

Transformation of any Adding Signal Technique in Tone Reservation Technique for PAPR Mitigation thanks to Frequency Domain Filtering

Désiré Guel, Jacques Palicot
 SUPELEC/IETR
 Avenue de la Boulaie CS 47601
 F-35576 Cesson-Sévigné cedex, France
 {desire.guel, jacques.palicot}@supelec.fr

Abstract—Orthogonal Frequency Division Multiplexing (OFDM) suffers from a high Peak-to-Average Power Ratio (PAPR). Tone Reservation (TR) is a popular PAPR reduction technique that uses a set of reserved subcarriers to carry the peak reducing signal. The major advantages of TR technique include no transmission performance degradation, no transmission of Side Information (SI) and downward compatibility. Because of all these benefits, TR seems to be promising for use in commercial standards such as Digital Video Broadcasting-Terrestrial (DVB-T2). Thanks to a frequency domain filtering, in this paper, which is an extension of [1], we propose Classical Transformation (CT) and Adaptive Transformation (AT) algorithms to transform Adding Signal techniques (like clipping techniques) to TR techniques in order to benefit of the TR advantages. As the transformation is a low-complexity process (about the FFT/IFFT complexity), the obtained technique results in a low-complexity TR technique. However, the transformation generates a loss of performance in PAPR reduction, which can be improved by iterating the process of transformation. Later in the paper, several Adding Signal techniques (as well-known clipping techniques) are transformed to TR techniques. Performance comparisons are done based on Complementary Cumulative Distribution Function (CCDF), Bit Error Rate (BER) and Power Spectral Density (PSD) metrics. The simulation results showed that, at the same PAPR reduction gain, CT algorithm is 2 times more complex than AT algorithm.

Keywords-Orthogonal Frequency Division Multiplexing (OFDM), Peak-to-Average Power Ratio (PAPR), Frequency Domain Filtering, Clipping Techniques.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM), although used in standards such as IEEE 802.11a/g, IEEE 802.16, HIPERLAN/2 and Digital Video Broadcasting (DVB) [2], suffers from high Peak-to-Average Power Ratio (PAPR). Large PAPR requires a linear High Power Amplifier (HPA), which is inefficient. Moreover, the combination of an insufficiently linear HPA range and large PAPR leads to in-band and out-of-band distortion [3]. Several PAPR reduction techniques have been proposed [4–7]. The simplest way to reduce PAPR is to deliberately clip and filter the OFDM signal before amplification. However, clipping is a nonlinear process and may cause significant distortion that degrades

the Bit Error Rate (BER) and increase adjacent out-of-band carriers [8].

Some techniques use coding, in which a data sequence is embedded in a larger sequence and only a subset of all the possible sequences is used to exclude patterns with high PAPR [9]. These techniques require receiver modifications to decode the received signal. Also, multiple signal representation techniques have been proposed. These include Partial Transmit Sequence (PTS) technique [10], Selected Mapping technique (SLM) [11] and interleaving technique [12]. These methods reduce the PAPR by controlling the phase of the data subcarriers, which provides an effective solution. However, they are computationally expensive due to multiple IFFTs and exhaustive search to find optimal phase sequences; also they require transmitting continuous SI to the receiver, which degrades the capacity of the system. The overall BER performance may also be degraded if there are errors in the SI [5].

PAPR can be reduced by the Adding Signal techniques [13], which are very simple techniques to implement and have become very attractive. Tone Reservation (TR) [14], which is a particular Adding Signal technique is a popular PAPR reduction technique that uses a set of reserved subcarriers to design a peak reducing signal. TR technique does not distort data-bearing subcarriers. Also, it not only eliminates the need for SI, but also prevents the BER degradation, as occurs with other techniques. However, TR technique requires an efficient generation of the peak-reducing signal. The optimal peak-reducing signal generation is obtained by solving a Quadratically Constrained Quadratic Program (QCQP), which is a type of convex optimization problem [14]. Although the optimum of a QCQP exists, it is shown in [14] that the solution requires a high computational cost of $\mathcal{O}(N_r N^2 L)$, where N_r is the number of the reserved subcarriers, N is the number of subcarriers and L is the oversampling factor. The authors of [15] propose an optimal peak-reducing signal based on Second Order Cone programming (SOCP) formulation of QCQP problem. Since finding the optimal solution to QCQP problem is computationally demanding, an iterative way to reduce PAPR was also proposed [14, 16].

In [1], we proposed a transformation algorithm to transform Classical clipping in TR clipping. This paper is an extension of [1], in which we propose two transformation algorithms the classical algorithm (algorithm from [1]) and an adaptative algorithm. Moreover, we apply these two transformations on several Adding Signal techniques like Geometrical method [17] and several types of clipping [18].

The remainder of this paper is organized as follows: Section II introduces the OFDM systems. Section III briefly reviews the Adding Signal techniques principle, gives some examples of these techniques and focuses on the TR techniques, which are specific Adding Signal techniques. Section IV describes the principle of the digital filter based on FFT/IFFT pair and derives the CT and AT algorithms. In Section V CT and AT algorithms are applied for PAPR reduction in a Wireless Local-Area-Network (WLAN) system, to two Adding Signal techniques and simulation results are provided, while in Section VI a conclusion is drawn.

II. OFDM SYSTEMS AND PAPR ISSUE

The basic idea underlying OFDM systems is the division of the available frequency spectrum into several subcarriers. To obtain a high spectral efficiency, the frequency responses of the subcarriers are overlapping and orthogonal, hence the name OFDM. This orthogonality can be completely maintained with a small price in a loss in SNR, even though the signal passes through a time dispersive fading channel, by introducing a cyclic prefix (CP).

The continuous-time baseband representation of an OFDM symbol is given by

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi f_k t}, \quad 0 \leq t \leq T_s, \quad (1)$$

where N data symbols X_k form an OFDM symbol $\mathbf{X} = [X_0, \dots, X_{N-1}]$, $f_k = \frac{k}{T_s}$ and T_s is the time duration of the OFDM symbol.

In practice, OFDM signals are typically generated by using an Inverse Discrete Fourier Transform (IDFT) as described by the block diagram in Fig. 1.

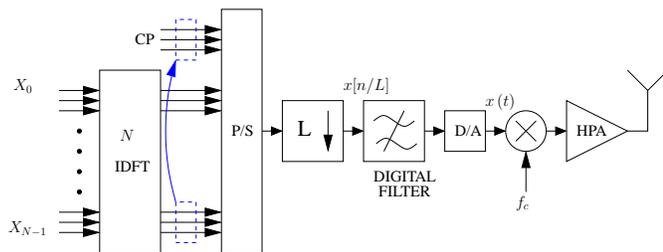


Figure 1: OFDM Transmitter Block Diagram.

The OFDM symbol represented by the vector $\mathbf{X} = [X_0 \dots X_{N-1}]^T$ is transformed via IDFT into T_s/N -spaced discrete-time vector $\mathbf{x} = x[n] = [x_0 \dots x_{N-1}]^T$, i.e.

$$x_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j2\pi \frac{n}{N} k}, \quad 0 \leq n \leq N-1. \quad (2)$$

In this paper, the discrete-time indexing $[n]$ denotes Nyquist Rate samples. Since oversampling may be needed in practical designs, we will introduce the notation $x[n/L]$ to denote oversampling by L . Several different oversampling strategies of $x[n/L]$ can be defined. From now on, the oversampled IDFT output will refer to oversample of (2), which is expressed as follows:

$$x[n/L] = \frac{1}{\sqrt{N}} \sum_{k=0}^{NL-1} X_k e^{j2\pi \frac{n}{NL} k}, \quad 0 \leq n \leq NL-1. \quad (3)$$

The above expression (3) can be implemented by using a length- (NL) IDFT operation with the input vector

$$\mathbf{X}^{(L)} = \left[X_0, \dots, X_{\frac{N}{2}-1}, \underbrace{0, \dots, 0}_{(L-1)N \text{ zeros}}, X_{\frac{N}{2}}, \dots, X_{N-1} \right].$$

Thus, $\mathbf{X}^{(L)}$ is extended from \mathbf{X} by using the so-called zero-padding scheme, i.e., by inserting $(L-1)N$ zeros in the middle of \mathbf{X} , i.e.,

$$X_k^{(L)} = \begin{cases} X_k, & k \in \mathcal{S}_1 \\ 0, & k \in \mathcal{S}_2 \end{cases},$$

where \mathcal{S}_1 and \mathcal{S}_2 are the set of in-band (IB) indices and out-of-band (OOB) indices respectively.

The cost of transceiver components depends on the dynamic range of the signals. In the literature, the envelope variations are often described in terms of the crest-factor (CF), peak-to-mean envelope power ratio (PMEPR) or simply peak-to-average power ratio (PAPR). In this paper, we adopt the terms PAPR to quantify the envelope excursions of the signal. The PAPR of the signal $x(t)$ may be defined as

$$\text{PAPR}_{[x]} \triangleq \frac{\max_{t \in [0, T_s]} |x(t)|^2}{\mathcal{P}_x}, \quad (4)$$

where $\mathcal{P}_x = E\{|x(t)|^2\}$ is the average signal power and $E\{\cdot\}$ is the statistical expectation operator. Note that, in order to avoid aliasing the out-of-band distortion into the data bearing subcarriers and in order to accurately describe the PAPR, an oversampling factor $L \geq 4$ is required.

In the literature, it is customary to use the Complementary Cumulative Distribution Function (CCDF) of the PAPR as a performance criterion. It is denoted as

$$\text{CCDF}_{[x]}(\psi) \triangleq \Pr\{\text{PAPR}_{[x]} \geq \psi\}.$$

If N is large enough, based on the central limit theorem, the real and imaginary parts of OFDM $x(t)$ have Gaussian distribution and its envelope will follow a Rayleigh distribution. This implies a large PAPR. In [19], it is shown that the mean of the PAPR, which is a random variable, can be approximated to

$$E[\text{PAPR}] \simeq C_{(\text{Euler})} + \ln[N], \quad (5)$$

where $C_{(\text{Euler})}$ is the Euler's constant defined below

$$C_{(\text{Euler})} = \lim_{N \rightarrow \infty} \left[\sum_{k=1}^N \frac{1}{k} - \ln[N] \right] \simeq 0.57721.$$

III. ADDING SIGNAL TECHNIQUES FOR PAPR REDUCTION

There are several different techniques for PAPR reduction, in this section, we present the Adding Signal techniques principle and then we focus on the TR techniques, which are specific Adding Signal techniques for PAPR reduction.

A. Adding Signal Techniques Principle

In Adding Signal context, the PAPR is reduced by adding a signal called sometimes "peak reducing signal" or "peak canceling signal". Many well known PAPR reduction techniques of the literature such as Tone Reservation (TR) [14], Tone Injection (TI) [14, 20], Geometric Approach for PAPR reduction method [17] are known as Adding Signal techniques. In [13], it is shown that any form of clipping can be formulated as an Adding Signal technique. The Adding Signal techniques consist of reducing the envelope of OFDM signal by adding a peak-reducing signal just before the HPA as shown in Fig. 2.

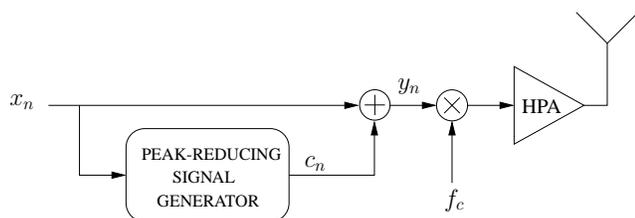


Figure 2: Adding Signal scheme for PAPR reduction.

Let x_n , $n = 0, \dots, NL - 1$ be the L -times oversampled time-domain OFDM signal, where N is the number of subcarriers. The PAPR reduced signal is therefore expressed by

$$y_n = x_n + c_n, \quad n = 0, \dots, NL - 1. \quad (6)$$

The peak reducing signal c_n is computed according to PAPR reduction techniques. In [15], c_n is computed based on an optimization algorithm (SOCP) in frequency domain, while in [17], c_n is computed in time domain based on

a Geometric approach. *S. Janaathanan et al.* propose, in [16], to compute c_n in frequency domain with the Gradient algorithm, which is a low-complexity algorithm. In [13], the reducing signal c_n is computed in time domain based on a nonlinear function $f(\cdot)$ called "function for PAPR reduction". Using $f(\cdot)$ to reduce the PAPR of $x[n/L]$, the peak reducing signal c_n is written as

$$c_n = f(|x_n|) e^{j\varphi_n} - x_n, \quad (7)$$

where φ_n is the x_n phase.

B. Some Examples of Adding Signal Techniques

As previously mentioned, depending on the way to generate the Adding Signal c_n , we obtain many different methods. In this section, we present the clipping techniques family [18], which could be easily seen as adding method and the geometrical method of [17].

1) *Clipping techniques family*: In the first subsection we formulate the classical clipping as an Adding Signal technique as described in Fig. 2, then in the second subsection we describe briefly four clipping techniques.

clipping technique formulated as an Adding Signal technique: Using the conventional clipping technique [8] to reduce OFDM PAPR, the output signal y_n , in terms of the input signal x_n is given as follows:

$$y_n = f(|x_n|) e^{j\varphi_n},$$

where φ_n is the x_n phase and $f(\cdot)$ is the clipping function. As $f(\cdot)$ is nonlinear function; according to Busgang theorem [21], the output signal y_n can be written as

$$y_n = \alpha x_n + d_n, \quad \text{where } \alpha = \frac{\mathcal{R}_{yx}(0)}{\mathcal{R}_{xx}(0)}. \quad (8)$$

$\mathcal{R}_{xx}(\tau)$ and $\mathcal{R}_{xy}(\tau)$ are autocorrelation and cross-correlation functions of the input signal and output signal. It is shown that the distortion term d_n is uncorrelated with the input signal x_n , i.e., $\mathcal{R}_{xd}(\tau) = 0$.

From (6) and (8), the peak-reducing signal c_n is expressed as

$$c_n = (\alpha - 1) x_n + d_n. \quad (9)$$

We see from (9) that, the peak-reducing signal depends on the distortion term resulting in the nonlinear process of the OFDM envelope.

several clipping techniques: It is obvious that in Eq. 9, the nonlinear clipping function is included in the α parameter. In this paragraph, we give some possible nonlinear function therefore some clipping techniques.

1) Classical Clipping (CC) technique

The Classical Clipping (CC) proposed in [8] is one of the most popular clipping technique for PAPR reduction known in the literature [8, 22]. It is sometimes

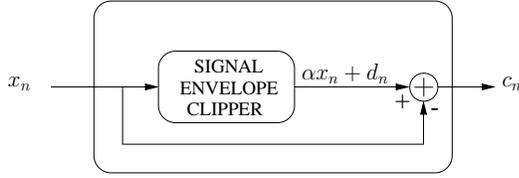


Figure 3: Peak-reducing signal generator block for clipping technique.

called hard clipping or soft clipping, to avoid any confusion, it is called Classical Clipping (CC) in this paper. In [8], its effects on the performance of OFDM, including the power spectral density, the PAPR and BER are evaluated. The function-based clipping used for CC technique is defined below and depicted in Fig. 5 (a).

$$f(r) = \begin{cases} r, & r \leq A \\ A, & r > A \end{cases} \quad (10)$$

Where A is the clipping threshold. We derive now, a bound for the α parameter depending on the clipping threshold. This derivation, could be performed for every type of clipping function, but we restrict here it to the classical clipping function. Let us consider the coefficient α defined in (8).

$$\begin{aligned} \alpha &= \frac{\mathcal{R}_{yx}(0)}{\mathcal{R}_{xx}(0)} \\ &= \frac{E\{rf(r)\}}{\mathcal{P}_x} = \frac{1}{\mathcal{P}_x} \int_0^{\infty} rf(r)p(r)dr, \end{aligned} \quad (11)$$

where $f(r)$ is the clipping function, \mathcal{P}_x is the OFDM signal power, $p(r)$ is the probability density function (PDF) of the OFDM envelope and $E\{\cdot\}$ is the statistical expectation operator. It can be shown that, for a large number of subcarriers, the OFDM envelope converges to Rayleigh envelope distribution. Therefore,

$$p(r) = \frac{2r}{\mathcal{P}_x} e^{-\frac{r^2}{\mathcal{P}_x}}, \quad r \geq 0. \quad (12)$$

Substituting the expressions of $f(r)$ and $p(r)$ given by (10) and (12) into (11), we show that

$$\begin{aligned} \alpha &= \frac{1}{\mathcal{P}_x} \int_0^A r^2 \frac{2r}{\mathcal{P}_x} e^{-\frac{r^2}{\mathcal{P}_x}} dr + \frac{1}{\mathcal{P}_x} \int_0^A Ar \frac{2r}{\mathcal{P}_x} e^{-\frac{r^2}{\mathcal{P}_x}} dr \\ &= 1 - \left(1 + \frac{A^2}{\mathcal{P}_x}\right) e^{-\frac{A^2}{\mathcal{P}_x}} + \frac{A^2}{\mathcal{P}_x} e^{-\frac{A^2}{\mathcal{P}_x}} + \frac{A}{\sqrt{\mathcal{P}_x}} \sqrt{\pi} Q\left(\frac{A\sqrt{2}}{\sqrt{\mathcal{P}_x}}\right) \\ &= 1 - e^{-\frac{A^2}{\mathcal{P}_x}} + \frac{A}{\sqrt{\mathcal{P}_x}} \sqrt{\pi} Q\left(\frac{A}{\sqrt{\mathcal{P}_x}} \sqrt{2}\right), \end{aligned} \quad (13)$$

where $\frac{A}{\sqrt{\mathcal{P}_x}}$ is the clipping ratio (CR) and $Q(\cdot)$ is the Q-function defined as

$$Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{\tau^2}{2}} d\tau.$$

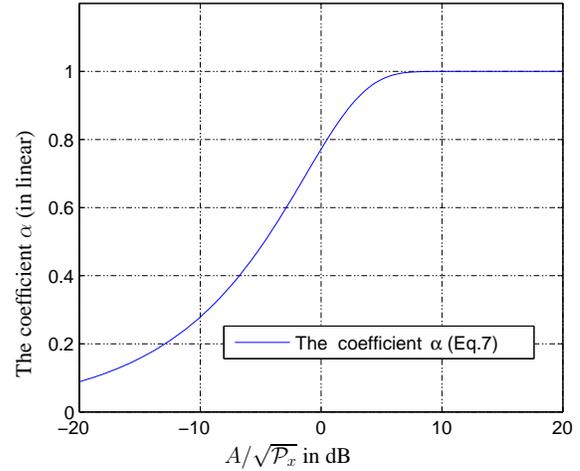


Figure 4: The coefficient α as a function of $\frac{A}{\sqrt{\mathcal{P}_x}}$.

Fig. 4 shows that the coefficient α expressed in (13) is an increasing function of $\frac{A}{\sqrt{\mathcal{P}_x}}$ and converges to 1 for $\frac{A}{\sqrt{\mathcal{P}_x}} \geq 5$ dB.

Now, let us consider the clipping threshold A sufficiently large such as $\alpha \approx 1$ but not very large, otherwise any peak will be reduced, i.e., $\frac{A}{\sqrt{\mathcal{P}_x}}$ approaches 5 dB. In this context, (9) becomes,

$$\begin{aligned} c_n &= (\alpha - 1)x_n + d_n \\ &\approx d_n. \end{aligned} \quad (14)$$

From (14), it can be concluded that, for $\frac{A}{\sqrt{\mathcal{P}_x}}$ approaches 5 dB, the peak-reducing signal is approximately equal to the distortion term resulting in the clipping of the OFDM envelope.

- 2) Heavyside Clipping (HC) technique: Often called hard clipping, HC is used in [23] as a baseband nonlinear transformation technique to improve the overall communication system performance. The heavyside function is expressed below and depicted in Fig. 5 (b).

$$f(r) = A, \quad \forall r \geq 0.$$

The HC technique is a case of school, it is widely used in theory but very rarely in practice. In [18] it is demonstrated, that HC has the worse performances of these four clipping techniques.

- 3) Deep Clipping (DC) technique

Deep Clipping has been proposed in [24] to solve the peaks regrowth problem due to the out-of-band filtering of the classical clipping and filtering method. So, in DC technique, the clipping function is modified in order to “deeply” clip the high amplitude peaks. A parameter called clipping depth factor has been introduced in order to control the depth of the clipping. The function-based clipping used for DC technique is defined below and depicted in Fig. 5 (c).

$$f(r) = \begin{cases} r & , \quad r \leq A \\ A - \beta(r - A) & , \quad A < r \leq \frac{1+\beta}{\beta}A \\ 0 & , \quad r > \frac{1+\beta}{\beta}A \end{cases}$$

where β is called the clipping depth factor.

4) Smooth Clipping (SC) technique

In [25], a Smooth Clipping technique is used to reduce the OFDM PAPR. In this paper, the function based-clipping for SC technique is defined below and depicted in Fig. 5 (d).

$$f(r) = \begin{cases} r - \frac{1}{b}r^3, & r \leq \frac{3}{2}A \\ A, & r > \frac{3}{2}A \end{cases}$$

where $b = \frac{27}{4}A^2$.

These four clipping functions are drawn on Fig. 5 and have been completely studied and compared in [18]. In the literature it exists other clipping function, among them we may cite the ‘invertible clipping’ of [26]. All these clipping techniques could be formulated as Adding Signal technique (as previously done in Section III-B1 and therefore could be transformed in TR techniques (see following sections).

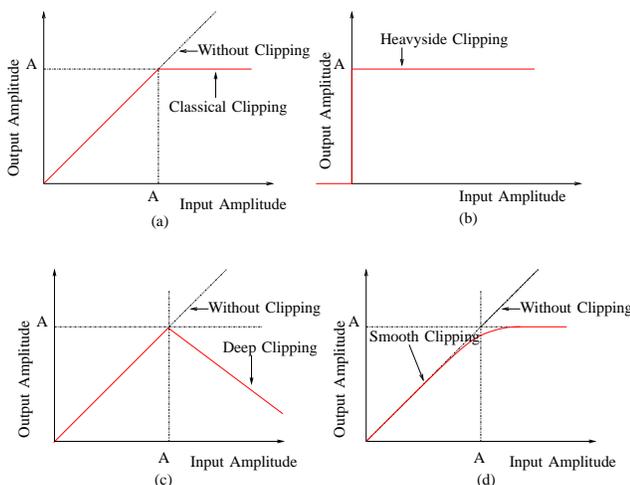


Figure 5: Functions-based clipping for PAPR reduction

C. Geometric Method for PAPR Reduction

The Geometric Method (GM) is an Adding Signal technique that was proposed for the first time in [17]. The GM technique is a backward-compatible technique, which means it does not require any additional information at the reception and the receiver should not be changed.

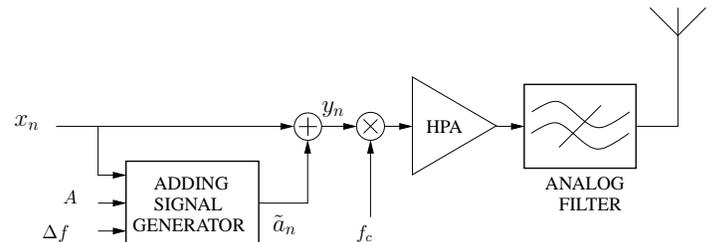


Figure 6: Principle of GM technique for OFDM PAPR reduction.

The principle of the technique is to first generate an “artificial signal” $a(t)$, which is then modulated into intermediate frequency Δf for give rise to an “adding signal” $c(t)$, which is in principle outside the useful band of the OFDM signal $x(t)$. The reduced signal $y(t) = x(t) + c(t)$ is modulated into RF frequency and then amplified. Immediately after amplification, the adding signal is removed by a bandpass analog filtering placed in the transmitter side. Fig. 6 shows the diagram of GM technique.

In [17], the “adding signal” $c(t)$ is determined using a geometric approach. It is expressed by the below equation

$$c(t) = \begin{cases} 0, & |x(t)| \leq A \\ [Ae^{j\varphi(t)} - x(t)] e^{j\Delta f t}, & |x(t)| > A \end{cases}$$

The mechanism of GM technique for PAPR reduction is described in Fig. 7.

D. Distortion Reduction in Adding Signal Techniques

Some of Adding Signal techniques can create the peak reducing signal without any in-band and out-of band distortion; this is the case of TR technique; we will go back to the details of this technique. However, when the peak reducing signal is generated based on nonlinear functions, some distortion are also generated. The well-know Adding Signal technique, in which some distortion are created is the clipping [8].

In [27], it is shown that, the peak reducing signal c_n calculated based on nonlinear functions, can be decomposed as follow

$$c_n = c_n^{(IB)} + c_n^{(OOB)}, \quad (15)$$

where, $c_n^{(IB)}$ is the peak reducing signal component created in the in-band of the OFDM signal, while $c_n^{(OOB)}$ is the peak reducing signal component created in the out-of-band of the OFDM signal.

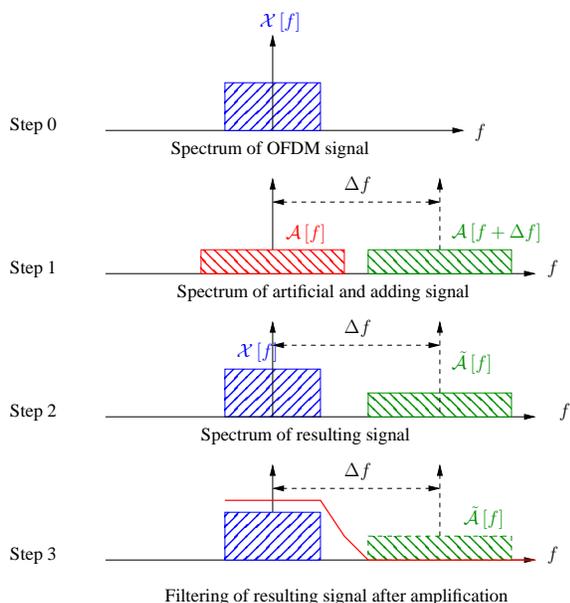


Figure 7: Mechanism of PAPR reduction.

Note that, $c_n^{(IB)}$ is responsible to in-band distortion that causes degradation of the BER whereas $c_n^{(OOB)}$ is responsible to out-of-band radiation that causes adjacent channel interference (ACI) and affects systems working in the neighbor bands.

In [8], the out-of-band distortion is mitigated by suppressing the out-of-band component $c_n^{(OOB)}$ of the peak reducing signal c_n . This distortion mitigation is done by a digital filter based on FFT/IFFT. But the in-band component $c_n^{(IB)}$ is kept, reason why in clipping and filtering technique of [8], the BER of the system is degraded.

The process of the suppression of the out-of-band distortion in Adding Signal is shown in Fig. 8 in frequency domain.

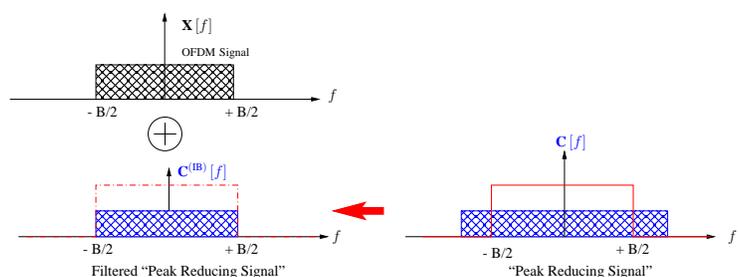


Figure 8: Out-of band distortion mitigation structure based on FFT/IFFT digital filter.

One drawback of out-of-band distortion suppression in Adding Signal techniques is peak regrowth due to the filtering of the peak reduction signal. Indeed, the digital filter based on FFT/IFFT truncates some of the information that is used to reduce PAPR and create some peak regrowth.

Some peak regrowth reduction methods have been proposed in the literature [22, 28]. A straightforward way of peak regrowth reduction is the repeated of Adding Signal and out-of-band distortion filtering. This method is proposed in [22] in the case of clipping and filtering. In [28], a peak regrowth reduction method based on the “deeply” clip the high amplitude peaks of the signal.

E. Overview of TR Techniques

The TR technique [14–16] is an Adding Signal technique. This technique has been studied mainly on the OFDM signal, without specification of a particular standard and can be generalized to all types of multicarrier systems. TR technique is a pioneering method, particularly since it was the first to be modeled as a convex optimization problem. The precursor of this technique is J. Tellado [14].

The principal idea of TR technique is to reserve N_r subcarriers in the OFDM symbol on which will be added a relevant information in order to change the time signal, so as to reduce the dynamics of the signal envelope. In this technique, the transmitter and receiver agree on the number and the positions of subcarriers, which are reserved to carry the corrective signal for decrease the PAPR.

It should be understood that at the beginning, TR technique is not backward compatible. Indeed when it was introduced for the first time by J. Tellado in [14], the positions of so-called “reserved subcarriers” are not fixed (known in advance), it assumes that the receiver must be informed by the transmitter on the positions dedicated subcarriers used to bring the “signal PAPR reduction”.

In this paper, the TR technique will be implemented using the “unused subcarriers” so-called “null subcarriers” of the standards in order to make the technique backward compatible. The schematic diagram of the method is given in Fig. 9.

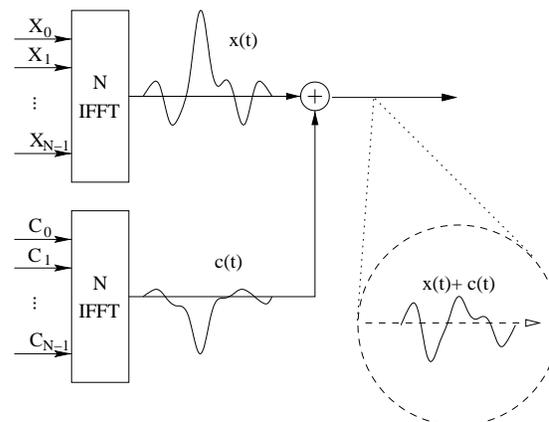


Figure 9: Illustration of Tone Reservation Structure.

The peak reducing signal c_n is carried by the reserved subcarriers and the peak-reduced signal is given by

$$y_n = x_n + c_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{NL-1} (X_k + C_k) e^{2j\pi \frac{kn}{NL}}, \quad (16)$$

where $0 \leq n \leq NL - 1$ and $\mathbf{C} = [C_0, \dots, C_{NL-1}]$ is the set of peak-reducing subcarriers.

Let $\mathcal{R} = \{i_0, \dots, i_{N_r-1}\}$ be the locations of the reserved subcarriers and let \mathcal{R}^c be the complement of \mathcal{R} in \mathcal{S}_1 . In TR technique, the constraint on c_n is that \mathbf{C} must satisfy $C_k \equiv 0$ for $k \in (\mathcal{R}^c \cup \mathcal{S}_2)$. On the other hand, \mathbf{X} must satisfy $X_k \equiv 0$ for $k \in \mathcal{R}$. \mathbf{X} and \mathbf{C} are not allowed to be nonzero on the same subcarriers, i.e.,

$$X_k + C_k = \begin{cases} X_k, & k \in \mathcal{R}^c \\ C_k, & k \in \mathcal{R} \end{cases}. \quad (17)$$

Because of $\mathcal{R} \cap \mathcal{R}^c = \emptyset$, the BER of the system is not degraded. Because of $C_k = 0$ for $k \in \mathcal{S}_2$, there is not out-of-band radiation.

The only drawback of TR technique is the loss of throughput due to the reserved carriers to carry out the ‘‘peak reducing signal’’ (which is the case, for example, in the DVBT2 standard [2]). To minimize this loss, we propose to use unused or nulls carriers of the standards as reserved carriers (see Section V).

IV. FROM ADDING SIGNAL TECHNIQUES FOR PAPR REDUCTION TO TR TECHNIQUES

In this section, we first describe the principle of the digital filter based on FFT/IFFT pair, which is used for the transformation of Adding Signal techniques in TR techniques. Then we derive the algorithms of classical and adaptive transformation of Adding Signal techniques in TR techniques. Finally, we evaluate their computational complexities.

Using the same principle of out-of band distortion suppression based on FFT/IFFT digital filtering, we have proposed in [1] for the first time the idea of the transformation of PAPR reduction techniques into TR techniques based on FFT/IFFT digital filtering. In [1], the classical clipping technique is transformed in TR technique for WLAN (IEEE 802.11 a/g) PAPR reduction. In this paper we propose an improvement of the classical transformation algorithm initially described in [1] by adaptively transform Adding Signal techniques to TR techniques. The new algorithm will be called adaptive transformation algorithm. Both algorithms are very similar and are both based on FFT/IFFT digital filtering.

A. The Digital Filter Based on FFT/IFFT Principle

Let \tilde{c}_n , the signal at the output of the IFFT/FFT pair based filter as shown in Fig. 10.

The FFT/IFFT pair-based filter consists of a FFT operation followed by an IFFT operation. The forward FFT transforms c_n back to the frequency-domain. The discrete frequency components of c_n on the reserved subcarriers are

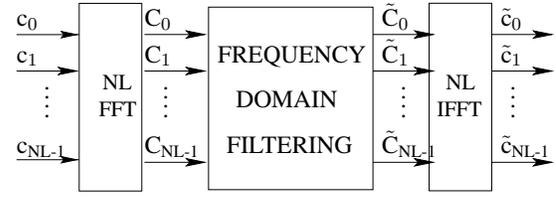


Figure 10: Digital filtering-based FFT/IFFT.

passed unchanged while the data subcarriers and the OOB components are set to zero, i.e.,

$$\tilde{C}_k = \mathcal{H}[C_k] = \begin{cases} C_k, & k \in \mathcal{R} \\ 0, & k \in (\mathcal{R}^c \cup \mathcal{S}_2) \end{cases}. \quad (18)$$

The IFFT operation transforms \tilde{C}_k , back to the time domain. This results in the filtered peak-reducing signal \tilde{c}_n at the output of the filter-based FFT/IFFT.

The relation between the input and the output of the FFT/IFFT pair-based filter is written as:

$$\tilde{c}_n = \mathcal{F}^{-1}(\mathcal{H}[\mathcal{F}(c_n)]), \quad (19)$$

where \mathcal{F} represents the FFT function, \mathcal{F}^{-1} is the IFFT function and \mathcal{H} is the digital filter response in frequency domain.

According to (18), only the components of c_n on the reserved subcarriers (\mathcal{R}) are used for OFDM PAPR reduction; that is why, the resulting PAPR reduction technique is a TR technique.

The FFT/IFFT pair-based filter complexity as well defined, depends only on the complexity of utilizing the FFT/IFFT pair and is approximated as $\mathcal{O}(NL \log_2 NL)$.

The principle of FFT/IFFT pair-based filter for transformation of Adding Signal techniques in TR techniques is shown in Fig. 11.

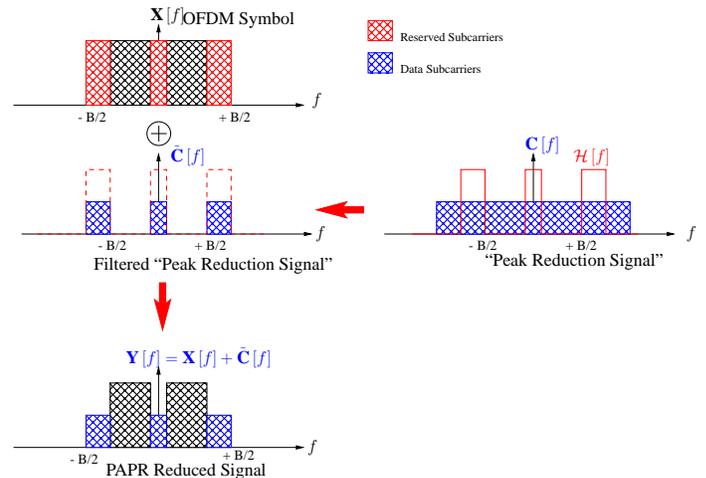


Figure 11: FFT/IFFT pair-based filter for transformation to TR technique.

In the same way as the out-of band distortion mitigation, the digital filter based on FFT/IFFT remove a part of the information, which is used for PAPR reduction and create some peak regrowth. For both classical and adaptive transformation algorithms, the repeated adding and filtering signal for PAPR reduction is used to reduce the peak regrowth phenomena.

Now let go to the details of classical and adaptive transformation algorithms.

B. Classical Transformation (CT) Algorithm

In this subsection, we describe the classical transformation algorithm of Adding Signal techniques for PAPR reduction to the TR techniques, which is based on the FFT/IFFT digital filter.

In order to reduce as much as possible the PAPR, the CT algorithm is based on an iterative algorithm, which the principle is as follow:

- Set up the locations of the reserved subcarriers \mathcal{R} and the maximum iteration number \mathcal{N}_{iter} , and choose the function for PAPR reduction $f(\cdot)$.
- Set up $i = 0$, where $x_n^{(0)} = x_n$ is the time-domain OFDM signal.
- Compute the (i) -iteration PAPR reduction signal as:

$$\tilde{c}_n^{(i)} = f_{\Delta} \left[f \left(x_n^{(i)} \right) - x_n^{(i)} \right], \quad (20)$$

where $f_{\Delta} = \mathcal{F}^{-1} \circ \mathcal{H} \circ \mathcal{F}$ is the FFT/IFFT based digital filter response in time domain.

- Compute the $(i + 1)$ -iteration PAPR reduced signal as:

$$x_n^{(i+1)} = x_n^{(i)} + \tilde{c}_n^{(i)} \quad (21)$$

It must bear in mind that, the system complexity grows linearly with the number of iterations. The data processed by this algorithm in this paper are L over-sampled OFDM symbols. The complexity of computational of $\tilde{c}_n^{(i)}$ for each iteration is $\mathcal{O}(NL \log_2 NL)$ because f_{Δ} , which is the digital filter response is based on a backward FFT followed by a forward IFFT. Assuming that \mathcal{N}_{iter} is the maximum number of iterations, the CT-algorithm complexity can be approximated to $\mathcal{O}(\mathcal{N}_{iter} NL \log_2 NL)$.

C. Adaptive Transformation (AT)-Algorithm

The AT-algorithm for the transformation of Adding Signal techniques to TR techniques is based on the CT-algorithm. In distinction of the CT-algorithm, the AT-algorithm scales the PAPR reduction signal $\tilde{c}_n^{(i)}$ by an optimal scaling factor $\beta_{opt}^{(i)}$ in order to outperform the PAPR reduction performance. Whereby the PAPR reduced signal for AT-algorithm, at $(i + 1)$ -iteration, is written as:

$$x_n^{(i+1)} = x_n^{(i)} + \beta_{opt}^{(i)} \tilde{c}_n^{(i)} \quad (22)$$

The scaling factor $\beta_{opt}^{(i)}$ is the solution of the optimization problem, which is formulated as

$$\beta_{opt}^{(i)} = \arg \min_{\beta} \left[\max_n \left| x_n^{(i)} + \beta \tilde{c}_n^{(i)} \right| \right]. \quad (23)$$

An exact solution of Eq. (23) exists but leads to a high computation complexity. An alternative solution to Eq. (23) is a low computation complexity suboptimal solution. In [29], it is shown that, a suboptimal solution of Eq. (23) is given by minimizing the total power of the samples with $\left| x_n^{(i)} + \tilde{c}_n^{(i)} \right| > A$, where A is the magnitude threshold. Solving Eq. (23) leads to

$$\beta_{opt}^{(i)} = \arg \min_{\beta} \left[\sum_{n \in \mathcal{S}_p^{(i)}} \left| x_n^{(i)} + \beta \tilde{c}_n^{(i)} \right| \right], \quad (24)$$

where, $\mathcal{S}_p^{(i)} = \left\{ n : \left| x_n^{(i)} + \tilde{c}_n^{(i)} \right| > A \right\}$. The above minimization problem is a linear least-squares problem and the solution is given by

$$\beta_{opt}^{(i)} = - \frac{\sum_{n \in \mathcal{S}_p^{(i)}} x_n^{(i)} \tilde{c}_n^{*(i)}}{\sum_{n \in \mathcal{S}_p^{(i)}} \left| \tilde{c}_n^{(i)} \right|^2}, \quad (25)$$

where $(\cdot)^*$ is the mathematical conjugate function.

The complexity of calculating $\beta_{opt}^{(i)}$ is $\mathcal{O}(\mathcal{N}_p)$, where \mathcal{N}_p is the size of $\mathcal{S}_p^{(i)}$. After \mathcal{N}_{iter} iterations, the AT-algorithm complexity can be approximated to $\mathcal{N}_{iter} [\mathcal{O}(NL \log_2 NL) + \mathcal{O}(\mathcal{N}_p)] \simeq \mathcal{O}(\mathcal{N}_{iter} NL \log_2 NL)$.

V. TRANSFORMATION OF ADDING SIGNAL TECHNIQUES FOR PAPR REDUCTION TO TR TECHNIQUES IN A WLAN SYSTEM CONTEXT

In this section, based on the above analysis, using the CT and AT algorithms, we propose to transform the classical clipping technique [8] and the Geometric PAPR reduction technique [17] into TR techniques for PAPR reduction of the Wireless Local-Area-Networks (WLAN) system based on IEEE 802.11a/g standards. In the first subsection we provide the characteristics of the IEEE 802.11a/g standards. Then using the two CT and AT algorithms in the second subsection, we give the obtained results .

A. The IEEE 802.11a/g standards based WLAN system

In WLAN IEEE 802.11a/g standard IFFT size (N) is 64. Out of these 64 subcarriers, 48 subcarriers are used for data, while 4 subcarriers are used for pilots. The rest 12 subcarriers are unused (null) subcarriers located at the positions $\mathcal{R} = \{0, 27, \dots, 37\}$ of the IFFT input.

The IEEE 802.11a/g Standard specifications are given in [30] and the transmit spectral mask requirements is shown in Fig. 12.

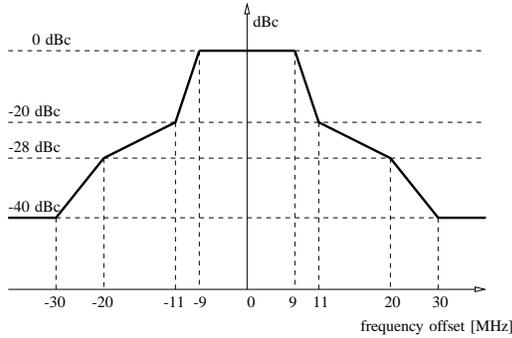


Figure 12: Spectral power mask of OFDM based WLAN.

B. Simulation results

- “Classical-TR-CC” and “Adaptive-TR-CC” are the TR techniques resulting of the transformation of classical clipping technique based on the CT and AT algorithms respectively.
- “Classical-TR-GM” and “Adaptive-TR-GM” are the TR techniques resulting of the transformation of Geometric PAPR reduction technique on the CT and AT algorithms respectively.

In this section, we evaluate the performance of the four TR techniques in a WLAN PAPR reduction context. For simulation results, only the unused subcarriers of the WLAN standard shall be utilized for PAPR reduction. The configuration, which is used for the simulations shown in Tab.I.

System Parameter	Parameter Value
Modulation Scheme	16-QAM
Number subcarriers	$N = 64$
Number of data subcarriers	48
Number of pilot subcarriers	4
Oversampling Factor	$L = 4$
Channel Model	AWGN
Clipping Ratio (CR)	$\frac{A}{\sqrt{P_x}} = 5$ dB

Table I: Simulation Environment

The distribution based on the CCDF is used to evaluate the performance in terms of PAPR reduction of the system, the BER metric is used to evaluate the transmission performance of the system over an AWGN channel and the Power Spectral Density (PSD) of signals will be evaluated.

We evaluate also the high signal fluctuations of the PAPR by determining the performance in terms of PAPR reduction defined as

$$\Delta\text{PAPR} = \text{PAPR}_{[y]} - \text{PAPR}_{[x]}, \quad [\text{in dB}]$$

where $\text{PAPR}_{[y]}$ is the required PAPR of the signal $y(t)$ to obtain a specific value of the CCDF, while $\text{PAPR}_{[x]}$ is the

required PAPR of the signal $x(t)$ to obtain a specific value of the CCDF. Another aspect of performance evaluated in this paper is the average power variation denoted ΔE and expressed as

$$\Delta E = 10 \log_{10} \left(\frac{\mathcal{P}_y}{\mathcal{P}_x} \right), \quad [\text{in dB}]$$

where \mathcal{P}_y is the average power of the signal $y(t)$, while \mathcal{P}_x is the average power of the signal $x(t)$.

We also draw PSD of the signal in order to check if, after the TR mitigation method, the PSD still respects the WLAN mask of Fig. 12.

Fig. 13 shows the peak power reduction results of Classical-TR-CC technique for different iterations. Simulation results from Fig. 13 shows that the reduction in PAPR increases with the iterations number. For example, at 10^{-2} of the CCDF, the reduction in PAPR for Classical-TR-CC technique is about 0.75 dB, 1.75 dB and 2.75 dB for $\mathcal{N}_{iter} = 1, 3$ and 5 respectively.

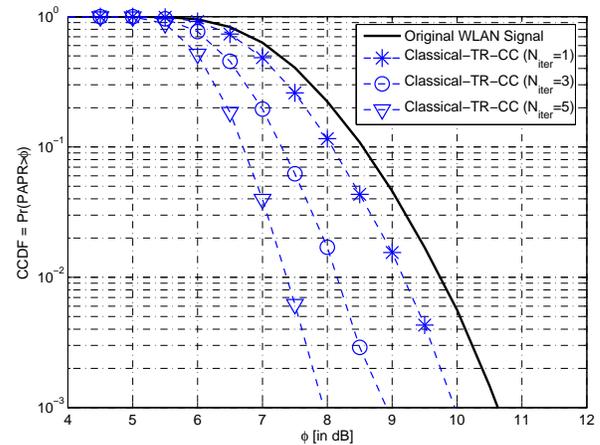


Figure 13: PAPR reduction performance for Classical-TR-CC technique for different iterations.

The reduction PAPR performance of the Adaptive-TR-CC technique for iterations number $\mathcal{N}_{iter} = 1, 3$ and 5 are plotted in Fig. 14 based on the CCDF curve. This figure provides the same conclusion as Fig. 13, i.e., the reduction in PAPR increases with the iterations number.

From Figs. 13 and 14, it is shown that the performance in PAPR reduction from CT algorithm as well from AT algorithm increases with the number of total iterations \mathcal{N}_{iter} .

Simulation results also shows that, for a given \mathcal{N}_{iter} , Adaptive-TR-CC technique is better than Classical-TR-CC technique in terms of PAPR reduction and Adaptive-TR-CC technique with $\mathcal{N}_{iter} = 3$ gives the same PAPR reduction performance as Classical-TR-CC technique with $\mathcal{N}_{iter} = 5$. It must bear in mind that, the system complexity grows linearly with the number of iterations.

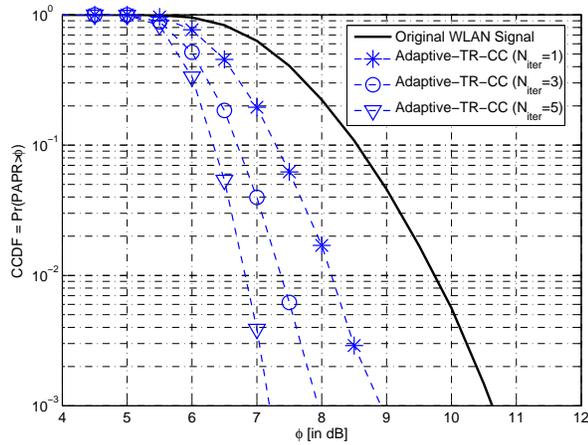


Figure 14: PAPR reduction performance for Adaptive-TR-CC technique for different iterations.

AT algorithm provides significant reduction in PAPR than CT algorithm at the same number of iterations; and AT algorithm with $N_{iter} = 3$ provides the same PAPR reduction performance as the CT algorithm with $N_{iter} = 5$. This means that at the same PAPR reduction gain, CT algorithm is $5/3 \sim 2$ times more complex than AT algorithm.

Fig. 15 shows the PAPR reduction performance according to A for different iterations. It shows that the reduction in PAPR increases with the number N_{iter} of iterations. It also shows that, for a given N_{iter} , the maximum in PAPR reduction is achieved for $\frac{A}{\sqrt{P_x}} \simeq 4$ dB and drops to 0 dB from $\frac{A}{\sqrt{P_x}} \geq 11$ dB. The maximum PAPR reduction of the Classical-TR-GM technique at the value of 10^{-2} of the CCDF is 2 dB, 2.5 dB, 2.75 dB and 3 dB for $N_{iter} = 1, 3, 5$ et 10 respectively.

The reduction in PAPR decreases when $\frac{A}{\sqrt{P_x}}$ increases for the simple reason that, when the “ threshold” A increases (i.e when $\frac{A}{\sqrt{P_x}}$ increases), there are fewer and fewer samples of the multicarrier signal that satisfy the condition $|x_n| > A$. Therefore, there will be fewer and fewer PAPR reduction.

The study of variation in the average power in the Classical-TR-GM technique (refer to Fig. 16) shows that the average power of the transmitted signal increases with the reduction of PAPR. It is clear that for $N_{iter} = 10$ where the PAPR reduction is most significant, the increasing of the average power is the most important.

Fig. 17 shows the transmission performance for Classical-TR-CC and Adaptive-TR-CC techniques over an AWGN channel. Fig. 18 is the BER performance for Classical-TR-GM and Adaptive-TR-GM techniques,

These simulation results show that the BER performance of the system using the different PAPR reduction techniques matches with the conventional BER; this means that the BER of the system is not degraded by the different techniques.

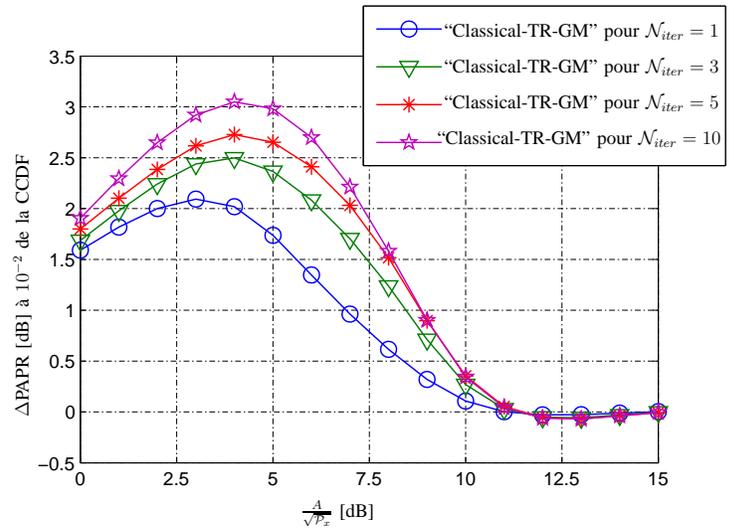


Figure 15: PAPR reduction performance of the Classical-TR-GM technique for different iterations number N_{iter} .

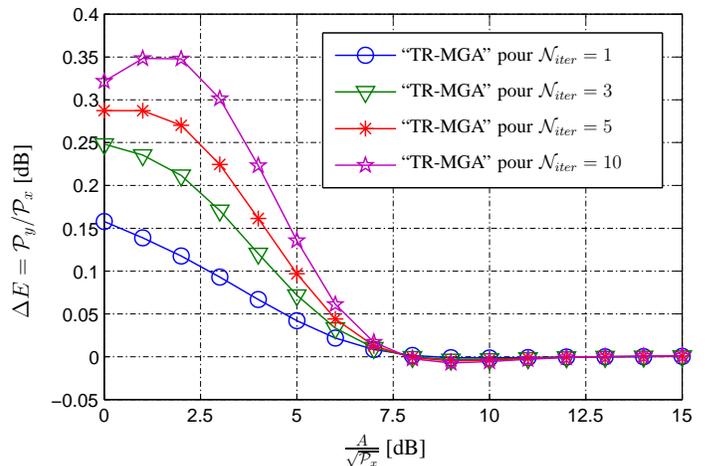


Figure 16: Average power ratio of the Classical-TR-GM technique for different iterations number N_{iter} .

Indeed, the different techniques used for PAPR reduction are TR techniques, as in TR techniques the data subcarriers and the PAPR reduction subcarriers are orthogonal, so the BER of the system is not corrupted.

Fig. 19 and Fig. 20 show the PSD of signals after investigating the PAPR-reduction algorithms. All the spectrums respect the WLAN spectral specifications. However, the level of spectrum under the PAPR reduction subcarriers with AT algorithm is higher than the level of spectrum with CT algorithm. Indeed at the same level of iteration, the power of the PAPR reduction signal with AT algorithm is higher than the PAPR reduction signal power with CT algorithm, that is why the performance in PAPR reduction with AT algorithm is better than those with CT algorithm at the same number

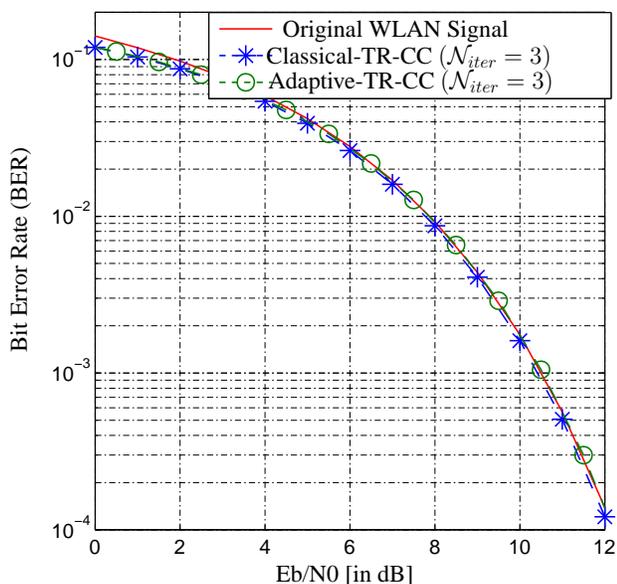


Figure 17: BER performance for Classical-TR-CC and Adaptive-TR-CC techniques over an AWGN channel.

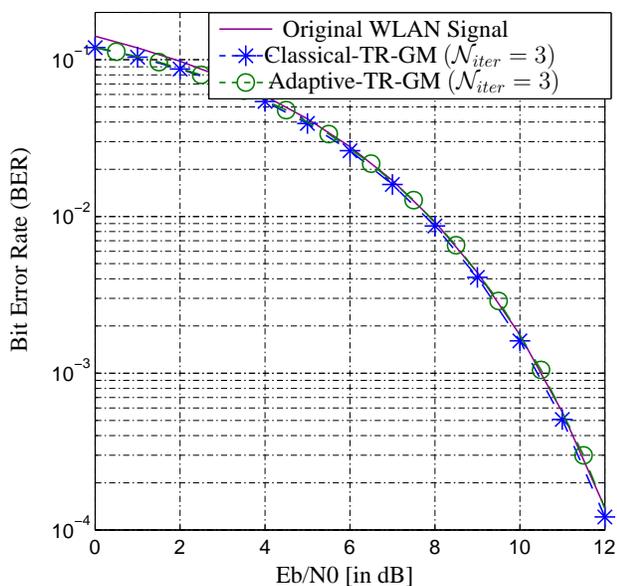


Figure 18: BER performance for Classical-TR-GM and Adaptive-TR-GM techniques over an AWGN channel.

of iterations.

Despite the high level of the PAPR reduction signal spectrum resulting to AT algorithm, the WLAN spectral specifications are respected.

VI. CONCLUSION

TR is a popular PAPR reduction technique that uses a set of reserved subcarriers to carry the peak reducing signal. Because of its many advantages, TR seems to be promising for use in commercial systems.

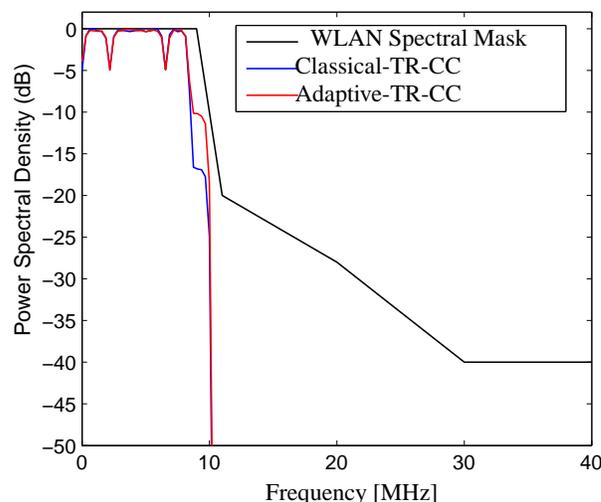


Figure 19: PSD of signals using "CT-CC" and "AT-CC" PAPR reduction technique ($N_{iter} = 3$).

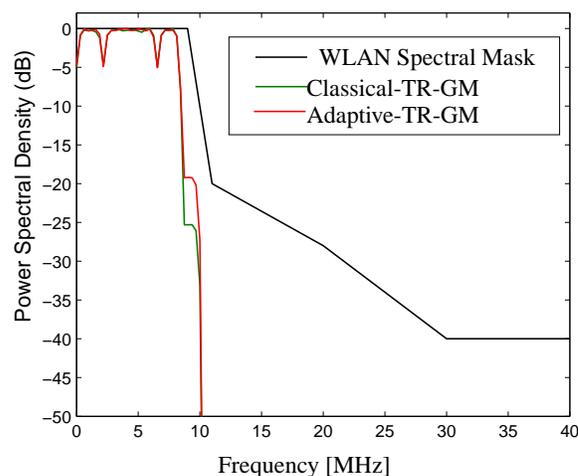


Figure 20: PSD of signals using "CT-MC" and "AT-MC" PAPR reduction technique ($N_{iter} = 3$).

In this paper, thanks to a Frequency Domain filtering, we have proposed two transformation algorithms to transform any Adding Signal techniques into TR techniques in order to benefit of the TR advantages. As the transformation in TR technique is a low-complexity process (about the FFT/IFFT complexity), the obtained technique results in a low-complexity TR technique. In order to increase the performance in PAPR of the obtained TR technique, the process of the peak reducing signal computation followed by filtering must be repeated several times.

Later in the paper, the classical clipping technique [8] and the Geometric PAPR reduction technique [17] are transformed into TR techniques for PAPR reduction. CT and AT algorithms proposed in this paper are applied and the performances of these two techniques are evaluated through

the BER, PAPR reduction, as well as DSP of resulting signals, in the WLAN context.

From simulation results, it is shown that, AT algorithm provides more reduction in PAPR than CT algorithm at the same computational complexity; but leads to an increase in the level of the PAPR reduction signal spectrum that nevertheless respects the standard spectral specifications.

We can conclude that: to transform any Adding Signal technique into TR technique, AT transformation algorithm should be preferably used.

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