Selfish Flow Games in Non-Cooperative Multi-Radio Multi-Channel Wireless Mesh Networks With Interference Constraint Topology

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Abstract—**Due to the advancement in technology, routers of Wireless Mesh Networks can be equipped with multiple interfaces to achieve parallel communication sessions among nodes. Assigning distinct non-overlapping channels to each set of communicating radios increases network connectivity and throughput. On the other hand, the performance of wireless networks is always limited by the interference phenomena among the concurrent transmission sessions. Combined with interference constraint topology due to limited available orthogonal channels, selfishness of end users further degrades individual fairness and affects overall network performance due to their protocol deviation in a non-cooperative environment. In this paper, we have proposed a non-cooperative game theoretical model in a multi-radio multi-channel Wireless Mesh Network based on the end users flows in an interference constrained topology. Necessary conditions for the existence of Nash Equilibrium have been derived. Our simulation results show that our distributed algorithm converges to a stable state in finite time where each node gets fair end to end throughput across multiple-collision domains at the end of the game. Further, the Price of Anarchy of the system was measured for several runs which is always near to one; showing the strength and stability of our proposed scheme.**

***Keywords-Multi-Radio Multi-Channel; Game Theory; Wireless Mesh Networks; Network Flows; Interference Constraint Topology; Price of Anarchy*.**

I. INTRODUCTION

In Non-Cooperative Networks, nodes behave selfishly to maximize their own benefit by deviating from the defined protocol [2], which leads to system-wide performance degradation, instability and individual unfairness. In Mobile Adhoc Networks (MANETs) [3], for example, each node acts as user of the network as well as rely data for others. A non-cooperative node can misbehave by dropping others packets to save its battery life while sending its own packets to be forwarded by other nodes. This selfish behavior of free riders leads to limited connectivity of the network and affects individual as well as network-wide performance. If all nodes behave selfishly in the same manner, the network will end up with each entity in isolation, as shown in Fig. 1, nodes c and g drop the incoming packets from other nodes

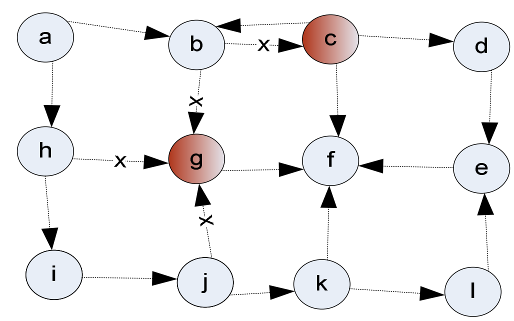

Figure 1. Non-Cooperative behaviour in MANETs

while send their packets to be forwarded by other nodes in the network. To cope up with these similar behaviors, multiple techniques have been used to enforce cooperation among the nodes for the stability of overall system [4]. Viewing this behavior from game theoretic prospective, a conflicting situation where each entity is self interested in the network resources or service leads to a non-cooperative game.

Like MANETs, Wireless Mesh Networks (WMNs) [5] have multi-hop topology spanning multiple collision domains. The inherited advantages of self configuration, self healing and self organization along with static nature of its backhaul routers make it a prime candidate for wireless broadband provisioning in users premises. However, unlike MANETs, WMNs routers can be equipped with multiple radios due to their static nature and the existence of permanent power supplies. Since multiple channels are available in the free Industrial, Scientific and Medical (ISM) band, multiple radios can be tuned simultaneously to exploit the free non-overlapping channels and hence increase the overall capacity, connectivity and resilience of the wireless mesh backhaul. Due to these characteristics, WMNs is a prime candidate to be deployed as a broadband wireless access network in the user premises. In WMNs, backhaul routers are divided into three types: Gateways, Access Points (APs) and core backbone routers. The Gateways have direct connection to the Internet while APs provide network access to the mesh backhaul users. The core backbone routers have the responsibilities of forwarding users traffic to/from the Internet via mesh gateways as shown in Fig. 2.

Due to the capabilities of meshing, IEEE has established subgroups in their existing network standards like IEEE 802.11s for WLAN based mesh networks, IEEE802.16e for Metropolitan Area Mesh Networks and IEEE 802.15 for Personal Area Mesh Networks. Since WMNs have the potential to be widely deployed as a broadband multi-hop wireless network [6], many vendors have invested in it and have deployed practical mesh topologies, e.g., Nortel [7], Motorola [8] and TroposNetworks [9].

Multi-Radio Multi-Channel (MRMC) in WMNs has gained a lot of research attention in the recent years [10]. Since wireless networks are always bandwidth constrained due to the shared wireless medium, interference from other transmissions, high bit error rates and retransmissions limit the capacity of wireless networks; multi radios tuned to multiple non-overlapping channels improve the overall capacity by decreasing the interference as the same channel can be reused multiple times in the same backhaul away from its transmission and interference range. Since designing a good MRMC algorithm is crucial for the WMNs performance, therefore a considerable amount of research has been done in this specific area.

Although, selfish routing and forwarding problems in MANETs have been well researched by providing solutions from Game Theory and considering all nodes as players of the game [11,12,13], the static infrastructure of WMNs shifts the set of players to the end users premises. Since forwarding nodes have no incentive to behave selfishly and there is no point to consider them in the set of players [14].

Game theory is a mathematical tool which is used in a situation when multiple entities interact with each other in a strategic setup. Formally, a game can be defined [15] as consisting of a non-empty finite set of $N=\{N_1,N_2,\ldots,N_{|N|}\}$ players, a complete set of actions/strategies $A_i=\{a_1,a_2,\ldots a_{|Ai|}\}$ for each $N_i \in N$. A set of all strategies space of all players, represented by the matrix $A=A_1xA_2x\ldots xA_{|N|}$. $A(a_i,a_{-i})$ is a strategy profile when a player $N_i \in N$ selects an action a_i from its action set Ai against the

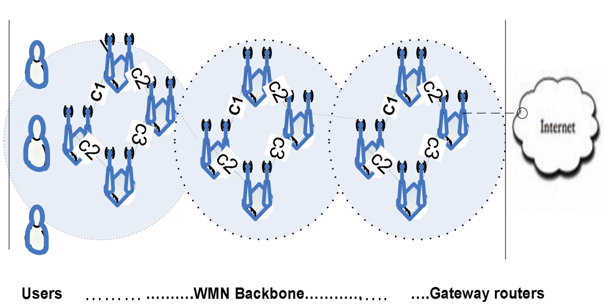

Figure 2. WMNs Components

actions of all other players N_{-i}. The notation $-i$ is a convenient way to represent a set of entities or set of events excluding a specific entity or event in a strategic setup. For example N_{-i} means set of all players excluding N_i and a_{-i} means set of actions of all players excluding action of N_i during a strategic interaction. At the end of the game, each player $N_i \in N$ gets benefit in the form of a real number (R) called outcome or payoff of the player which is determined by the utility function U_i as: $U_i=A_i \rightarrow R$.

Depending on the player's knowledge about each other's strategies, payoffs, and past histories; games can be subdivided into different categories. When players have complete information about each other's strategies and payoffs, such type of game is called a game with complete information. In games of incomplete information, players have partial or no information about each other strategies and payoffs. Games can be simultaneous or sequential depending upon the occurrence of individual players actions. When players interact with each other and take their decisions simultaneously, such games are called simultaneous move games. When the players take decision one after the other, such type of games are called sequential. When all players have information about each other past moves and actions, such type of games are called games with perfect information. All the simultaneous move games are games with imperfect information. Games where cooperation is enforced among players outside the pre-defined rules of the game are called cooperative games. In non-cooperative games, players cannot communicate with each other through some enforceable agreement other than the rules of the game [16].

In this paper, which is the extension of our previous work [1], we address end users flow game across non-cooperative multi-radio multi-channel WMNs in a selfish environment by considering and interference constrained topology. We prove analytically the existence of Nash Equilibrium (NE) under certain conditions.

The rest of our paper is organized as follows. In Section II, related research work is presented. Section III provides an introduction to our game theoretical model along with some essential concepts. In Section IV, we present our analytical results and necessary condition for the existence of Nash Equilibrium. In Section V, we discuss the convergence algorithm. In Section VI, we present our simulation results and conclude our paper in Section VII with future directions and recommendations.

II. RELATED WORK

Application of game theory to networks is not new and a huge amount of literature can be found at different layers of the protocol stack. In [17], for example, congestion control has been analyzed using game theory while [18, 19, 20] have addressed routing games. Power control games have been extensively studied in [21, 22] while Medium Access Control has been analyzed by using game theoretical analysis in [23, 24]. A detailed survey targeting the telecommunication problems using game theory can be found in [25], while game theory applications in wireless networks can be specifically found in [26].

Due to the practical importance of WMNs, considerable research efforts have been put in the designing of an intelligent MRMC technique. In [27], authors have addressed MRMC with a graph theoretic approach while A. Raniwala *et al.* [28] have presented MRMC models based on flows. The work of M. Alicherry *et al.* [29] addresses routing and channel assignment as a combined problem. Although all of the above research work have tackled MRMC from different aspects but they consider that all the nodes cooperate with each other for system wide throughput optimization and selfish behavior has been explicitly ignored.

In one of their pioneering work, Felegyhazi *et al*. [30] have proven the existence of Nash Equilibrium in a non-cooperative multi radio multi channel assignment. They have formulated channel assignment as a game where nodes, equipped with multiple radios, compete for shared multiple channels in a conflict situation and the result shows that the system converges to a stable Nash Equilibrium where each player gets equal and fair share of the channel resources. The work of Chen et *al.* [31] is an extension of [30] where perfect fairness has been provided to all players by improving the max-min fairness. Despite the interesting results, their work is limited to single collision domain while multi-hop networks like WMNs span multiple collision domains and hence all the above cited work cannot be applied to this specific scenario as discussed. In one of the recent study Gao *et al.* [32] have provided a more practical approach by extending the number of hops in the mesh backbone. They have proved that allowing coalition among players can lead to node level throughput improvement. They have provided a coalition-proof Nash Equilibrium and algorithms to reduce the computational complexity of equilibrium convergence; their solution considers cooperation among the nodes inside the coalition and hence cannot be applied to a fully non-cooperative WMNs environment. More importantly, it will be more apposite to consider end users generating flows as players of the game [31] because of their competition for the common channel resource across the wireless mesh backhaul. In such a situation, channel assignment and flow routing may be tackled simultaneously. A class of game theoretical model for routing in transportation networks has been presented by Rosenthal [33]. The author have considered *n* players in a competitive environment, each wanted to ship one unit from source to destination while minimizing its transportation cost. They have proven the existence of pure strategy Nash Equilibrium. In [34, 35], authors have provided game theoretic solutions based on end users flows to control congestion inside the communication network. Routing in general wired networks has been studied as a non-cooperative game in [36, 37, 38, 39, 40], where the conditions for the existence of Nash Equilibrium has been derived. Banner *et al.* [41] have extensively studied the non-cooperative routing problem in wireless networks based on splittable and unsplittable flows. Although, they have proven the existence of Nash Equilibrium for both classes of flow problems; their solution is not applicable to MRMC WMNs. In [14], selfish routing and channel assignment in wireless mesh networks is formulated as a Strong Transmission Game where it is assumed that selfish nodes at the user premises assign channels, in a strategic setup, to their end to end paths. While they have solved channel assignment and routing problem in a non-cooperative environment from the end users selfish prospective, the strong assumption of non-interference among channels need a large set of orthogonal frequencies which is limited by the fewer channels available in the IEEE 802.11 a/b/g/n standards [33, 34, 35, 36]. In practice, channel assignment is always an interference constrained phenomena due to the availability of fewer channels in the orthogonal frequency set of ISM band [46] and large backbone size of WMNs. In one of our recent work [1], a single stage selfish flow game was formulated in a MRMC multiple collision domain and fairness of individual nodes was investigated with the assumption of an interference free topology. In this paper, we extend our previous work by considering channel interference during game formulation. To the best of our knowledge, this is the first work in the area of competitive flow routing in a MRMC WMNs with interference constrained topology.

III. SYSTEM MODEL AND CONCEPTS

As shown in Fig. 2, mesh routers having multi radio capabilities reside in multiple collision domains. We assume that there is always a chance of channel usage conflict across the mesh backbone.

A. Network Model

We represent multi-hope WMNs spanning multiple collision domains with a Unit Disk Graph (UDG) G (V, E) [47], where the sets V and E represent mesh backhaul routers and their associated links accordingly in the graph G. We assume that each mesh router uses same transmission power as in IEEE 802.11 a/b/g/n [42, 43, 44, 45] standards. Any two mesh routers v_i and v_j can communicate with each other successfully, if the Euclidian Distance between them is less than the sum of their radii i-e for any two routers (v_i, v_j) ∈ V:

$$\mathrm{d}\left(v_i, v_j\right) < r_{v_i} + r_{v_j} \tag{1}$$

where r_{vi} and r_{vj} are the radii of vertices v_i and v_j respectively. In other words, they are in the transmission range of each other as shown in Fig. 3 by smaller circles around the vertices. Let the interference range of a node is represented by the outer circle, whose radii is twice that of smaller circle, then two set of nodes (v_1,u_1), (v_2,u_2) cannot communicate with each other if either:

$$\mathrm{d}(u_1,v_2)<2(r_{u1}+r_{v2}) \parallel \mathrm{d}(v_1,v_2)<2(r_{v1}+r_{v2}) \parallel \mathrm{d}(u_1,u_2)<2(r_{u1}+r_{u2}) \parallel \mathrm{d}(v_1,u_2)<2(r_{v1}+r_{u2}) \tag{2}$$

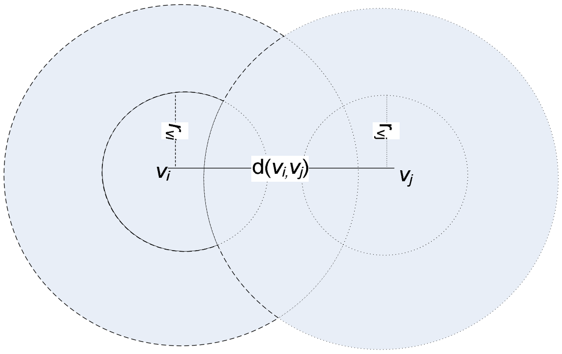

Figure 3. Transmission and Interference Range

Channel assignment to nodes links is essentially same as colouring the edges of UDG with appropriate colours such that two edges e_i, e_j belonging to any two pair of nodes (u_i,v_i), (u_j,v_j) satisfying any of the condition in (2) get distinct colours. Refer to Fig. 3, where colouring of UDG means assigning distinct colours to interfering edges. However, due to the multi-radio nature of mesh routers, interference constraint and limited available non-overlapping channels; we assume relaxation in the colouring assignment where two interfering edges can be assigned with same colour. We will discuss it in more detail in section B.

B. Game Theoritic Model

We formulate our game theoretic model by considering selfish end users as players of the game with imperfect information in non-cooperative multi-collision domain mesh network as follows. We divide the core of the mesh network in a set of multiple collision domains D={1, 2, 3,…, |d|} and the set of non-overlapping orthogonal channels, as present in IEEE 802.11a/b [33, 34, 35, 36], are represented by C={C_1,C_2,C_3,….$C_{|C|}$} as shown in Fig. 4. We refer to any channel C_i in a specific collision domain as C_{ij} where C_i∈C and $j \in$ D , respectively. The maximum achievable data rate on a channel $C_i \in$ C is represented by R_{Ci}. We assume that the maximum achievable capacity on all the channels is the same, i-e:

$$R_{Ci} = R_{Cj}\,,\ \forall\ C_i,\ C_j \in \mathrm{C} \qquad (3)$$

We assume that channels set is limited according to the IEEE 802.11 a/b/g/n [31, 32, 33, 34] standards and there is a chance that a channel reused can interfere according to the condition given in (2). We define the degree of interference of a channel C_k ∈C as Φ_{Ck} , showing the number of links which have been assigned the same channel C_k in the same collision domain *j.* In the UDG, it is the number of incident edges having same colours as defined in (2).

Nodes originating flows from the user premises are the players of the game represented by a finite non-empty set $\mathrm{N} = \{N_1, N_2, N_3, \ldots . N_{|\mathrm{N}|}\}$, where N_n is any player belonging to the set N. The set of flows generated by any

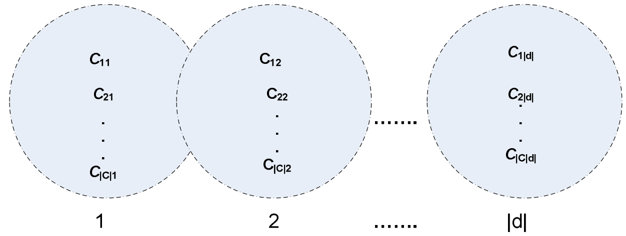

Figure 4. Channel distribution in multiple collision domains

player Nn∈ N is represented by a non-empty set f={f_1, f_2, …, $f_{|f|}$}, where f_n ∈ f represents any flow generated by player N_n∈ N. We define the strategy of a player N_n∈ N as the channel selection vector for each of its flow across the multiple collision domains. i.e,:

$$A_n = \left\{ f_{1,Ci}\,,\ f_{2,Ci}\,,\ \ldots..,\ f_{|f|,Ci} \right\} \qquad (4)$$

where f_1, f_2,…$f_{|f|}$ ∈ f are the flows of player N_n and C_i ∈C is the arbitrary channel in the channel set across collision domains 1, 2, …., |d| ∈D.

Accordingly, the strategy profile of all players is represented by:

$$\mathrm{A} = \left(A_1, A_2, \ldots., A_{|\mathrm{N}|}\right)^T \qquad (5)$$

The n^{th} row of the vector in (5) shows the strategy of player N_n, i-e, A_n as in (4). Each player N_n ∈ N takes a rational decision by selecting an end to end path across the core of the network towards the gateway of the mesh by maximizing its utility function. We formulate the utility function of players N_n as:

$$U_n = \sum_{f_n \in \mathrm{f}\,,\,C_i \in C\,,\,j=1}^{|d|} \left(\frac{1}{\Phi_{C_{ij}}} \left(\frac{f_n}{F_{ij}} \right) R_{C_{ij}} \right) \qquad (6)$$

where C_{ij} denotes the channel C_i selected by player N_n ∈ N for its flow f_n ∈ f in j^{th} collision domain and F_{ij} is the total number of flows on channel C_{ij} as defined in (7). R_{Cij} is the maximum achievable data rate on channel C_i ∈C in collision domain j∈D, which is equal for all channels as in (3). We assume that all users generate CBR flows and define the parameter, F_{ij}, representing the number of flows on a specific channel, C_i∈C, in any collision domain, j∈D, as:

$$F_{ij} = \frac{Q(C_{ij})}{\tau(CBR)} \qquad (7)$$

where $Q(C_{ij})$ is the queue length associated with the link which has assigned channel C_{ij} and τ (CBR) is the constant bit rate of any flow. Ideally, F_{ij} determines the number of flows or load on a specific channel. Naturally, a rational player strategy will be to select an end-to-end path having channels which are least loaded by other flows and the channels are least interfered. We define the term Φ_{Ci} as the degree of interference on a specific link to which channel C_i has been assigned. In such a selfish environment, game theory provides a realistic solution towards the stability of the system by reaching a point where no flow can move to any other channel across the whole end to end path unilaterally. This stable point is called Nash Equilibrium (NE) and is defined below [15].

A strategy profile A^* is called Nash Equilibrium if for each player $N_n \in$ N:

$$U_n\left(A_n^*,\ A_{-n}\right) \geq\ U_n\left(A_n, A_{-n}\right),\ \forall A_n \in \mathrm{A} \tag{8}$$

where $U_n(A_n^*, A_{-n})$ is the payoff, according to the utility function defined in (6), of player N_n by selecting the strategy A_n^* against the strategies of all other players A_{-n} as in (5). In other words, in NE, everyone is playing its best response to everyone else in a game. It is the point where no player can get any benefit by unilaterally deviating from its strategy.

IV. Existence of Nash Equilibrium

To check the existence of Nash equilibrium in our proposed model, we assume two types of channels in any collision domain. Channels having maximum number of flows are represented by C_{max} and the category of channels having minimum number of flows as C_{min}. We define a parameter ℓ_{ik} which is the difference of number of flows on any two channels C_i, $C_k \in$ C within a specific collision domain $j \in$ D i-e:

$$\ell_{ik} = F_{ij\,(max)} - F_{kj\,(min)} \tag{9}$$

where $F_{ij(max)}$ and $F_{ij(min)}$ are the number of flows on C_{max} and C_{min}, respectively.

We define another term $\psi(C_i, C_k)$, which defines the difference in degree of interference between two channels C_i, $C_k \in$C within a specific collision domain $j \in$ D as:

$$\psi(C_i, C_k) = \Phi_{Ci} - \Phi_{Ck} \tag{10}$$

Where Φ_{Ci} and Φ_{Ck} are the degrees of interference on channel C_i and C_k, respectively as defined in section III.

Being rational, the objective of each player is to maximize its utility by selecting an end to end path with channels having minimum number of load and minimum interference on it. We define some necessary conditions for such a selfish environment and prove the existence of Nash Equilibrium.

Lemma 1: For a MRMC multi-collision domain mesh network, for $\psi(C_i,C_k)$=0, if $\left(\frac{nf_n}{F_{ij}}\right).R_{C_{ij}} = 1\ \forall\ f_n \in \mathrm{N}_n$ AND $\forall\ f_m \neq f_n \in \mathrm{N}_m, \left(\frac{f_m}{F_{ij}}\right).R_{C_{ij}} = 0$ with $\ell_{ik} \geq 1\ \forall\ C_i \in \mathrm{C}_{\max}, C_k \in \mathrm{C}_{\min}$ for any $j \in$ D, then the strategy profile A^* is not a Nash Equilibrium.

If all the flows of any player N_n selects any channel $C_i \in C_{max}$ in any collision domain while flows of all other users N_m put their flows on $C_m \in C_{min}$, then player N_n will have an incentive to unilaterally deviate from her strategy and this can no longer be a Nash Equilibrium. As shown in Fig. 5, player N_1 selects Channel 1 for all its four flows in collision domain j. Its utility can be increased if one of the flow is transferred to other channels (C_2, C_3 or C_4).

Proof: Let α_n be the gain of player N_n deviating from its current strategy which has defined all its flows $n.f_n$ on one of channel $C_{ij} \in C_{max}$. Let F_{ij} be the total flows on $C_{ij} \in C_{max}$ and F_{kj} be the total flows on $C_{kj} \in C_{min}$.
then:
$\alpha_n = U_n' - U_n$, where U_n' is the new payoff of player N_n after deviating from its current strategy.
Let $\theta \in N_n$ is one of the flow, which player N_n redirect to another channel $C_k \in C_{min}$ in any collision domain $j \in$D and calculate the benefit of change along the path as follows.

$$\alpha_n = \sum_{f_n \in \mathrm{f}, C_i \in \mathrm{C}, j=1}^{|j-1|} \left(\frac{1}{\Phi_{C_{ij}}}\left(\frac{f_n}{F_{ij}}\right) R_{C_{ij}}\right) + \left(\frac{1}{\Phi_{C_{ij}}}.\frac{(n-\theta)f_n}{(F_{ij}-\theta)f_n}\right) R_{C_{ij}}$$

$$+\left(\frac{1}{\Phi_{C_{kj}}}.\frac{\theta f_n}{(F_{kj}+\theta)}\right) R_{C_{kj}} - \left(\frac{1}{\Phi_{C_{ij}}}.\frac{nf_n}{F_{ij}}\right) R_{C_{ij}}$$

$$+ \sum_{f_n \in \mathrm{f}, C_i \in \mathrm{C}, j=j+1}^{|d|} \frac{1}{\Phi_{C_{ij}}}\left(\frac{f_n}{F_{ij}}\right) R_{C_{ij}} \tag{11}$$

By considering only $j \in$D collision domain:

$$= \frac{1}{\Phi_{C_{ij}}}.\left(\frac{(n-\theta)f_n}{(F_{ij}-\theta)}\right) R_{C_{ij}} + \frac{1}{\Phi_{C_{kj}}}.\left(\frac{\theta f_n}{(F_{kj}+\theta)}\right) R_{C_{kj}}$$

$$- \frac{1}{\Phi_{C_{ij}}}.\left(\frac{nf_n}{F_{ij}}\right) R_{C_{ij}}$$

Since R_{Cij}=R_{Ckj}, F_{kj}+θ=F_{ij} and $\Phi_{Cij} = \Phi_{Ckj}$ as $\psi(C_i,C_k)$=0,
Therefore:

$$= \frac{1}{\Phi_{C_{ij}}}\left[\left(\frac{(n-\theta)f_n}{(F_{ij}-\theta)}\right) R_{C_{ij}} + \left(\frac{\theta f_n}{F_{ij}}\right) R_{C_{ij}} - \left(\frac{nf_n}{F_{ij}}\right) R_{C_{ij}}\right]$$

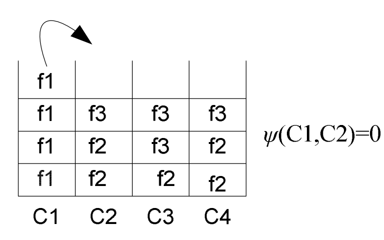

Figure 5. Example of homogenous flows on one channel

After simplification:

$$\alpha_n = \frac{1}{\Phi_{C_{ij}}}\left[\left(\frac{\theta f_n (n-\theta}{F_{ij}(F_{ij}-\theta)}\right) R_{C_{ij}}\right] \tag{12}$$

> 0 as $(n-\theta)$ and (F_{ij}- θ) are positive.

Hence the strategy profile A* cannot be a Nash Equilibrium.

Lemma 2: In a MRMC multiple-collision domain mesh network, for $\psi(C_i,C_k)$=0, in any collision domain j ∈D along the end to end path of flows, if ℓ_{ik}>1for any $C_i \in C_{max}, C_k \in C_{min}$ then the strategy profile A* is not a Nash Equilibrium. Let ℓ_{ik}>1in any collision domain j ∈D along the end to end path of flows then it essentially means that there exists a channel $C_{kj} \in C_{min}$ in j ∈D for which $\frac{Q(Cij)}{\tau\,(CBR)} - \frac{Q(C_{kj})}{\tau\,(CBR)} > 1$, and hence at least one of the flow $f_n \in N_n$ on $C_{ij} \in C_{max}$ has incentive to change for her benefit. As shown in Fig. 6, N_2 can unilaterally switch one of its flows to C_1, C_2 or C_4.

Proof: Let a user N_n changes its flow, f_n, from $C_{ij} \in C_{max}$ to $C_{kj} \in C_{min}$ in j∈D collision domain along the end to end path. The gain of change is calculated as follows:

$$\alpha_n = \sum_{f_n \in f, Ci\in C, j=1}^{|j-1|}\left(\frac{1}{\Phi_{C_{ij}}}.\left(\frac{f_n}{F_{ij}}\right)R_{C_{ij}}\right) + \frac{1}{\Phi_{C_{kj}}}.\left(\frac{f_n}{\frac{Q(C_{kj})}{\tau(CBR)}+1}\right)R_{C_{kj}}$$

$$-\frac{1}{\Phi_{C_{ij}}}\left(\frac{f_n}{\frac{Q(C_{ij})}{\tau(CBR)}}\right)R_{C_{ij}} + \sum_{f_n \in f, Ci\in C, j=j+1}^{|d|}\frac{1}{\Phi_{C_{ij}}}\left(\frac{f_n}{F_{ij}}\right)R_{C_{ij}} \tag{13}$$

By considering the j^{th} collision domain only, the first and last summation terms become irrelevant and hence by simplification, we get:

$$= \frac{1}{\phi Ckj}\left(\frac{f_n}{\left(\frac{Q(Ckj)}{\tau\,(CBR)}+1\right)}\right)RC_{kj} - \frac{1}{\phi Cij}\left(\frac{f_n}{\frac{Q(Cij)}{\tau\,(CBR)}}\right)R_{C_{ij}}$$

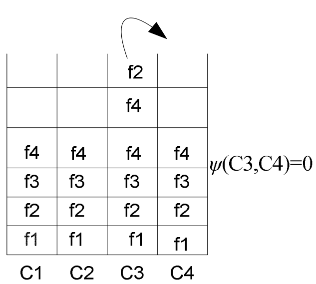

Figure 6. Flow distribution on channels where ℓ>1

Since $\Phi_{Cij} = \Phi_{Ckj}$ as $\psi(C_i,C_k)$=0, therefore:

$$= \frac{1}{\phi Cij}\left[\left(\frac{f_n}{\left(\frac{Q(Ckj)}{\tau\,(CBR)}+1\right)}\right)R_{C_{kj}} - \left(\frac{f_n}{\frac{Q(Ckj)}{\tau\,(CBR)}+\ell}\right)R_{C_{kj}}\right]$$

$$\alpha_n = \frac{1}{\phi Cij}\left[\left(\frac{\ell *\ \tau\,(\mathrm{CBR}) - \tau\,(\mathrm{CBR})}{\tau(Ckj)+\tau(\mathrm{CBR})*(\tau(Ckj)+\ell*\tau(\mathrm{CBR}))}\right) f_n R_{C_{kj}}\right] \tag{14}$$

>0, as ℓ>1

Since player N_n has an incentive to change from its current strategy to the new one, and hence the current strategy profile A* cannot be a Nash Equilibrium.

Lemma 3: In a MRMC Multiple-Collision domain mesh network, for $\psi(C_i,C_k)$=0, for any player N_n if $\forall f_n C_{ij}$-$\forall f_n C_{kj}$>2 and $\ell_{ik} \geq 1$ in any collision domain j ∈D, $\forall C_{ij} \in C_{max}$, $\forall C_{kj} \in C_{min}$, then the strategy profile A* is not a Nash Equilibrium.

As shown in Fig. 7, difference of flows of N_2 on C_2 and C_3>2, although it does not deviate from lemma2, player N_2 has incentive to switch one of its flow from C_2 to C_3.

Proof: Let θ_1, $\theta_2 \in N_n$ are the number of flows of player N_n on C_{max} and C_{min} respectively. We define $\Delta\theta_{1,2} = \theta_1 - \theta_2$ as the flow difference of any player on two *max*, *min* channels. Let N_n redirects one of its flow from C_{max} to C_{min} in any collision domain j ∈D along the end to end path. Then the gain of change is given by:

$$\alpha_n = \sum_{f_n \in f, Ci\in C, j=1}^{|j-1|}\frac{1}{\Phi_{C_{ij}}}.\left(\left(\frac{f_n}{F_{ij}}\right)R_{C_{ij}}\right) + \frac{1}{\Phi_{C_{ij}}}.\left(\frac{(\theta_1-1)f_n}{(F_{ij}-1)}\right)R_{C_{ij}}$$

$$+\frac{1}{\Phi_{C_{kj}}}\left(\frac{(\theta_2+1)f_n}{(F_{kj}+1)}\right)R_{C_{kj}} - \frac{1}{\Phi_{C_{ij}}}\left(\frac{\theta_1 f_n}{F_{ij}}\right)R_{C_{ij}} - \frac{1}{\Phi_{C_{kj}}}\left(\frac{\theta_2 f_n}{F_{kj}}\right)R_{C_{kj}}$$

$$+\sum_{f_n \in f, Ci\in C, j=j+1}^{|d|}\frac{1}{\Phi_{C_{ij}}}\left(\frac{f_n}{F_{ij}}\right)R_{C_{ij}} \tag{15}$$

We suppose the change of end to end path for N_n accurse in the j^{th} collision domain only and hence by eliminating the first and last summation terms. Since F_{kj}=F_{ij}- ℓ before flow switch from C_{ij} to C_{kj} and F_{kj}+1=F_{ij} after flow switch by assuming $\ell = 1$, by substituting appropriate terms:

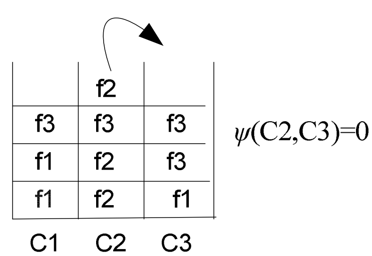

Figure 7. Homogenous flows difference on two channels>2

$$\alpha_n = \frac{1}{\Phi_{C_{ij}}}\left(\frac{(\theta_1-1)f_n}{(F_{ij}-1)}\right)R_{Cij} - \frac{1}{\Phi_{C_{ij}}}\left(\frac{\theta_1 f_n}{F_{ij}}\right)R_{Cij}$$
$$+\frac{1}{\Phi_{C_{ij}}}\left(\frac{(\theta_2+1)f_n}{F_{ij}}\right)R_{Cij} - \frac{1}{\Phi_{C_{ij}}}\left(\frac{\theta_2 f_n}{(F_{ij}-\ell)}\right)R_{Cij}$$

Since $\phi C_{ij}=\phi C_{kj}$ as $\psi(C_i,C_k)=0$ and by further simplification:

$$\alpha_n = \frac{1}{\Phi_{C_{ij}}}\left(\frac{\theta_1-(\theta_2+1)f_n}{F_{ij}\left(F_{ij}-1\right)}\right)R_{Cij} \tag{16}$$

The term $\theta_1-(\theta_2+1)$ and $F_{ij}-1>0$ as $\Delta\theta_{1,2}>2$
$\therefore\ \alpha_n>0$ and A^* cannot be a Nash Equilibrium under such condition.

Lemma 4: In a MRMC Multiple-Collision domain mesh network, for $\psi(C_i,C_k)\geq 1$, for any player N_n if $\forall f_n C_{ij}-\forall f_n C_{kj}=1$ and $\ell_{ik}\geq 1$ in any collision domain $j\in D$, $\forall C_{ij}\in C_{max}$, $\forall C_{kj}\in C_{min}$, then the strategy profile A^* is not a Nash Equilibrium.

As shown in Fig. 8, difference of flows of N_2 on C_2 and $C_3>2$, although it does not deviate from lemma 2, player N_2 has incentive to switch one of its flow from C_2 to C_3.

Proof: The proof of this lemma is straightforward. Let $f_n\in$ f be the only flow of player $N_n\in$ N on C_{max}. It essentially means that there are no flows defined by player N_n on channel C_{min} in any collision domain $j\in D$ along the end to end path. The gain of change is given by:

$$\alpha_n = \sum_{f_n\in \mathrm{f},\, Ci\in \mathrm{C},\, j=1}^{|j-1|}\frac{1}{\Phi_{C_{ij}}}.\left(\left(\frac{f_n}{F_{ij}}\right)R_{C_{ij}}\right)+\frac{1}{\Phi_{C_{kj}}}.\left(\frac{f_n}{\left(F_{ij}+1\right)}\right)R_{C_{kj}}$$
$$-\frac{1}{\Phi_{C_{ij}}}\left(\frac{f_n}{F_{ij}}\right)R_{C_{kj}}+\sum_{f_n\in \mathrm{f},\, Ci\in \mathrm{C},\, j=j+1}^{|d|}\frac{1}{\Phi_{C_{ij}}}\left(\frac{f_n}{F_{ij}}\right)R_{C_{ij}} \tag{17}$$

We suppose the change of end to end path for N_n accurse in the j^{th} collision domain only and hence by eliminating the first and last summation terms.

$$=\frac{1}{\Phi_{C_{kj}}}\left(\frac{f_n}{F_{kj}+1}\right)R_{C_{kj}}-\frac{1}{\Phi_{C_{ij}}}\left(\frac{f_n}{F_{ij}}\right)R_{C_{ij}}$$

Since $F_{kj}+1\leq F_{ij}$ and $R_{Cij}=R_{Ckj}$, by substituting the appropriate terms and further simplification:

$$=\frac{1}{\Phi_{C_{kj}}}\left(\frac{f_n}{F_{ij}}\right)R_{C_{ij}}-\frac{1}{\Phi_{C_{ij}}}\left(\frac{f_n}{F_{ij}}\right)R_{C_{ij}}$$
$$=\left(\frac{\Phi_{C_{ij}}-\Phi_{C_{kj}}}{\Phi_{C_{kj}}.\Phi_{C_{ij}}}\right)\left(\frac{f_n}{F_{ij}}\right)R_{C_{ij}}$$
$$\alpha_n=\left(\frac{\psi(C_{ij},C_{kj})}{\Phi_{C_{kj}}.\Phi_{C_{ij}}}\right)\left(\frac{f_n}{F_{ij}}\right)R_{C_{ij}} \tag{18}$$

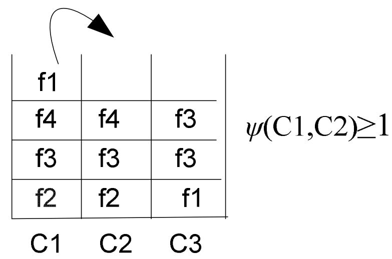

Figure 8. An interfarence difference

Here we only consider the case $F_{kj}+1==F_{ij}$ for simplicity, we can prove that $F_{kj}+1<F_{ij}$ have the same results.

The term $\psi(C_{ij},C_{kj})\geq 1$, therefore:
$\alpha_n>0$ and A^* cannot be a Nash Equilibrium under such a condition.

Proof of NE existence: In a MRMC Multiple-Collision domain mesh network, if $\ell<=1$ for all $j\in D$ and Lemma 1 and 3 do not hold then the strategy profile A^* is a Nash Equilibrium. From (14) of lemma 2, let $\ell=1$ and we assume that lemma1 and 3 do not hold, then:

$$\alpha_n=\left(\frac{\tau(CBR)-\tau(CBR)}{\left(Q(C_{kj})+\tau(CBR)\right)*\left(Q(C_{kj})+\ell*\tau(CBR)\right)}\right)f_n.R_{Ckj}=0 \tag{19}$$

Also for $\ell=0$:

$$\alpha_n=\left(\frac{-\tau(CBR)}{\left(Q(C_{kj})+\tau(CBR)\right)*\left(Q(C_{kj})\right)}\right)f_n.R_{Ckj}<0 \tag{20}$$

Also from (18) of lemma4, let $\psi(C_{ij},C_{kj})\geq 1$:

$$\alpha_n=\left(\frac{-\psi(C_{ij},C_{kj})}{\Phi_{C_{kj}}.\Phi_{C_{ij}}}\right)\left(\frac{f_n}{F_{ij}}\right)R_{C_{ij}}<0 \tag{21}$$

Both the negative sign and zero result show that each player has no incentive to deviate from its current strategy and hence the current strategy profile is a Nash Equilibrium.

Theorem 1: A strategy profile A^* is Nash Equilibrium, if:

1) $\forall f_n\in N_n, \left(\frac{nf_n}{F_{ij}}\right)RC_{max}<1, \exists\left(\frac{f_m}{Fij}\right)R_{C_{max}}>0\ for\ \ell\geq 1, \forall j, \forall C_{max}, C_{min}, \psi(C_{max},C_{min})=0$
2) $\forall C_{max}, C_{min}, 1\geq\ell\geq 0, \forall j\in D, \psi(C_{max},C_{min})=0$
3) $\Delta\theta_{C_{max},C_{min}}\leq 2$ for $\ell\geq 1, \forall f_n\in N_n, \forall N_n\in N, \forall j\in D, \psi(C_{max},C_{min})=0$
4) $\psi(C_{max},C_{min})<1, \forall C_{max},C_{min}, if\ \ell\geq 1,\ if\ \theta 1 f_n-\theta 2 f_n\geq 1$ for θ_1 on C_{max}, θ_2 on C_{min} for any player $N_n\in$ N.

V. CONVERGENCE TO NASH EQUILIBRIUM

In the previous section, we have proven that NE exists in a multi-radio multi-channel flow game with interference constraint in a non-cooperative environment. In this section we present a distributed algorithm running on each end node with imperfect information. As shown in Fig. 9, each player has information about each channel usage inside all collision domains but no player knows the strategy of her opponent players, thus a game of imperfect information.

Using Algorithm 1, each player $N_n \in \mathrm{N}$ selects channels for all its flows $f_n \in \mathrm{f}$ in each collision domain across the end to end path in a distributed manner. Lines 5, 9, 13 and 17 of the algorithm are sufficient conditions where it converges and ends up with an NE. Furthermore, each node keeps a record of its channel usage, $C_{ij}\_f_{nCount}$, in each collision domain for a necessary check at lines 9 and 14. Players in this game move simultaneously without having information about other players past histories. With this imperfect information, the game converges to stable NE in a non-cooperative environment.

Algorithm 1: Nash Equilibrium in MRMC multiple-collision domain game with interference constraint.

1. for each $N_n \in \mathrm{N}$
2. for each $f_n \in \mathrm{f}$ (do)
3. for j=1 to |d|
4. for i=1 to $C_{|c|}$
5. if $Q(C_{ij}) \leq Q(C_{min(j)})$ & $\phi_{Cij} \leq \phi_{Cmin(j)}$
6. Select channel C_i for flow f_n
7. $C_{ij}\_f_{nCount}$=$C_{ij}\_f_{nCount}$+1
8. exit;
9. elseif($C_{ij}\_f_{nCount}$ - $C_{(rem)}\_f_{nCount}$) ≤ 2 & $(\frac{nf_n}{F_{ij}}) R_{Cij} \neq 1$ & C_{ij}-$C_{(rem)j} \leq 1$
10. Select channel C_i for flow f_n
11. $C_{ij}\_f_{n\mathrm{Count}}$= $C_{ij}\_f_{n\mathrm{Count}}$ +1
12. exit;
13. elseif($Q(C_{rem(j)})$-$Q(C_{ij}) \geq 1$ & $\psi(C_{rem(j)}, C_{ij}) \geq 1$
14. Select channel C_i for flow f_n
15. $C_{ij}\_f_{n\mathrm{Count}}$= $C_{ij}\_f_{n\mathrm{Count}}$ +1
16. exit;
17. else
18. next i
19. next j
20. while(f_n)

Figure 9. Algorithm for Nash Equilibrium convergence using imperfect information

VI. PERFORMANCE EVALUATION

In this section, we evaluate our proposed algorithm and show its results in terms of individual fairness and price of anarchy. In the second subsection, we investigate and compare the throughput difference of our scheme with a random channel selection scheme by varying the number of players. All the experiments were conducted in MATLAB to test the performance and effectiveness of the proposed scheme.

A. Price of Anarchy and Individual Fairness

In this subsection, we investigate the Individual fairness of end user nodes and Price of Anarchy (*PoA*). We formulate the *PoA* as the ratio of throughput achieved by individual players in case of worst NE and best NE i-e

$$PoA = \frac{\Gamma_n(NE_{worst})}{\Gamma_n(NE_{best})}, n = 1, 2..., N_{|N|} \tag{22}$$

where Γ_n is the end-to-end throughput of player $N_n \in \mathrm{N}$.

In the simulation, 40 nodes are deployed randomly in a rectangular area of 600X600 units. The transmission range is considered 50 units and interference range is taken as twice of the transmission range. We configure 12 nodes on the left hand side of topology as players of the game. At the right hand side of the topology, 3 nodes are configured as the gateways. Each node generates 10 CBR flows of 64Kbps during each run of the game. Simulation was carried out by considering IEEE 802.11a [31], where 8 non-overlapping channels were selected for parameter C across 5 collision domains. Each node is configured with two radios, each for transmission and reception. All players move simultaneously having no information of one another past histories. With this imperfect information, we investigate the performance of the proposed scheme in terms of *PoA*. Since multiple NEs exist for this game, the mechanism for selecting the best NE by players is beyond the scope of this paper. We aim to solve this problem in a separate study. As shown in Fig. 10, the *PoA* is in the range of 0.71 and 1 for all the players. This shows a very strong indication that the individual throughput is not degraded even if the system converges to a worst NE.

Fairness among players was measured at the end of the game. As shown in Fig. 11, individual players achieve end to end data rate with a standard deviation of 2.19Mbps, with imperfect information, at the end of the game. This shows that using our algorithm, players achieve fair end to end data rate across multiple collision domains when game converges to NE. The reason is that when the game ends up with NE, each player is playing its best response as its strategy to every other player of the game and hence has no incentive to deviate individually from its current strategy. We carried out simulation by considering the same set of parameters, where nodes deviate from the proposed algorithm with selfish behavior. As shown in Fig. 12, although some nodes perform better comparatively to our scheme by achieving high end to end throughput; the standard deviation is

3.74Mbps. This shows that some of the selfish nodes get access to less interfered channels while leaving the crowded channels for others. This selfish behavior leads to individual unfairness of the system as compared to the proposed scheme.

B. Standard Deviation and Max-Min throughput Difference

In second scenario, 100 nodes are deployed in a rectangular area of 1000X1000 units. The transmission range is taken 25 units and the interference range as twice of the transmission range. Variance among players throughputs was measured by varying the set of players, N, from 5 to 40 in 5 steps. Since the previous work done in this area is either on selfish routing in wired networks or in wireless networks without considering multiple radio multiple channels in the core network. Therefore, we compared our proposed scheme with random channel selection where flows select channels across multiple collision domains arbitrarily. Fig. 13 compares max-min throughput difference by using our scheme with random channel selection. Max-min throughput difference is the difference between the flow which gets maximum throughput and flow that gets minimum throughput, as in Chen *et al.* [31]. It can be observed that our scheme outperforms random channel selection for each set of players by having minimum max-min throughput difference. The max-min difference is higher for both systems at beginning but as the number of players increases max-min throughput difference of our proposed system either decreases or remains constant when the game ends up with NE. This shows the stability of our scheme at NE. The max-min throughput difference for random selection does not remain stable with varying number of players, as shown in Fig. 8. This is because some of the selfish end nodes, being rational, select less crowded channels across multiple collision domains and thus increase their end to end throughput while leaving more crowded channels for other nodes.

Fig. 14 shows the standard deviation comparison of players throughputs in our proposed scheme against that of random selection. The values for collision domains (D), Channels (C), number of flows per node and CBR were kept same in both schemes while number of players/nodes was varied from 5 to 40 in step 5. Results in Fig. 14 suggest that our proposed scheme always performed better than random selection irrespective of the number of players. When number of nodes is low, some selfish players have always incentive to select less crowded channels across multiple collision domains and hence variance among players throughputs is high leading to high standard deviation. With increase in number of players, our proposed system shows a constant and predictable decrease in standard deviation while random selection scheme is unpredictable. This means that our proposed system achieves good fairness in long run when the system converges to NE.

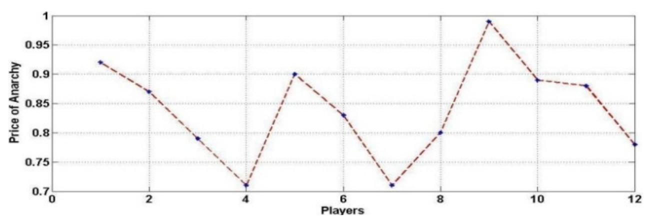

Figure 10. Price of Anarchy with D=5,C=8, N=12, f=10 and CBR=64Kbps

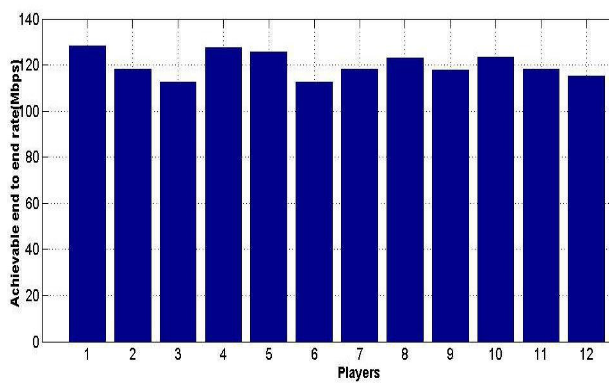

Figure 11. Total End to End rate of individual players with imperfect information with D=5,C=8, N=12, f=10 and CBR=64Kbps

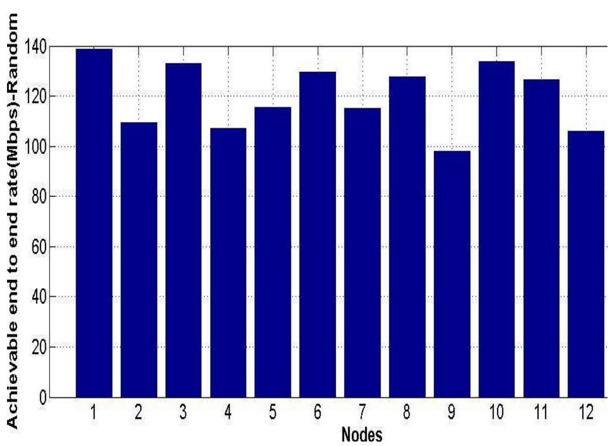

Figure 12. Total End to End rate of individual players with imperfect information with D=5,C=8, N=12, f=10 and CBR=64Kbps

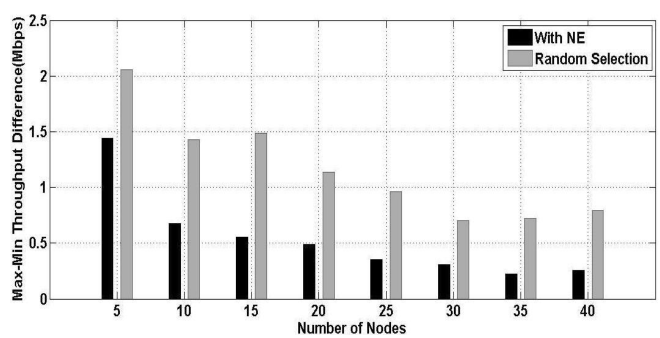

Figure 13. Max-Min Throughput Difference by varying number of players with imperfect information. D=5,C=8,N=5:5:40, f=10,CBR=64Kbps

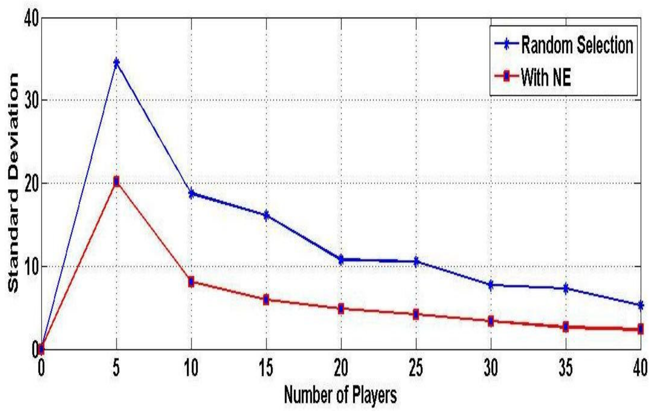

Figure 14. Standard Deviation among players throughput with D=5,C=8,N=5:5:40, f=10,CBR=64Kbps

VII. Conclusion

In this paper, we have developed a multiple-collision domain MRMC game theoretic model based on end user flows in a non-cooperative environment. An interference constrained topology was considered due to the limited available orthogonal channels. Our analytical results have proven that Nash Equilibrium exists with proposed necessary conditions in a game of imperfect information. Based on a distributed algorithm, our game theoretic model converges to stable state in finite time and all channels are perfectly load-balanced at the end of the game. Simulation results show that standard deviation of players throughputs is less than that of random channel selection scheme in long run. Furthermore, the Price of Anarchy of the system is close to one showing the efficiency of the proposed scheme.

We have considered single stage static game where players move simultaneously and once NE is established there is no incentive for any player to deviate from its current strategy, individually.

The work done in this paper can be extended to incorporate routing along with channel assignment in a non-cooperative environment by considering co-channel interference in the game formulation. It can be an interesting future research direction to investigate the effect of the coalition of flows on the overall fairness of players in repeated games.

In future, we are working to extend our proposed model to investigate the different mechanisms for selecting the best NE among the players. Further, we are working on coalition resistance game theoretic models for joint selfish routing and MRMC in multi-collision domain WMNs in an interference constraint non-cooperative environment.

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