+E-SDM method. At each user, the received signals during the 1st and the 2nd time slots are combined using the bit LLR addition and the combined signal is demodulated. We show the effectiveness of the proposed DF relaying system in MU-MIMO downlink through computer simulations.

The paper is organized as follows. In Section II, the DF relay model in MU-MIMO downlink is introduced. In Section III, we design the downlink transmission during the

TABLE I LIST of PARAMETERS

N_{S} :Number of transmission antennas of BS
N_{u} : Number of users
m_i : Number of reception antennas of user i $(i = 1, \cdots, N_{\mathrm{u}})$
N_{D} : Total number of reception antennas of all users
M_{R} : Number of reception antennas of relay
N_{R} : Number of transmission antennas of relay
$\boldsymbol{C}_{\mathrm{SD}}$: Optimum power assignment matrix to each user on SD link
$\boldsymbol{V}_{\mathrm{SD}}$: Precoding matrix of E-SDM to each user on SD link
$\boldsymbol{N}_{\mathrm{SD}}$: Precoding matrix of BD on SD link
$\boldsymbol{B}_{\mathrm{SD}i}$: Block channel matrix of user i on SD link
nl_i : Nullity of user i on SD link
$L_{\mathrm{SD}i}$: Number of transmit streams of user i on SD link

1st and the 2nd time slots. In Section IV, we clarify the BER characteristics through computer simulations. The paper is concluded with Section V with the most important results.

II. DF RELAY MODEL IN MU-MIMO DOWNLINK

The proposed DF relaying model in MU-MIMO downlink system is shown in Figure 1. The Base Station is equipped with N_{S} transmission antennas. There exist total N_{u} users and the user $i\,(=1,\cdots,N_{\mathrm{u}})$ has m_i reception antennas. Thus, there are total $N_{\mathrm{D}}=\sum_{i=1}^{N_u} m_i$ reception antennas at all users. The DF relay, which locates between BS and user terminals, has M_{R} reception and N_{R} transmission antennas. At Base Station, the transmit signal $\boldsymbol{s}=[\boldsymbol{s}_1^T \ \cdots \ \boldsymbol{s}_i^T \ \cdots \ \boldsymbol{s}_{N_{\mathrm{u}}}^T]^T$ to each user is firstly multiplied by the precoding matrix $\boldsymbol{C}_{\mathrm{SD}}$, where $\boldsymbol{C}_{\mathrm{SD}}$ is the optimum power assignment matrix for multiple different modulation streams in E-SDM of each user. The power assignment in E-SDM of each user is done to minimize the BER by using Lagrange multiplier method [10].

Secondary, the precoding matrix $\boldsymbol{V}_{\mathrm{SD}}$ for making the multiple stream transmission using E-SDM is multiplied by $\boldsymbol{C}_{\mathrm{SD}}\boldsymbol{s}$. The transmit signal to user i is expressed as $\boldsymbol{s}_i=[s_i^{(1)} \ \cdots \ s_i^{(L_{\mathrm{SD}i})}]^T$ where $L_{\mathrm{SD}i}$ is the number of modulation streams of user i in E-SDM. The matrix $\boldsymbol{V}_{\mathrm{SD}}$ is expressed as

$$\boldsymbol{V}_{\mathrm{SD}}=\begin{bmatrix} \boldsymbol{V}_{\mathrm{SD}1} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \ddots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{V}_{\mathrm{SD}N_{\mathrm{u}}} \end{bmatrix} \tag{1}$$

where the diagonal element $\boldsymbol{V}_{\mathrm{SD}i},\ i=1\sim N_{\mathrm{u}}$ is the precoding matrix of E-SDM for each user. Third, the precoding matrix $\boldsymbol{N}_{\mathrm{SD}}$ for BD is multiplied by $\boldsymbol{V}_{\mathrm{SD}}\boldsymbol{C}_{\mathrm{SD}}\boldsymbol{s}$. The transmit signal from BS is then given by $\boldsymbol{x}=\boldsymbol{N}_{\mathrm{SD}}\boldsymbol{V}_{\mathrm{SD}}\boldsymbol{C}_{\mathrm{SD}}\boldsymbol{s} \ =[x_1 \ x_2 \ \cdots \ x_{N_{\mathrm{S}}}]^T$. During the 1st time slot, the transmit signal $\boldsymbol{x}$ is broadcasted both to user terminals and the DF relay. The precoding matrix $\boldsymbol{N}_{\mathrm{SD}}$ makes the channel matrix $\boldsymbol{H}_{\mathrm{SD}}$ from BS to each user block diagonal. The receive signal vector $\boldsymbol{y}_{\mathrm{SD}}$ at destination users is expressed as

$$\begin{cases} \boldsymbol{y}_{\mathrm{SD}}=\boldsymbol{H}_{\mathrm{SD}}\boldsymbol{x}+\boldsymbol{n}_{\mathrm{SD}}=\boldsymbol{B}_{\mathrm{SD}}\boldsymbol{V}_{\mathrm{SD}}\boldsymbol{C}_{\mathrm{SD}}\boldsymbol{s}+\boldsymbol{n}_{\mathrm{SD}} \\ \boldsymbol{B}_{\mathrm{SD}}=\boldsymbol{H}_{\mathrm{SD}}\boldsymbol{N}_{\mathrm{SD}}, \quad \boldsymbol{x}=\boldsymbol{N}_{\mathrm{SD}}\boldsymbol{V}_{\mathrm{SD}}\boldsymbol{C}_{\mathrm{SD}}\boldsymbol{s} \end{cases} \tag{2}$$

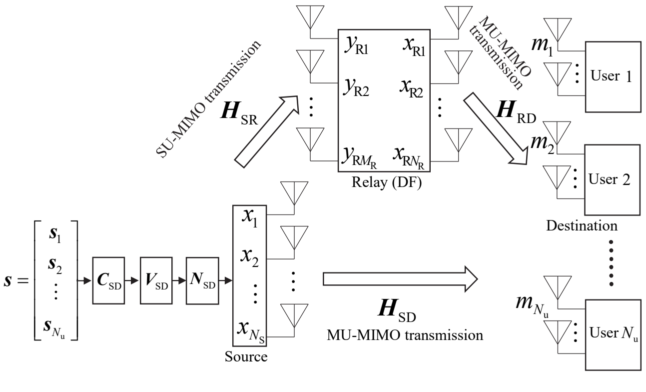

Figure 1. DF relay model in MU-MIMO downlink system

$\boldsymbol{B}_{\mathrm{SD}}$ in (2) is the block diagonalized channel matrix and is expressed as

$$\boldsymbol{H}_{\mathrm{SD}}\boldsymbol{N}_{\mathrm{SD}}=\boldsymbol{B}_{\mathrm{SD}}=\begin{bmatrix} \boldsymbol{B}_{\mathrm{SD}1} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{B}_{\mathrm{SD}2} & \cdots & \vdots \\ \vdots & \cdots & \ddots & \boldsymbol{0} \\ \boldsymbol{0} & \cdots & \boldsymbol{0} & \boldsymbol{B}_{\mathrm{SD}N_{\mathrm{u}}} \end{bmatrix} \tag{3}$$

where $\boldsymbol{B}_{\mathrm{SD}i},\ i=1,\cdots,N_{\mathrm{u}}$ is the block channel matrix for user i and $\boldsymbol{n}_{\mathrm{SD}}=[\boldsymbol{n}_{\mathrm{SD}1}^T \ \cdots \ \boldsymbol{n}_{\mathrm{SD}i}^T \ \cdots \ \boldsymbol{n}_{\mathrm{SD}N_{\mathrm{u}}}^T]^T$ in (2) is the receive noise vector for each user.

The precoding matrix $\boldsymbol{N}_{\mathrm{SD}}$ for BD is derived as follows. Denoting the channel matrix for user i as $\boldsymbol{H}_{\mathrm{SD}i}\,(m_i\times N_{\mathrm{S}})$, the entire channel matrix from BS to users is expressed as

$$\boldsymbol{H}_{\mathrm{SD}}=\begin{bmatrix}\boldsymbol{H}_{\mathrm{SD}1}^T & \cdots & \boldsymbol{H}_{\mathrm{SD}i}^T & \cdots & \boldsymbol{H}_{\mathrm{SD}N_{\mathrm{u}}}^T\end{bmatrix}^T \tag{4}$$

We denote the matrix $\tilde{\boldsymbol{H}}_{\mathrm{SD}i}$ in which the channel matrix $\boldsymbol{H}_{\mathrm{SD}i}$ of user i is excluded from the entire channel matrix $\boldsymbol{H}_{\mathrm{SD}}$ as

$$\tilde{\boldsymbol{H}}_{\mathrm{SD}i}=\begin{bmatrix}\boldsymbol{H}_{\mathrm{SD}1}^T & \cdots & \boldsymbol{H}_{\mathrm{SD}i-1}^T & \boldsymbol{H}_{\mathrm{SD}i+1}^T & \cdots & \boldsymbol{H}_{\mathrm{SD}N_{\mathrm{u}}}^T\end{bmatrix}^T \tag{5}$$

with the size of $((N_{\mathrm{D}}-m_i)\times N_{\mathrm{S}})$ [2],[19]. By making the Singular Value Decomposition (SVD) on $\tilde{\boldsymbol{H}}_{\mathrm{SD}i}$, we get

$$\tilde{\boldsymbol{H}}_{\mathrm{SD}i}=\tilde{\boldsymbol{U}}_{\mathrm{SD}i}\tilde{\boldsymbol{\Lambda}}_{\mathrm{SD}i}\begin{bmatrix}\tilde{\boldsymbol{V}}_{\mathrm{SD}i}^{(1)} & \boldsymbol{N}_{\mathrm{SD}i}\end{bmatrix}^H \tag{6}$$

where $\boldsymbol{N}_{\mathrm{SD}i}$ is the null space of $\boldsymbol{H}_{\mathrm{SD}i}$ and it orthogonalizes $\boldsymbol{H}_{\mathrm{SD}i'}\,(i'\neq i)$, i.e., $\boldsymbol{H}_{\mathrm{SD}i'}\boldsymbol{N}_{\mathrm{SD}i}=\boldsymbol{0}\ (i'\neq i)$. By obtaining all the null spaces of every user, the precoding matrix $\boldsymbol{N}_{\mathrm{SD}}$ for BD is evaluated as

$$\boldsymbol{N}_{\mathrm{SD}}=\begin{bmatrix}\boldsymbol{N}_{\mathrm{SD}1} & \cdots & \boldsymbol{N}_{\mathrm{SD}i} & \cdots & \boldsymbol{N}_{\mathrm{SD}N_{\mathrm{u}}}\end{bmatrix} \tag{7}$$

The size of block channel matrix for user i becomes $\boldsymbol{B}_{\mathrm{SD}i}\ (m_i\times n\ell_i)$, where the nullity $n\ell_i$ of the matrix $\tilde{\boldsymbol{H}}_{\mathrm{SD}i}$ is defined by

$$n\ell_i=nullity(\tilde{\boldsymbol{H}}_{\mathrm{SD}i})=N_{\mathrm{S}}-rank(\tilde{\boldsymbol{H}}_{\mathrm{SD}i}) \tag{8}$$

As the rank of $\tilde{\boldsymbol{H}}_{\mathrm{SD}i}$ is given by

$$rank(\tilde{\boldsymbol{H}}_{\mathrm{SD}i})=\min(N_{\mathrm{D}}-m_i,\ N_{\mathrm{S}}) \tag{9}$$

the nullity $n\ell_i$ is expressed as

$$n\ell_i=N_{\mathrm{S}}-rank(\tilde{\boldsymbol{H}}_{\mathrm{SD}i})=N_{\mathrm{S}}-N_{\mathrm{D}}+m_i>0 \tag{10}$$

In E-SDM, the receive weight $\boldsymbol{U}_{\mathrm{SD}i}^H$ which satisfies

$$\boldsymbol{B}_{\mathrm{SD}i}=\boldsymbol{U}_{\mathrm{SD}i}\boldsymbol{\Lambda}_{\mathrm{SD}i}\boldsymbol{V}_{\mathrm{SD}i}^H \tag{11}$$

is multiplied by $\boldsymbol{y}_{\mathrm{SD}i}$ and

$$\begin{aligned}\boldsymbol{z}_{\mathrm{SD}i}&=\boldsymbol{U}_{\mathrm{SD}i}^H\boldsymbol{y}_{\mathrm{SD}i}=\boldsymbol{U}_{\mathrm{SD}i}^H\boldsymbol{B}_{\mathrm{SD}i}\boldsymbol{V}_{\mathrm{SD}i}\boldsymbol{C}_{\mathrm{SD}i}\boldsymbol{s}_i+\boldsymbol{U}_{\mathrm{SD}i}^H\boldsymbol{n}_{\mathrm{SD}i}\\&=\boldsymbol{\Lambda}_{\mathrm{SD}i}\boldsymbol{C}_{\mathrm{SD}i}\boldsymbol{s}_i+\tilde{\boldsymbol{n}}_{\mathrm{SD}i}\end{aligned} \tag{12}$$

is obtained. As it follows $\boldsymbol{\Lambda}_{\mathrm{SD}i}=diag(\sqrt{d_{\mathrm{SD}i}^{(1)}},\cdots,\sqrt{d_{\mathrm{SD}i}^{(L_{\mathrm{SD}i})}})$ and $\boldsymbol{C}_{\mathrm{SD}i}=diag(\sqrt{\zeta_{\mathrm{SD}i}^{(1)}},\cdots,\sqrt{\zeta_{\mathrm{SD}i}^{(L_{\mathrm{SD}i})}})$, (12) is expanded as

$$\begin{bmatrix} z_{\mathrm{SD}i}^{(1)} \\ \vdots \\ z_{\mathrm{SD}i}^{(L_{\mathrm{SD}i})} \end{bmatrix}=\begin{bmatrix} \sqrt{d_{\mathrm{SD}i}^{(1)}\zeta_{\mathrm{SD}i}^{(1)}}\,s_{\mathrm{SD}i}^{(1)}+\tilde{n}_{\mathrm{SD}i}^{(1)} \\ \vdots \\ \sqrt{d_{\mathrm{SD}i}^{(L_{\mathrm{SD}i})}\zeta_{\mathrm{SD}i}^{(L_{\mathrm{SD}i})}}\,s_{\mathrm{SD}i}^{(L_{\mathrm{SD}i})}+\tilde{n}_{\mathrm{SD}i}^{(L_{\mathrm{SD}i})} \end{bmatrix} \tag{13}$$

where parallel $L_{\mathrm{SD}i}=\min(m_i,n\ell_i)$ AWGN channels independent of each other are obtained.

We assume that the DF relay knows the precoding matrix $\boldsymbol{N}_{\mathrm{SD}}\boldsymbol{V}_{\mathrm{SD}}\boldsymbol{C}_{\mathrm{SD}}$ of BS. Therefore, the BS has to inform $\boldsymbol{N}_{\mathrm{SD}}\boldsymbol{V}_{\mathrm{SD}}\boldsymbol{C}_{\mathrm{SD}}$ to the DF relay before the data transmission starts. The transmission from BS to DF relay during the 1st time slot is done by Single User-MIMO (SU-MIMO) environment. The receive signal $\boldsymbol{y}_{\mathrm{SR}} = [y_{\mathrm{R}1} \;\; y_{\mathrm{R}2} \;\; \cdots \;\; y_{\mathrm{R}M_{\mathrm{R}}}]^T$ at DF relay is expressed as

$$\boldsymbol{y}_{\mathrm{SR}} = \boldsymbol{H}_{\mathrm{SR}}\boldsymbol{x} + \boldsymbol{n}_{\mathrm{SR}} = (\boldsymbol{H}_{SR}\boldsymbol{N}_{\mathrm{SD}}\boldsymbol{V}_{\mathrm{SD}}\boldsymbol{C}_{\mathrm{SD}})\boldsymbol{s} + \boldsymbol{n}_{\mathrm{SR}} \tag{14}$$

where $\boldsymbol{n}_{\mathrm{SR}}\,(M_{\mathrm{R}} \times 1)$ is the receive noise vector at DF relay. The effective channel matrix $\boldsymbol{H}_{\mathrm{SR}}\boldsymbol{N}_{\mathrm{SD}}\boldsymbol{V}_{\mathrm{SD}}\boldsymbol{C}_{\mathrm{SD}}$ in (14) is assumed to be known at the DF relay and the demodulation of transmit signal $\boldsymbol{s}$ is done by MMSE (Minimum Mean Squared Error) nulling, MLD (Maximum Likelihood Detection) or Sphere Decoding. This means that the DF relay does not need to inform the channel matrix $\boldsymbol{H}_{\mathrm{SR}}$ between BS and DF relay to the BS. The demodulated signal $\hat{\boldsymbol{s}}$ at DF relay is transmitted to each user during the 2nd time slot using MU-MIMO with BD+E-SDM which is the same as in the Source-Destination (SD) link, i.e., the E-SDM is employed for the multiple stream transmission from the DF relay to each user. At each user terminal, the received signals during the 1st and the 2nd time slots are combined using bit LLR addition and demodulated. When the errors are detected at the DF relay through the CRC (Cyclic Redundancy Check), erroneous user does not employ the Relay-Destination (RD) transmission to prevent error propagation, instead the SD link transmission is repeated in the second time slot.

III. DESIGN OF DOWNLINK TRANSMISSION

A. *Transmission during the 1st time slot*

1) *Design of SD Link*

The Source-Destination (SD) link transmission during the 1st time slot is done by using BD+E-SDM. The size of block channel matrix $\boldsymbol{B}_{\mathrm{SD}i}$ is given by $m_i \times n\ell_i$ and the number of streams of E-SDM (eigen mode transmission) becomes $L_{\mathrm{SD}i}$=$\min(m_i, n\ell_i)$. When the number of streams is one, BD+E-SDM is referred to as BD+MRT (Maximum Ratio Transmission) [20]. From (10), the nullity $n\ell_i$ of user i can be arbitrary chosen by increasing or decreasing the number of transmission antennas N_{S} at BS. If the elements of channel matrix $\boldsymbol{H}_{\mathrm{SD}}$ follow the i.i.d (independent and identically distributed) complex Gaussian random variables, i.e., if $\boldsymbol{H}_{\mathrm{SD}}$ is the MIMO channel matrix of quasi-static flat Rayleigh fading, the diversity order of the first eigen mode stream in E-SDM for the block channel matrix $\boldsymbol{B}_{\mathrm{SD}\,i}$ is given by $m_i \cdot n\ell_i$ [21]. When the minimum number of reception antennas among users is $m_{\min}$, from (10) the nullity $n\ell_{\min}$ of minimum antenna user is given by

$$n\ell_{\min} = N_{\mathrm{S}} - N_{\mathrm{D}} + m_{\min} > 0 \tag{15}$$

Hence, the number of transmission antennas N_{S} at BS which enables the BD must satisfy

$$N_{\mathrm{S}} > N_{\mathrm{D}} - m_{\min} \tag{16}$$

The nullity $n\ell_i$ of user i other than the minimum antenna user becomes

$$n\ell_i = N_{\mathrm{S}} - N_{\mathrm{D}} + m_i > n\ell_{\min} \tag{17}$$

The size of block channel matrix $\boldsymbol{B}_{\mathrm{SD}\,i}$ of user i is determined as $m_i \times n\ell_i$. In this design method, firstly the nullity $n\ell_{\min}$ of the user which has the minimum number of reception antennas $m_{\min}$ is determined and secondary the nullity $n\ell_i$ of the other user i is derived. The diversity orders of the first eigen mode stream of minimum antenna user and other user i are given by $m_{\min} \cdot n\ell_{\min}$ and $m_i \cdot n\ell_i \geq m_{\min} \cdot n\ell_{\min}$, respectively.

For example, we consider the case where the number of total users is $N_{\mathrm{u}} = 3$, the numbers of reception antennas of three users are $m_1 = 3$, $m_2 = 2$, $m_3 = 1$ respectively, and the total number of reception antennas is $N_{\mathrm{D}} = m_1 + m_2 + m_3 = 6$. In this case, $m_3 = 1$ is minimum and it holds $m_{min} = m_3 = 1$. If the expected diversity order of the first eigen mode stream of user 3 is set to 3, for example, then we obtain $n\ell_{min} = 3/m_{min} = 3/1 = 3$. With these parameters, the total number of transmission antennas at BS N_{S} is determined as $N_{\mathrm{S}} = n\ell_{min} + N_{\mathrm{D}} - m_{min} = 3 + 6 - 1 = 8$ and the size of block matrix $\boldsymbol{B}_{\mathrm{SD}3}$ of user 3 becomes $m_{min} \times n\ell_{min} = 1 \times 3$. Thus, the nullity of user 2 is determined as $n\ell_2 = N_{\mathrm{S}} - N_{\mathrm{D}} + m_2 = 8 - 6 + 2 = 4$, the size of $\boldsymbol{B}_{\mathrm{SD}2}$ becomes $m_2 \times n\ell_2 = 2 \times 4$, and the diversity order of the first eigen mode stream of user 2 is given as $m_2 \cdot n\ell_2 = 2 \cdot 4 = 8$. Likewise, the nullity $n\ell_1$ of user 1 is given by $n\ell_1 = N_{\mathrm{S}} - N_{\mathrm{D}} + m_1 = 8 - 6 + 3 = 5$, $\boldsymbol{B}_{\mathrm{SD}1}$ becomes $m_1 \times n\ell_1 = 3 \times 5$, and the diversity order of the first eigen mode stream of user 1 is determined as $m_1 \cdot n\ell_1 = 3 \cdot 5 = 15$.

2) *Design of SR Link*

For the Source-Relay (SR) link from BS to DF relay, the precoding for DF relay at BS is not employed. This means no degree of freedom of transmission antennas (number of transmission antennas) at BS is consumed for the SR link. This is because if the transmission antennas at BS are assigned to the DF relay using BD, in the absence of DF relay the assigned transmission antennas to DF relay are of no use. Therefore, the extra transmission antennas are then used for the users to enhance the SD link quality. In this case, the effect of using DF relay does not become obvious compared with the enhanced SD link quality. That is, the DF relay should be employed when the SD link quality is poor and the additional DF relay usage brings the great effect to the overall performance. As the demodulation at DF relay is done by only using receive CSI, the precoding matrix $\boldsymbol{N}_{\mathrm{SD}}\boldsymbol{V}_{\mathrm{SD}}\boldsymbol{C}_{\mathrm{SD}}$ at BS needs to be informed to the DF relay in advance before the data transmission from BS to DF relay starts. The DF relay demodulates the receive signal with MMSE nulling, MLD or Sphere Decoding by using effective channel matrix of $\boldsymbol{H}_{\mathrm{SR}}\boldsymbol{N}_{\mathrm{SD}}\boldsymbol{V}_{\mathrm{SD}}\boldsymbol{C}_{\mathrm{SD}}$. So, the SR link transmission is done under SU-MIMO environment and not MU-MIMO. The SR link quality directly affects the subsequent Relay-Destination (RD) link quality very much, because the poor SR link quality causes the error propagation on the RD link. Thus, we must raise the SR link quality as much as possible. To solve this problem, increasing the number of reception antennas at DF relay or increasing the number of transmission antennas at BS is considered. Also at DF relay, MLD or Sphere Decoding with better BER characteristic than MMSE nulling is considered. But when the number of total transmit streams of L from BS to DF relay is large, the exponential increase of complexity in MLD becomes a problem. In such case, we can resort the problem to employ the Sphere Decoding with less complexity or to use the MMSE nulling with far less complexity but degraded performance for demodulating SU-MIMO signals. When the number of reception antennas at DF relay is given by M_{R} , the effective channel matrix for demodulating the transmit signal $\boldsymbol{s}$ at DF relay link becomes $\boldsymbol{H}_{\mathrm{SR}}\boldsymbol{N}_{\mathrm{SD}}\boldsymbol{V}_{\mathrm{SD}}\boldsymbol{C}_{\mathrm{SD}}$ $(M_{\mathrm{R}} \times L)$. Accordingly, to apply MMSE nulling at DF relay it needs $M_{\mathrm{R}} \geq L$. Even though the elements of $\boldsymbol{H}_{\mathrm{SR}}$ are i.i.d. complex Gaussian random variables, the row elements of effective channel matrix $\boldsymbol{H}_{\mathrm{SR}}\boldsymbol{N}_{\mathrm{SD}}\boldsymbol{V}_{\mathrm{SD}}\boldsymbol{C}_{\mathrm{SD}}$ of SR link do not always become i.i.d. complex Gaussian random variables. This channel element correlation deteriorates the BER characteristic of MMSE nulling, MLD (or Sphere Decoding) compared with i.i.d. random variable case. Also, if errors are detected at the DF relay with CRC code, the subsequent RD transmission is not employed for the erroneous users, instead the SD link transmission is repeated using the 2nd time slot. For error free users on SR link, the RD link transmission is done. The received signals at each user during the 1st and 2nd time slots are then combined through the bit LLR addition and the multiple streams to each user are demodulated.

B. Transmission during the 2nd time slot

For the Relay-Destination (RD) link transmission during the 2nd time slot, we use BD+MRT or BD+E-SDM, which is the same as in the 1st time slot. In this case, the number of streams to each user on Source-Destination (SD) link does not need to coincide with the one on RD link, because the signal combining between SD link and RD link is done using bit LLR addition. However, the transmission rate (bps/Hz) must be the same between SD link and RD link for the bit LLR combining. We can design the RD link quality as in the SD link. Thus, we can improve the RD link quality by increasing the number of transmission antennas N_{R} at DF relay. As already stated in Section III *A)-2)*, in order to prevent the error propagation in the RD link, we adopt the protocol in which the data from DF relay are only forwarded to each user in case of no error detected at the DF relay. When the error is detected at the DF relay and the RD link is not employed during the 2nd time slot, in order to prevent the degradation of receive signal quality, we repeat the SD link transmission using the vacant 2nd time slot. The signals received in the 1st and 2nd time slots are bit LLR combined and demodulated at each user as mentioned in Section III *A)-2)*.

At each user terminal, each stream of MRT or E-SDM is equivalently represented as the AWGN channel with the positive real gain h . When the received signal in each stream is denoted as r and the corresponding transmitted signal $s_l,\ l = 0,\cdots,Q-1$ has Q modulation levels, the equivalent AWGN channel of each stream in each user is expressed as

$$r = hs_l + n,\ l = 0,1,\cdots,Q-1 \tag{18}$$

The symbol LLR is the extension of bit LLR and is defined as

$$\begin{aligned}\lambda_l &= \log_e\left\{\frac{p(s_l \mid r)}{p(s_0 \mid r)}\right\} = \log_e\left\{\frac{[p(r \mid s_l)p(s_l)]/p(r)}{[p(r \mid s_0)p(s_0)]/p(r)}\right\}\\ &= \log_e\left\{\frac{p(r \mid s_l)}{p(r \mid s_0)}\right\}\end{aligned} \tag{19}$$

where we have assumed the priori probabilities are all equal, i.e., $p(s_l) = 1/Q,\ l = 0,1,\cdots,Q-1$. The transition probability density function $p(r \mid s_l)$ is expressed as

$$p(r \mid s_l) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{|r - hs_l|^2}{2\sigma^2}\right] \tag{20}$$

where $\sigma^2 = (1/2)E\{|n|^2\}$. From (19) and (20), it holds

$$\lambda_l = \frac{-|r - hs_l|^2 + |r - hs_0|^2}{2\sigma^2} \tag{21}$$

When the modulation level is $Q = 2$, the symbol LLR coincides with the bit LLR.

The bit LLR is obtained easily from the symbol LLR. When the data bits assigned to a signal point is $(c_1,\cdots,c_k,\cdots,c_N)$, the *k*-th bit LLR $\lambda(c_k)$ is given by

$$\begin{aligned}\lambda(c_k) &= \log_e\left\{\frac{p(c_k = 0 \mid r)}{p(c_k = 1 \mid r)}\right\} = \log_e\left\{\frac{p(c_k = 0, r)/p(r)}{p(c_k = 1, r)/p(r)}\right\}\\ &= \log_e\left\{\frac{\sum\limits_{\{s \mid c_k = 0\}} p(s, r)}{\sum\limits_{\{s' \mid c_k = 0\}} p(s', r)}\right\} = \log_e\left\{\frac{\sum\limits_{\{s \mid c_k = 0\}} p(s)p(r \mid s)}{\sum\limits_{\{s' \mid c_k = 0\}} p(s')p(r \mid s')}\right\}\end{aligned} \tag{22}$$

where $\{s \mid c_k = 0\}$ and $\{s' \mid c_k = 1\}$ denote the transmit symbols s and s' with the *k*-th bit being $c_k = 0$ and $c_k = 1$ respectively. As each bit is generated equally, it holds $p(s) = p(s')$. From (19), we can say

$$p(r \mid s_l) = \exp(\lambda_l)\,p(r \mid s_0) \tag{23}$$

and (22) is expressed as

$$\begin{aligned}\lambda(c_k) &= \log_e\left\{\frac{\sum_{s \mid c_k = 0}\exp(\lambda)\,p(r \mid s_0)}{\sum_{s' \mid c_k = 1}\exp(\lambda')\,p(r \mid s_0)}\right\}\\ &= \log_e\left\{\frac{\sum_{\lambda \mid c_k = 0}\exp(\lambda)}{\sum_{\lambda' \mid c_k = 1}\exp(\lambda')}\right\}\end{aligned} \tag{24}$$

where $\lambda \mid c_k = 0$ and $\lambda' \mid c_k = 1$ denote the symbol LLR’s of s and s' in which the *k*-th bits are $c_k = 0$ and $c_k = 1$ respectively. Next, we define the symbol LLR’s

$s_\alpha (= s_0 - \alpha)$ and $s_\beta (= s_0 - j\beta)$ as λ_α and λ_β respectively, where s_α and s_β denotes the transmit signal points displaced by α and $j\beta$ from s_0 respectively. It then holds

$$\lambda_\gamma = \lambda_\alpha + \lambda_\beta \tag{25}$$

where $\gamma = \alpha + j\beta$. By using (25), we do not need to obtain every symbol LLR's when calculating the bit LLR's resulting in simplifying the bit LLR calculation. The bit LLR addition between the 1st time slot and 2nd time slot is done at each user stream and finally the bit decision is made based on the added bit LLR value.

IV. INVESTIGATION OF BER CHARACTERISTICS THROUGH COMPUTER SIMULATIONS

We have checked the BER characteristics of the proposed MU-MIMO downlink transmission using DF relay. The abscissa of BER characteristic is taken as the transmit SNR [22], which is defined as the ratio of transmission power P from BS for a user to the receive noise power σ^2 at each user reception antenna and is given as

$$\left(\frac{S}{N}\right)_{\text{transmit}} = \frac{P}{\sigma^2} \tag{26}$$

where the transmission power P is equal among every user. The channel from BS to each user, the channel from BS to DF relay and the channel from DF relay to each user are all assumed to be quasi-static Rayleigh fading channel. That is, the element h_{ij} of channel matrix $\boldsymbol{H}\,(\boldsymbol{H}_{\text{SD}}, \boldsymbol{H}_{\text{SR}}, \boldsymbol{H}_{\text{RD}})$ is an i.i.d. complex Gaussian random variable with the variance of $E\{|h_{ij}|^2\} = 1$. We consider the distance from BS to each user is equal among users and the DF relay locates at the middle point between BS and users. We also set the power decaying exponent of propagation loss as $\alpha = 3.5$ regarding the different distances from BS to DF relay and from BS to users. The BER characteristics in Figure 2~Figure 13 are averaged over N_u users. Thus, the BER shows the average BER among all users.

First, we investigate the case where the number of transmission antennas of BS is $N_{\text{S}} = 2$, the number of user terminals $N_{\text{u}} = 2$, the number of reception antennas of each user $m_1 = m_2 = 1$, the number of reception antennas of DF relay $M_{\text{R}} = 4$, and the number of transmission antennas of DF relay $N_{\text{R}} = 2$. We call this as $2\times(4,2)\times(1,1)$ model. In this model, for the SD link during the 1st time slot, BD+MRT scheme is used as the MU-MIMO down link transmission. QPSK modulation is used for each user stream. The BER characteristic is shown in Figure 2. "SD link 2 times w/o relay" means the scheme in which the MU-MIMO transmission on SD link is repeated twice without using the relay. "$2\times(4,2)\times(1,1)$ MMSE" and "$2\times(4,2)\times(1,1)$ MLD" mean the schemes in which the receive signal at DF relay is demodulated using MMSE nulling and using MLD respectively.

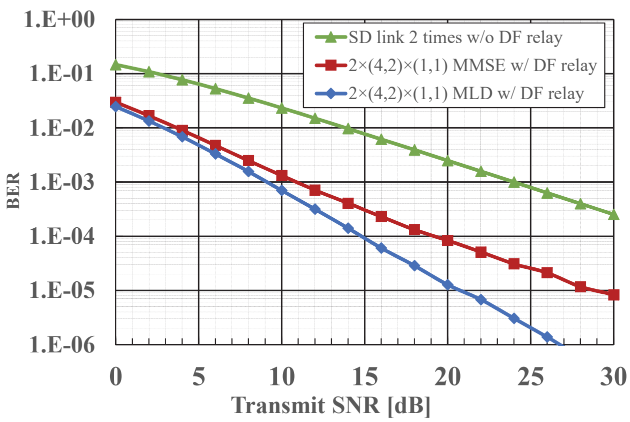

Figure 2. BER characteristics of $2\times(4,2)\times(1,1)$ model (Transmission rate to each user is 2 (bps/Hz).)

From Figure 2, we see that by using the DF relay and by combining the bit LLR's of SD link and RD link signals, the BER characteristic with using DF relay is very much improved compared with the 2 times transmission on SD link without using DF relay. From Figure 2, "SD link 2 times w/o DF relay" shows the BER slope of about $10^{-1}/10\,(\text{dB})$ for the transmit SNR=10~20 (dB) and we see that the diversity order of 1 $(\text{BER} = 10^{-1}/10\ \text{dB})$ is almost obtained. As stated in Section III. *A)-2)*, as the row elements of channel matrix $\boldsymbol{H}_{\text{SR}}\boldsymbol{N}_{\text{SD}}\boldsymbol{V}_{\text{SD}}\boldsymbol{C}_{\text{SD}}$ on the SR link do not always become independent, the SR link quality is degraded. Accordingly, the diversity order of 2 at each user is not achieved especially when the MMSE nulling is used at the DF relay. However, by using the MLD at DF relay, the SR link quality is improved. So "MLD w/ DF relay" in Figure 2 shows the BER slope of about $10^{-2}/10\,(\text{dB})$ for the transmit SNR=10~20 (dB) and we see that the diversity order of 2 is almost obtained.

Next, we consider the case where the number of transmission antennas at BS is $N_{\text{S}} = 4$, the number of users $N_{\text{u}} = 2$ and the numbers of reception antennas of each user are $m_1 = m_2 = 1$. Compared with the previous case of $N_{\text{S}} = 2$, more transmission antennas are assigned to each user. The numbers of reception and transmission antennas at DF relay are $M_{\text{R}} = 4$ and $N_{\text{R}} = 4$, respectively. We call this as $4\times(4,4)\times(1,1)$ model. The transmission protocols of SD, SR and RD links are the same as the previous $2\times(4,2)\times(1,1)$ model. The modulation format is QPSK. We show the simulation results in Figure 3. The channel matrix $\boldsymbol{H}_{\text{SD}}$ on SD link is 2×4 and the channel matrix $\tilde{\boldsymbol{H}}_{\text{SD}i}$ in which the channel matrix for user i is excluded from $\boldsymbol{H}_{\text{SD}}$ becomes 1×4 in Figure 3. From (17), the nullity of $\tilde{\boldsymbol{H}}_{\text{SD}i}$ is calculated as $n\ell_i = 4 - 2 + 1 = 3,\ (i = 1,2)$. The

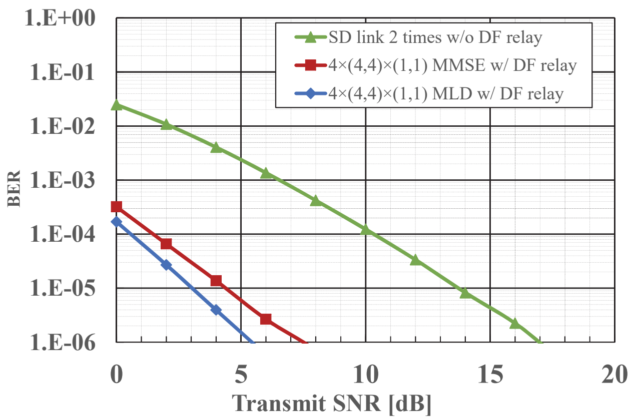

Figure 3. BER characteristics of $4\times(4,4)\times(1,1)$ model (Transmission rate to each user is 2 (bps/Hz).)

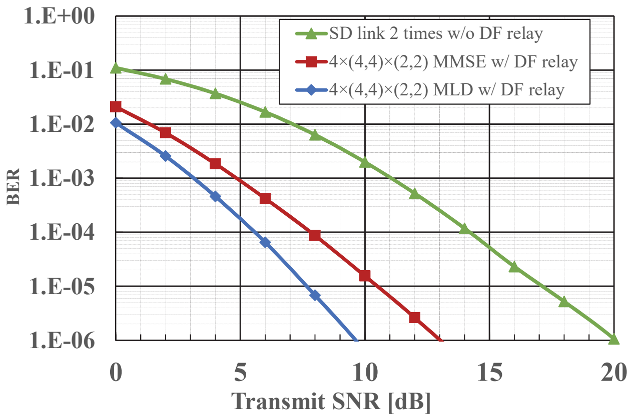

Figure 4. BER characteristics of $4\times(4,4)\times(2,2)$ model (Transmission rate to each user is 4 (bps/Hz) and 16QAM or QPSK is optimally selected to minimize the BER.)

size of block matrix $\boldsymbol{B}_i$ of user i becomes $m_i \times n\ell_i = 1\times 3, (i=1,2)$ and the diversity order of each user stream is given by $m_i \cdot n\ell_i = 3, (i=1,2)$. From Figure 3, "SD link 2 times w/o DF relay" shows the BER slope of about $10^{-3}/10\,(\mathrm{dB})$ for the transmit SNR=5~15 (dB) and we see that the diversity order of 3 is almost obtained. When using the DF relay, we can also expect the diversity order of 3 on the RD link as in the SD link. Thus, if there is no error on the SR link, then we can say the diversity order of 6 is obtained on the BER after the bit LLR addition at each user. As the transmit SNR becomes higher, the errors on SR link decrease and the diversity order of 6 is more easily achievable at high SNR region. But in Figure 3, "MMSE w/ DF relay" shows the BER slope of about $10^{-1}/2\,(\mathrm{dB})$ for the transmit SNR=4~6 (dB) and "MLD w/ DF relay" shows the BER slope of about $10^{-1}/2\,(\mathrm{dB})$ for the transmit SNR=3~5 (dB). So by using DF relay, we know the diversity order of 5 is achieved in this SNR region.

Next, we consider the case where the number of transmission antennas at BS is $N_{\mathrm{S}}=4$, the number of users $N_{\mathrm{u}}=2$ and the numbers of reception antennas of each user are $m_1=m_2=2$. The numbers of reception and transmission antennas at DF relay are $M_{\mathrm{R}}=4$ and $N_{\mathrm{R}}=4$, respectively. We call this as $4\times(4,4)\times(2,2)$ model. In this case, the optimum modulation formats which minimize the BER characteristics are selected under the constant transmission rate of 4 (bps/Hz) on the SD link. This means one stream transmission with BD+MRT using 16QAM or two stream transmission with BD+E-SDM using two QPSK's is adaptively selected for given $\boldsymbol{H}_{\mathrm{SD}}$ [10]. Also, in case of two stream transmission using two QPSK's, the optimum power assignment to the 1st and 2nd eigen mode channels which minimizes the BER is employed [10]. In this two stream transmission, the DF relay needs to know in advance the precoding matrix $\boldsymbol{N}_{\mathrm{SD}}$ for BD, the matrix $\boldsymbol{V}_{\mathrm{SD}}$ for the eigen mode transmission in E-SDM and the matrix $\boldsymbol{C}_{\mathrm{SD}}$ for the power allocation factor to the 1st and 2nd eigen mode channels. Also, in order to make the bit LLR addition at each user, the transmission rate on the RD link must coincide with the one on the SD link. Hence, BD+E-SDM transmission on RD link adopts the same transmission rate as in the SD link. The simulation results are shown in Figure 4. BD+E-SDM scheme is employed on the SD link and the size of block channel matrix for user i becomes $\boldsymbol{B}_i = 2\times 2\ (i=1,2)$ in Figure 4. The transmission to each user is done by one stream transmission with the maximum eigen value using 16QAM or two stream transmission with two different eigen values using two QPSK's. Figure 4 shows the average BER characteristics. In this $4\times(4,4)\times(2,2)$ model, like in Figure 2 and Figure 3, the use of DF relay and the bit LLR addition at each user during the 1st and 2nd time slots improve the diversity order and the BER characteristic when compared with the 2 times transmission on the SD link without using relay.

Next, we consider the case where the number of transmission antennas at BS is $N_{\mathrm{S}}=6$, the number of users $N_{\mathrm{u}}=2$ and the numbers of reception antennas of each user are $m_1=m_2=3$. The numbers of reception and transmission antennas at DF relay are $M_{\mathrm{R}}=6$ and $N_{\mathrm{R}}=6$, respectively. We call this as $6\times(6,6)\times(3,3)$ model. This model is the extension of previous $4\times(4,4)\times(2,2)$ model. The transmission protocols are the same as the previous $4\times(4,4)\times(2,2)$ model. On the SD link, the size of block channel matrix for user i becomes $\boldsymbol{B}_i=3\times 3\ (i=1,2)$. For each user, one stream transmission with BD+MRT using 64QAM, two stream transmission with BD+E-SDM using 16QAM and QPSK, or three stream transmission with BD+E-SDM using three QPSK's is adaptively selected for $\boldsymbol{H}_{\mathrm{SD}}$ under the constant transmission rate of 6 (bps/Hz). The RD link transmission uses the same bit transmission rate as in the SD link. We show the simulation results in Figure 5. When comparing the BER characteristics at $\mathrm{BER}=10^{-6}$, "MMSE w/ DF relay" and "MLD w/ DF relay" improve the BER by 5 (dB) and 8 (dB), respectively compared with "SD link 2 times w/o DF relay." We also

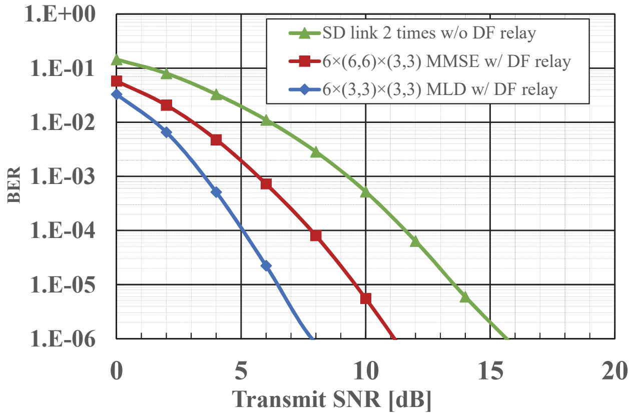

Figure 5. BER characteristics of 6×(6,6)×(3,3) model (Transmission rate to each user is 6 (bps/Hz) and 64QAM, 16QAM or QPSK is optimally selected to minimize the BER.)

find the slope of BER curve of “MLD w/ DF relay” is steeper than “SD link 2 times w/o DF relay” and know the effectiveness of using DF relay as in the previous cases.

Next, we consider the case where the number of transmission antennas at BS is $N_{\mathrm{S}}=8$, the number of users $N_{\mathrm{u}}=4$ and the numbers of reception antennas of each user are $m_1=m_2=m_3=m_4=2$. The numbers of reception and transmission antennas at relay are $M_{\mathrm{R}}=8$ and $N_{\mathrm{R}}=8$, respectively. We call this as $8\times(8,8)\times(2,2,2,2)$ model. This model is the extension of previous $6\times(6,6)\times(3,3)$ model to $N_{\mathrm{u}}=4$ users. The transmission protocols are the same as the previous $4\times(4,4)\times(2,2)$ and $6\times(6,6)\times(3,3)$ models. On the SD link, the size of block channel matrix for user i becomes $\boldsymbol{B}_i=2\times 2$ $(i=1,2,3,4)$. For each user, one stream transmission with BD+MRT using 16QAM or two stream transmission with BD+E-SDM using two QPSK’s is adaptively selected for $\boldsymbol{H}_{\mathrm{SD}}$ under the constant transmission rate of 4 (bps/Hz). The RD link transmission uses the same transmission rate as in the SD link. We show the simulation results in Figure 6. In this $8\times(8,8)\times(2,2,2,2)$ model, the demodulation using MLD at relay becomes difficult because the number of searches in MLD becomes $16^4=4^8=65536$ for the total 4~8 streams from the BS. So instead of using MLD, we employed the Sphere Decoding SE algorithm [23],[24] that can obtain the same Maximum Likelihood (ML) solution as the MLD with far less computational complexity. Like in Figures 2-5, the use of DF relay and the bit LLR addition at each user during the 1st and 2nd time slots improve the diversity order and the BER characteristic compared with the 2 times transmission on SD link without using DF relay. When comparing the BER characteristics at BER $=10^{-6}$, “MMSE w/ DF relay” and “MLD w/ DF relay” improve the BER by 6 (dB) and 11 (dB) respectively compared with “SD link 2 times w/o DF relay.” We also find the slope of BER curve of “MLD w/ DF relay” is steeper than “SD link 2 times w/o DF relay” and know the effectiveness of using DF relay as in the previous cases.

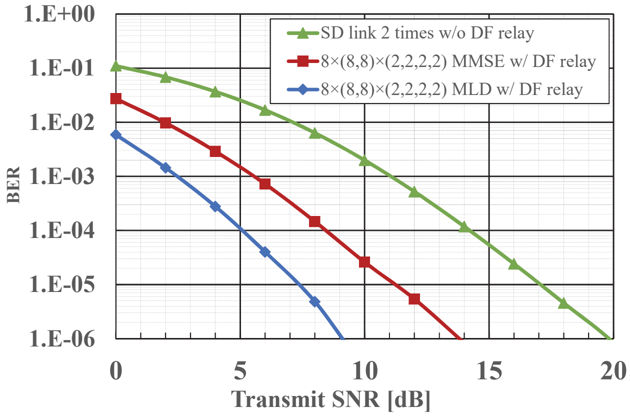

Figure 6. BER characteristics of 8×(8,8)×(2,2,2,2) model (Transmission rate to each user is 4 (bps/Hz) and 16QAM or QPSK is optimally selected to minimize the BER.)

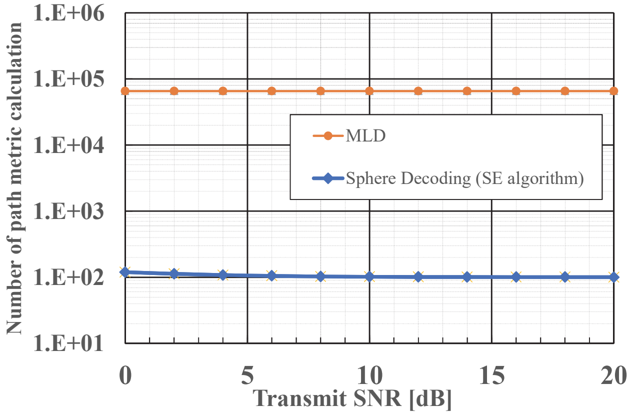

Figure 7. Comparison of number of path metric calculation at DF relay between MLD and Sphere Decoding in 8×(8,8)×(2,2,2,2) model (Transmission rate to each user is 4 (bps/Hz) and 16QAM or QPSK is optimally selected to minimize the BER.)

The effect of complexity reduction by using Sphere Decoding (SE algorithm in [23],[24]) in place of MLD at DF relay is shown in Figure 7. When MLD is employed at DF relay, the required number of path metric calculation for 4 users is $16^4=65536$, however, by using Sphere Decoding it can be reduced to almost 100 and the large complexity reduction has been achieved. Also, the complexity reduction by Sphere Decoding becomes more prominent at higher transmit SNR.

Next, we consider the case where the number of transmission antennas at BS is $N_{\mathrm{S}}=6$, the number of users $N_{\mathrm{u}}=2$ and the numbers of reception antennas of each user are $m_1=3, m_2=2$. The numbers of reception and transmission antennas at relay are $M_{\mathrm{R}}=5$ and $N_{\mathrm{R}}=4$, respectively. We call this as $6\times(5,4)\times(3,2)$ model. On the SD link, the sizes of block channel matrices for user 1 and user 2 become $\boldsymbol{B}_{\mathrm{SD1}}=3\times 4$ and $\boldsymbol{B}_{\mathrm{SD2}}=2\times 3$, respectively. For user 1, one stream transmission with BD+MRT using 64QAM, two stream transmission with BD+E-SDM using

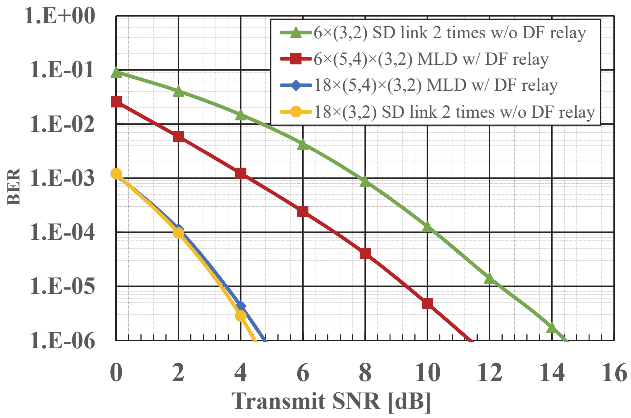

Figure 8. BER characteristics of $6\times(5,4)\times(3,2)$ or $18\times(5,4)\times(3,2)$ model (Transmission rate to user 1 is 6 (bps/Hz) and 64QAM, 16QAM or QPSK is optimally selected to minimize the BER. Transmission rate to user 2 is 4(bps/Hz) and 16QAM or QPSK is optimally selected to minimize the BER.)

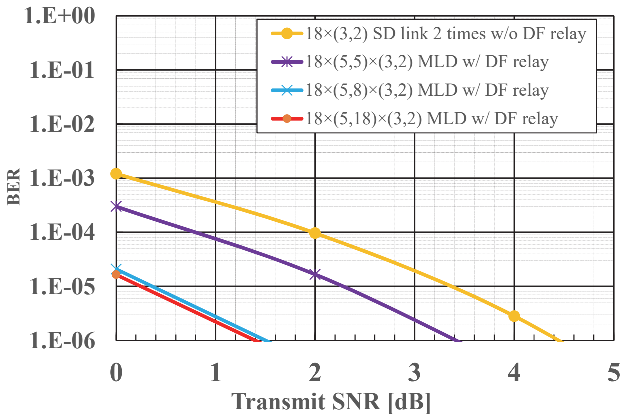

Figure 9. BER characteristics of $18\times(5,5)$ or $(5,8)$ or $(5,18)\times(3,2)$ model (Transmission rate to user 1 is 6 (bps/Hz) and 64QAM, 16QAM or QPSK is optimally selected to minimize the BER. Transmission rate to user 2 is 4(bps/Hz) and 16QAM or QPSK is optimally selected to minimize the BER.)

16QAM and QPSK, or three stream transmission with BD+E-SDM using three QPSK's is adaptively selected for $\boldsymbol{H}_{\mathrm{SD}}$ under the constant transmission rate of 6 (bps/Hz). For user 2, one stream transmission with BD+MRT using 16QAM or two stream transmission with BD+E-SDM using two QPSK's is adaptively selected for $\boldsymbol{H}_{\mathrm{SD}}$ under the constant transmission rate of 4 (bps/Hz). The numbers of reception and transmission antennas at relay is selected as $M_{\mathrm{R}}=5$ and $N_{\mathrm{R}}=4$, respectively and these antenna numbers are minimum required ones. This is because as the numbers of eigen streams for user 1 and user 2 on the SD link are $L_{\mathrm{SD1}}=\min(3,4)=3$ and $L_{\mathrm{SD2}}=\min(2,3)=2$, respectively, the minimum required number of reception antennas at relay should be $M_{\mathrm{R}}=3+2=5$ for making MMSE nulling at relay. From (16) the number of transmission antennas at relay has to satisfy the condition $N_{\mathrm{R}}>N_{\mathrm{D}}-m_{\min}=(m_1+m_2)-\min(m_1,m_2)=5-2=3$ and the minimum number of $N_{\mathrm{R}}=4$ is employed. On the RD link, the sizes of block channel matrices for user 1 and user 2 become $\boldsymbol{B}_{\mathrm{RD1}}=3\times 2$ and $\boldsymbol{B}_{\mathrm{RD2}}=2\times 1$, respectively. For user 1, one stream transmission with BD+MRT using 64QAM or two stream transmission with BD+E-SDM using 16QAM and QPSK is adaptively selected for $\boldsymbol{H}_{\mathrm{RD}}$ under the constant transmission rate of 6 (bps/Hz). For user 2, one stream transmission with BD+MRT using 16QAM is only selected for $\boldsymbol{H}_{\mathrm{RD}}$ under the constant transmission rate of 4(bps/Hz). When the number of transmission antennas at BS is increased, the receive gain of each user on the SD link raises. Accordingly, we increased the number of transmission antennas at BS from $N_{\mathrm{S}}=6$ to $N_{\mathrm{S}}=18$ on this $6\times(5,4)\times(3,2)$ model and compared the BER performance between $6\times(5,4)\times(3,2)$ and $18\times(5,4)\times(3,2)$ models. On the SD link of $18\times(5,4)\times(3,2)$ model, the sizes of block channel matrices for user 1 and user 2 become $\boldsymbol{B}_{\mathrm{SD1}}=3\times 16$ and $\boldsymbol{B}_{\mathrm{SD2}}=2\times 15$, respectively. The numbers of eigen streams on the SD link of user 1 and user 2 become $L_{\mathrm{SD1}}=\min(3,16)=3$ and $L_{\mathrm{SD2}}=\min(2,15)=2$, respectively and these numbers of eigen streams are the same between $6\times(5,4)\times(3,2)$ and $18\times(5,4)\times(3,2)$ models. We show the simulation results in Figure 8. From Figure 8, we know that "$6\times(5,4)\times(3,2)$ MLD w/ DF relay" improves the BER by 2 (dB) at $\mathrm{BER}=10^{-6}$ compared with "$6\times(3,2)$ SD link 2 times w/o DF relay" and the effect of using DF relay is verified. On the other hand, regarding $18\times(5,4)\times(3,2)$ model, we observe that there is almost no BER difference between "$18\times(5,4)\times(3,2)$ MLD w/ DF relay" and "$18\times(3,2)$ SD link 2 times w/o DF relay," thus we could not identify the effectiveness of using DF relay. This is considered that when the number of transmission antennas N_{S} is increased from 6 to 18, the transmission quality of SD link surpasses the RD link and two times transmission on the SD link is enough even though the DF relay is not used.

Next, we increased the number of transmission antennas of DF relay from $N_{\mathrm{R}}=4$ to a larger number on $18\times(5,4)\times(3,2)$ model and examined the effectiveness of using DF relay. When $N_{\mathrm{R}}=5$, the number of eigen streams of user 1 and user 2 on the RD link become $L_{\mathrm{RD1}}=3$ and $L_{\mathrm{RD2}}=2$, respectively and the same pattern of adaptive modulation as the SD link can be employed. We show the simulation results in Figure 9. From Figure 9, we observe that the BER performance of "$18\times(5,5)\times(3,2)$ MLD w/ DF relay" is better than "$18\times(3,2)$ SD link 2 times w/o DF relay" and can verify the effectiveness of using DF relay even when the number of BS antennas is $N_{\mathrm{S}}=18$. When the number of transmission antennas at DF relay N_{R} is further increased, we compared the BER performance between "$18\times(5,8)\times(3,2)$ MLD w/ DF relay" and "$18\times(5,18)\times(3,2)$ MLD w/ DF relay." However, the BER characteristics are almost equal between them and $N_{\mathrm{R}}=8$ seems enough and saturated number for the DF relay.

Next, under the condition of $N_{\mathrm{S}}=18$ and with the same transmission rate, we investigated the required number of

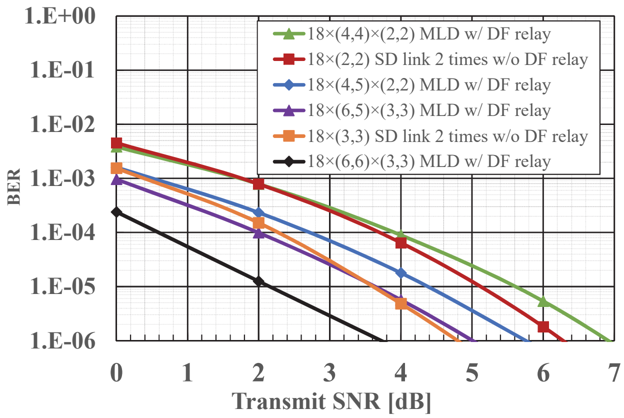

Figure 10. BER characteristics of $18\times(4,4)$ or $(4,5)\times(2,2)$ and $18\times(6,5)$ or $(6,6)\times(3,3)$ model (Transmission rate to user 1 is 6 (bps/Hz) and 64QAM, 16QAM or QPSK is optimally selected to minimize the BER. Transmission rate to user 2 is 4 (bps/Hz) and 16QAM or QPSK is optimally selected to minimize the BER.)

transmission antennas $N_{\rm R}$ at DF relay to improve the BER performance. We considered $18\times(2,2)$ and $18\times(3,3)$ models on the SD link where two users have equally 2 and 3 reception antennas, respectively. For $18\times(2,2)$ SD link model, the block diagonalized matrix size of user i becomes $\boldsymbol{B}_{{\rm SD}i}=2\times16$ $(i=1,2)$ and the number of eigen streams is given by $L_{{\rm SD}i}=\min(2,16)=2$ $(i=1,2)$. This means the required number of reception antennas at DF relay is greater than $M_{\rm R}=2+2=4$ for the MMSE nulling. On the other hand, for $18\times(3,3)$ SD link model, the block diagonalized matrix size of user i becomes $\boldsymbol{B}_{{\rm SD}i}=3\times15$ $(i=1,2)$ and the number of eigen streams is given by $L_{{\rm SD}i}=\min(3,15)=3$ $(i=1,2)$. This means the required number of reception antennas at DF relay is greater than $M_{\rm R}=3+3=6$. On the SD link of $18\times(2,2)$, for user 1, one stream transmission with BD+MRT using 64QAM or two stream transmission with BD+E-SDM using 16QAM and QPSK is adaptively selected for $\boldsymbol{H}_{\rm SD}$ under the constant transmission rate of 6 (bps/Hz). For user 2, one stream transmission with BD+MRT using 16QAM or two stream transmission with BD+E-SDM using two QPSK's is adaptively selected for $\boldsymbol{H}_{\rm SD}$ under the constant transmission rate of 4 (bps/Hz). On the SD link of $18\times(3,3)$, for user 1, one stream transmission with BD+MRT using 64QAM, two stream transmission with BD+E-SDM using 16QAM and QPSK, or three stream transmission with BD+E-SDM using three QPSK's is adaptively selected for $\boldsymbol{H}_{\rm SD}$ under the constant transmission rate of 6 (bps/Hz). For user 2, one stream transmission with BD+MRT using 16QAM, two stream transmission with BD+E-SDM using two QPSK's or three stream transmission with BD+E-SDM using QPSK and two BPSK's is adaptively selected for $\boldsymbol{H}_{\rm SD}$ under the constant transmission rate of 4 (bps/Hz). We show the simulation results in Figure 10. From Figure 10, regarding the $18\times(2,2)$ SD link model, we observe the BER is improved in the order of "$18\times(4,5)\times(2,2)$ MLD w/ DF relay", "$18\times(2,2)$ SD link 2 times w/o DF relay" and "$18\times(4,4)\times(2,2)$ MLD w/ DF relay." This observation means that $N_{\rm R}=5$ is required to show the effectiveness of using DF relay rather than $N_{\rm R}=4$. Regarding the $18\times(3,3)$ SD link model, "$18\times(6,5)\times(3,3)$ MLD w/ DF relay" shows almost the same BER characteristics as "$18\times(3,3)$ SD link 2 times w/o DF relay" and "$18\times(6,6)\times(3,3)$ MLD w/ DF relay" shows the best BER characteristics. Accordingly we can say that minimum $N_{\rm R}=6$ is needed to show the effectiveness of using DF relay in this case.

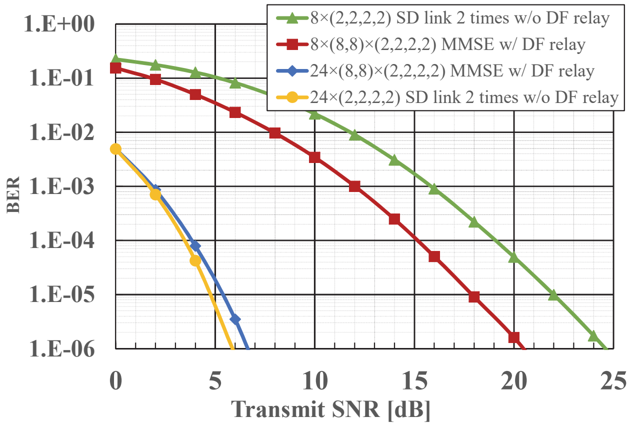

Figure 11. BER characteristics of $8\times(8,8)\times(2,2,2,2)$ or $24\times(8,8)\times(2,2,2,2)$ model (Transmission rate to each user is 6(bps/Hz) and 64QAM, 16QAM or QPSK is optimally selected to minimize the BER.)

Next, we consider the case where the number of transmission antennas at BS is $N_{\rm S}=8$, the number of users $N_{\rm u}=4$ and the numbers of reception antennas of each user are $m_1=m_2=m_3=m_4=2$. The numbers of receive and transmission antennas at relay is $M_{\rm R}=8$ and $N_{\rm R}=8$, respectively. We call this as $8\times(8,8)\times(2,2,2,2)$ model. This model is the same as the previous Figure 6, but this time we extended the transmission rate from 4 (bps/Hz) to 6 (bps/Hz). On the SD link, the size of block channel matrix for user i becomes $\boldsymbol{B}_i=2\times2$ $(i=1,2,3,4)$. For each user, one stream transmission with BD+MRT using 64QAM or two stream transmission with BD+E-SDM using 16QAM and QPSK is adaptively selected for $\boldsymbol{H}_{\rm SD}$ under the constant transmission rate of 6 (bps/Hz). The RD link transmission uses the same transmission rate 6 (bps/Hz) as in the SD link. Moreover, regarding on $8\times(8,8)\times(2,2,2,2)$ model, we increased the number of transmission antennas at BS from $N_{\rm S}=8$ to $N_{\rm S}=24$. We call this as $24\times(8,8)\times(2,2,2,2)$ model. The size of block diagonalized matrix of user i on the SD link becomes $\boldsymbol{B}_{{\rm SD}i}=2\times18$ $(i=1,2,3,4)$ and the number of eigen streams of user i is given by $L_{{\rm SD}i}=\min(2,18)=2$ $(i=1,2,3,4)$. Accordingly, the adaptive modulation is the same between $8\times(8,8)\times(2,2,2,2)$ and $24\times(8,8)\times(2,2,2,2)$. We show the simulation results in Figure 11.

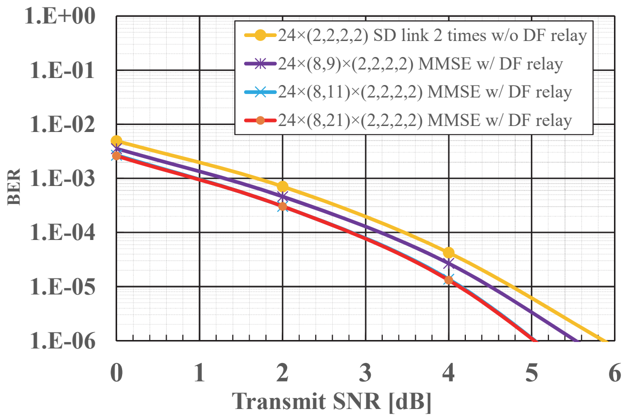

Figure 12. BER characteristics of $24\times(8{,}9)$ or $(8{,}11)$ or $(8{,}21)\times(2{,}2{,}2{,}2)$ model (Transmission rate to each user is 6 (bps/Hz) and 64QAM, 16QAM or QPSK is optimally selected to minimize the BER.)

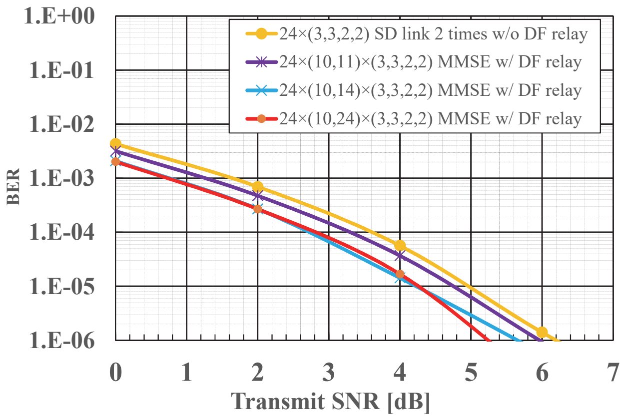

Figure 13. BER characteristics of $24\times(10{,}11)$ or $(10{,}14)$ or $(10{,}24)\times(3{,}3{,}2{,}2)$ model (Transmission rate to each user is 6 (bps/Hz) and 64QAM, 16QAM or QPSK is optimally selected to minimize the BER.)

From Figure 11, the BER of $8\times(8{,}8)\times(2{,}2{,}2{,}2)$ is superior to "$8\times(2{,}2{,}2{,}2)$ SD link 2 times w/o DF relay" by 4 (dB) at $\mathrm{BER}=10^{-6}$ and we see the effectiveness of using the DF relay. On the other hand, there is almost no BER difference between "$24\times(8{,}8)\times(2{,}2{,}2{,}2)$ MMSE w/ DF relay" and "$24\times(2{,}2{,}2{,}2)$ SD link 2 times w/o DF relay," and we could not see any effect of using the DF relay in this case. As stated before, this phenomenon comes from the fact that the SD link quality is better than the RD link due to the increased number of transmission antennas $N_{\mathrm{S}}=24$ at BS.

Next, regarding $24\times(8{,}8)\times(2{,}2{,}2{,}2)$ model, we increased the number of transmission antennas N_{R} at DF relay. We show the simulation results in Figure 12. From Figure 12, compared with "$24\times(2{,}2{,}2{,}2)$ SD link 2 times w/o DF relay," the BER is improved by using DF relay. But unlike the cases of "$8\times(2{,}2{,}2{,}2)$ SD link 2 times w/o DF relay" and "$8\times(8{,}8)\times(2{,}2{,}2{,}2)$ MMSE w/ DF relay" in Figure 11, the effect of using relay is not so large and there is no BER difference between "$24\times(8{,}11)\times(2{,}2{,}2{,}2)$ MMSE w/ DF relay" and "$24\times(8{,}21)\times(2{,}2{,}2{,}2)$ MMSE w/ DF relay." From these observations, we know that $N_{\mathrm{R}}=11$ is the saturated number of transmission antennas at DF relay and when the number of transmission antennas at BS is large such as $N_{\mathrm{S}}=24$, the effect of using DF relay is limited.

Next, we try to vary the number of reception antennas of each user under the condition that the number of transmission antennas at BS and the transmission rate of each user are unchanged from the previous figure, i.e., $N_{\mathrm{S}}=24$ and 6 (bps/Hz), respectively. On the SD link, we consider $24\times(3{,}3{,}2{,}2)$ model. The sizes of block diagonalized matrices of SD link are given by $\boldsymbol{B}_{\mathrm{SD}i}=3\times17$ $(i=1{,}2)$ and $\boldsymbol{B}_{\mathrm{SD}i}=2\times16\,(i=3{,}4)$, respectively. The number of eigen streams of each user becomes $L_{\mathrm{SD}i}=\min(3{,}17)=3\ (i=1{,}2)$ and $L_{\mathrm{SD}i}=\min(2{,}16)=2\ (i=3{,}4)$, respectively. For $L_{\mathrm{SD}i}=3\ \ (i=1{,}2)$, one stream transmission with BD+MRT using 64QAM, two stream transmission with BD+E-SDM using 16QAM and QPSK, or three stream transmission with BD+E-SDM using three QPSK's is adaptively selected for $\boldsymbol{H}_{\mathrm{SD}}$ under the constant transmission rate of 6 (bps/Hz). For $L_{\mathrm{SD}i}=2\ \ (i=3{,}4)$, one stream transmission with BD+MRT using 64QAM or two stream transmission with BD+E-SDM using 16QAM and QPSK is adaptively selected for $\boldsymbol{H}_{\mathrm{SD}}$ under the constant transmission rate of 6 (bps/Hz). The size of $\boldsymbol{B}_{\mathrm{SD}i}=3\times17\ \ (i=1{,}2)$ in $24\times(3{,}3{,}2{,}2)$ model is greater than $\boldsymbol{B}_{\mathrm{SD}i}=2\times18\ \ (i=1{,}2{,}3{,}4)$ in $24\times(2{,}2{,}2{,}2)$ model. On the other hand, $\boldsymbol{B}_{\mathrm{SD}i}=2\times16\,(i=3{,}4)$ in $24\times(3{,}3{,}2{,}2)$ model is a little bit less than $\boldsymbol{B}_{\mathrm{SD}i}=2\times18\,(i=1{,}2{,}3{,}4)$ in $24\times(2{,}2{,}2{,}2)$ model. The minimum number of reception antennas at DF relay for MMSE nulling becomes $M_{\mathrm{R}}=3+3+2+2=10$. We show the simulation results in Figure 13. From Figure 13, we see almost the same observations as in Figure 12 and when the number of transmission antennas at BS is large such as $N_{\mathrm{S}}=24$, the effect of using DF relay is limited.

V. CONCLUSIONS

In this paper, we discussed the improvement of transmission quality when the DF relay is applied to the MU-MIMO down link system with the linear Block Diagonalization plus MRT or E-SDM scheme. By knowing the precoding matrix of BS at DF relay, the DF relay can demodulate the transmit signals from BS with the receive CSI only. As the existence of DF relay does not affect the design of precoding matrix at BS, we can add the DF relay only when the transmission quality between BS and user terminals is insufficient. By adding the DF relay and utilizing the 2nd time slot, we can improve the receive quality or diversity order of each user terminal. We made the designs of SD, SR and RD links during the 1st and the 2nd time slots and verified the effectiveness of using DF relay. Although the SR link quality affects the total BER performance very much, we can utilize the simple MMSE nulling by increasing the number of reception antennas at DF relay to improve the SR link quality. We observed that even though the DF relay is equipped with the minimum number of reception and transmission antennas required, there exists the effect of using DF relay. But by increasing the number of transmission antennas at BS, the SD link quality is improved gradually resulting in the better SD link quality than the relaying SRD link. This large number of transmission antennas at BS finally limits the effect of using DF relay.

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