A Step Forward on Adaptive Iterative Clipping Approach for PAPR Reduction in OFDM System

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Abstract—**The Orthogonal Frequency Division Multiplexing (OFDM) is the most commonly used multicarrier modulation in telecommunication systems because of its efficient use of frequency resources and its robustness to multipath fading channels. However, as any multicarrier signal, the Peak-to-Average-Power Ratio (PAPR) is one of the major drawbacks of OFDM signals. Many research papers have dealt with the PAPR mitigation methods, such as clipping methods, tone reservation based approaches, and partial transmit signals. However, in this paper we focus on the clipping method. This method is one of the most efficient adding signal techniques for PAPR reduction in terms of complexity. Nevertheless, clipping presents many drawbacks such as a bit error rate degradation, an out-of-band emission, and a mean power degradation. In this paper, a clipping method featuring a threshold that adapts to the desired upper bounded output PAPR is presented. Once the desired output PAPR has been predefined, the proposed AC approach consists of clipping each OFDM symbol with an adaptive threshold so that the PAPR value of the clipped symbol is equal to this desired output PAPR. This paper proposes three different ways to compute the adaptive threshold of each OFDM symbol we want to clip according to the desired output PAPR. The theoretical analysis and the simulation results validate the interest and potential of this new clipping method.**

Keywords–Orthogonal Frequency Division Multiplexing;High Power Amplifier; Peak-to-Average Power Ratio; Complementary Cumulative Distribution Function; Clipping; Adaptive Clipping

I. INTRODUCTION

The work presented by the authors in the previous AICT 2016 conference in Bruxelles [1], was focusing on the reduction of the complexity of the Adaptive Clipping (AC) method, previously presented in [2]. The present paper extends the work in [1] by introducing a new way to compute the adapted algorithm to reach the targeted PAPR. The proposed work, allows to reach the exact solution of the problem, whereas the computed threshold in [1] and in [2] just allows to get an approximation. Furthermore, we illustrate, in this paper, the achieved performances by extensive results, which prove the interest of our proposal.

The Peak-to-Average Power Ratio (PAPR) is one of the main issues of the Orthogonal Frequency Division Multiplex (OFDM) signal [3]. Many works such as the coding based techniques [4], [5], the probabilistic based approaches [6], [7] and the "adding signal for high peak cancellation" based techniques [8], [9], [10] have been documented in the literature for the purpose of PAPR mitigation. The clipping method [11], [12] is an efficient technique for PAPR mitigation where the peak-cancellation signal is computed thanks to the clipping of the signal amplitudes that exceed a predefined threshold A. This paper focuses on classical clipping method (CC), detailed in [11]. The main objective of the presented is to propose a new clipping technique that offers better outcomes than that of the CC methods in terms of signal degradation, and similar outcomes in terms of PAPR reduction. In others words, the proposed clipping method achieves:

1) A better bit error rate (BER), a lesser out-of-band (OOB) emission and mean power degradation.
2) Same performances in terms of PAPR reduction.

Note that, in practice, in CC method, a normalized threshold is used: $\rho = 10\log_{10}(\frac{A^2}{\mathrm{P}_{\mathbf{x}}})$, where $P_{\mathbf{x}}$ represents the mean power of the discrete signal $\mathbf{x}$, whose PAPR has to be reduced. It can be noted that the normalized threshold defines the PAPR, below which the signal is not clipped. Due to the large amplitude variations of the OFDM signals in the time domain, the PAPR value of each OFDM symbol highly depends on its content. Therefore, after achieving the PAPR mitigation thanks to the CC method [11] by featuring a predefined normalized threshold, the obtained PAPR value also depends on its content. Therefore, the upper bounded PAPR of the clipped signal, at each value of its Complementary Cumulative Distribution Function (CCDF), increases when the CCDF decreases. This is illustrated by the left curve depicted in Fig. 1. Note that this is also the case for the original OFDM CCDF curve. That means that there is no deterministic upper bounded PAPR for CC method. This is exactly what we target in this work. This deterministic value corresponds to the vertical solid blue line depicted in Fig. 1.

In practice, the suitable upper bounded PAPR value of the signal for the Input Back Off (IBO) definition at the High Power Amplifier (HPA) is chosen where CCDF(Φ) is close to zero (usually 10^{-4}). In this paper, which is an extension of the work presented in [1], this value is called the desired upper bounded PAPR and denoted as PAPR_0. Thus, in [1], [2] the authors have shown that in CC method many OFDM symbols are either severely clipped or unnecessarily clipped with respect to this desired upper bounded PAPR_0. To illustrate

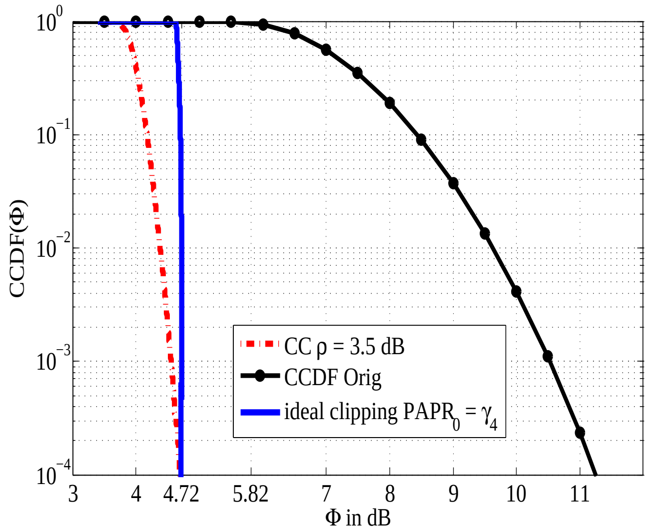

Figure 1. Scenario of the CCDF curves of a CC and Ideal Clipping.

this assertion, let us depict in Fig. 2 the result of zooming the zone around 10^{-1} for the CCDF in Fig. 1. Note that our main objective is to have a PAPR clipping output whose value is close to 4.72 dB (the vertical blue line). Therefore, all the symbols that have a PAPR value between 4.1 dB and 4.72 dB are clipped unnecessarily (see Δ_1 in Fig. 2). Besides this, all the symbols whose PAPR values are between 4.72 dB and 8.4 dB are too severally clipped by the CC technique in comparison with the ideal clipping (indicated by Δ_2 in Fig. 2). If we extend these considerations to all CCDF values, we then obtain the two areas indicated in Fig. 3 as:

- Area1: symbols are unnecessarily clipped
- Area2: Symbols are clipped more severely than necessary

In order to avoid these drawbacks, the authors have pro-

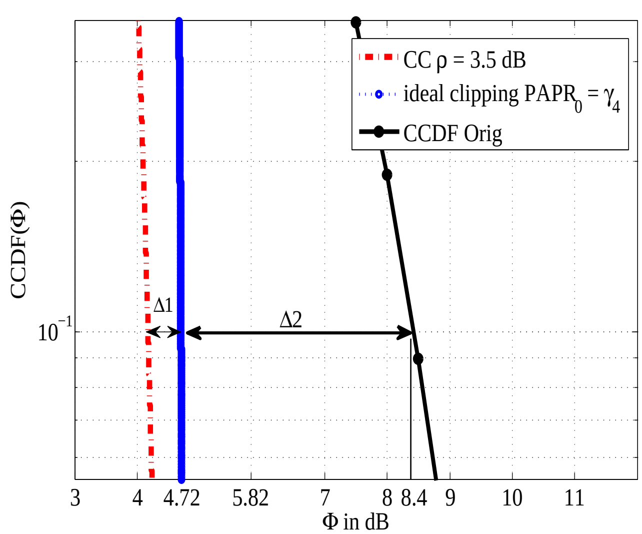

Figure 2. Zoom at CCDF=10^{-1} to illustrate symbols too much clipped by CC.

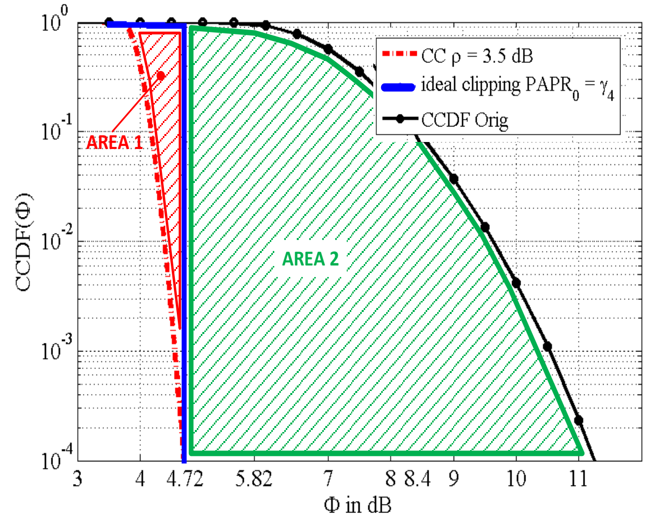

Figure 3. Scenario of the CCDF curves of a CC and Ideal Clipping. AREA1: symbols unnecessary clipped, AREA2: symbols clipped more severaly than necessary.

posed an AC algorithm in [1], [2], where the threshold is adapted to the content of each OFDM symbol, according to the desired upper bounded PAPR value PAPR_0. Other AC methods exist in the literature [13], [14]. In [14], the authors proposed to adapt the normalized threshold ρ depending on the mapping constellation of the OFDM signal for a better compromise between the PAPR reduction and the BER degradation. In [13], the authors proposed an adaptive iterative clipping and filtering scheme [15], in which the computation of the amplitude threshold A, is first computed on the basis of the predefined normalized threshold, and done at each iteration. This latter approach improves the performances of the system in terms of PAPR reduction but further degrades the signal. In contrast, in [2], the proposed AC approach and the CC method [11] achieve similar performance in terms of PAPR reduction. However, a better bit error rate (BER), a lesser out-of-band (OOB) emission, and a lesser mean power degradation are achieved. Nevertheless, the computational complexity of the proposed algorithm is high. In fact, an exhaustive search, by means of a constant step (ϵ) is performed, so as to find the optimal threshold. Therefore, the number of required iterations to find the optimal threshold $\rho^{(\mathbf{x})}$ will depend on both the content of each OFDM symbol and the step ϵ.

In [1], a fast AC method has been proposed to compute the value of $\rho^{(\mathbf{x})}$. Presented approach consists of adapting the step ϵ at each iteration to the content of each OFDM symbol, this is equivalent to clipp the signal iteratively by adapting the clipping magnitude at each iteration as a function of PAPR_0 and the content of the clipped signal at the previous iteration. Therefore, this approach requires less iterations to find the adaptive threshold [2]. This approach is called the Iterative Adaptive Clipping (IAC). The AC and IAC methods give both an approximation of the adaptive threshold $\rho^{\mathbf{x}}$ via an exhaustive search featuring respectively a constant and a non constant step. This paper is an extension of the analysis presented in [1], where we will first provide a complete analysis of both the AC and IAC methods. Then, we will extend our analysis to present a new approach that allows us to find the exact adaptive threshold $\rho^{\mathbf{x}}$. This new approach is named power approximation based method for adaptive clipping (PAC).

The paper is organized as follows: In Section II, the problem's formulation of the AC principle will be briefly presented. In Sections III-A and III-B, we will review the AC and the IAC approaches and we will show that IAC method needs fewer iterations than AC approach so as to reach $\rho^{(\mathbf{x})}$. In Section (III-C) the new proposed approach will be presented. A comparative study pertaining to signal degradation will then be conducted in Section IV. Finally, conclusions will be presented in Section V.

II. Proposed approach principle

In this section, after reminding some definitions and notations, the AC method is hereafter described.

A. Notations and definitions

Throughout this paper an OFDM symbol $x(t)$ of duration T_u is given by the following equation

$$x(t) = \sum_{m=0}^{M-1} X_m \, e^{j2\pi mFt}, \text{ with } 0 \leq t \leq T_u, \tag{1}$$

where M is the total number of carriers, $F = \frac{1}{T_u}$ is the inter-carrier space, mF the m^{th} frequency, and X_m the symbol carried out by the m^{th} carrier at time T_u.

We denote $\mathbf{x} = [x_0, \ldots, x_{LM-1}]$ the vector containing the discrete samples of $x(t)$ after the oversampling operation. It has been demonstrated that it can be efficiently computed thanks to use of Inverse Fast Fourier Transform (IDFT)[1]. As described in [16], since the PAPR mitigation operation is generally undertaken in the discrete time domain, the oversampling factor L should be greater than 4 in order to get a good approximation of the PAPR of the analogue OFDM symbol $x(t)$. The PAPR value of $\mathbf{x}$ is given by the following expression

$$\text{PAPR}_{[\mathbf{x}]} \quad = \quad \frac{\max\limits_{m=0,\ldots,ML-1} \left\{ |x_m|^2 \right\}}{P_{\mathbf{x}}}, \tag{2}$$

where $P_{\mathbf{x}}$ is the mean power of the discrete OFDM symbol $\mathbf{x}$.

Since $\mathbf{x}$ is a random variable, the Complementary Cumulative Distribution Function (CCDF) defined by (3) is used to characterize the useful upper bounded PAPR for the IBO's characterization. In practice, this upper bounded PAPR value can be chosen at $\text{CCDF}_{(.)}(\Phi) = 10^{-4}$ as

$$\text{CCDF}_{\mathbf{x}}(\Phi) = \mathbb{P}\text{rob}\left[\text{PAPR}_{[\mathbf{x}]} \geq \Phi\right] \tag{3}$$

In this paper, this upper bounded PAPR value will be denoted by γ_4 and will be referred to as the achieved PAPR or output PAPR. Note that, in our proposed approach, this value will be denoted by PAPR_0. More generally, the positive scalar γ_e will represent in this paper the upper bounded PAPR of symbols obtained at a clip rate of 10^{-e}, i.e., at the CCDF value equal to 10^{-e} with $e \geq 0$

$$\gamma_e = \max_{\Phi} \left\{ \text{CCDF}_{\mathbf{y}}(\Phi) \geq 10^{-e} \right\}, \tag{4}$$

where $\text{CCDF}_{\mathbf{y}}(\Phi) = \mathbb{P}rob[\text{PAPR}_{[\mathbf{y}]} \geq \Phi]$ and $\mathbf{y} = [y_0, \ldots, y_{ML-1}]$ is the OFDM symbol after clipping.

B. The proposed AC approach principle

The CC proposed in [11] is one of the most popular clipping technique for PAPR reduction known in the literature. It is sometimes called hard clipping or soft clipping. To avoid any confusion, the term CC will be used thereafter in this paper. In [11], its effects on the performance of OFDM, which include the determination of the power spectral density, the PAPR and the BER, are evaluated. The function-based clipping, used in CC technique, is defined as

$$f(r, \mathrm{A}) = \begin{cases} r, & r \leq \mathrm{A} \\ A, & r > \mathrm{A} \end{cases}, \tag{5}$$

where A is the magnitude clipping threshold. From (5) we can say that, if some samples of $\mathbf{x}$ are greater than the clipping threshold A, then the PAPR value of the output signal $\mathbf{y}$, obtained after PAPR reduction that uses the CC method, can be expressed as follows

$$\text{PAPR}_{\mathbf{y}} \quad = \quad \frac{A^2}{P_{\mathbf{y}}}. \tag{6}$$

Given, $A = \left(10^{\frac{\rho}{20}}\right)\sqrt{P_{\mathbf{x}}}$, the PAPR of $\mathbf{y}$ can be rewritten as follows:

$$\begin{aligned} \text{PAPR}_{\mathbf{y}} \quad &= \quad \left(10^{\frac{\rho}{10}}\right)\left(\frac{P_{\mathbf{x}}}{P_{\mathbf{y}}}\right) \\ \text{then } \text{PAPR}_{\mathbf{y}}(\text{ in dB }) \quad &\geq \quad \rho(\text{ in dB}). \end{aligned} \tag{7}$$

Therefore, it can be noticed that $\gamma_e \geq \rho$ for any $e \geq 0$. Thus, γ_e increases where e increases. In practice, the desired γ_e for the IBO parametrization at the HPA is generally chosen where the CCDF value is equal to 10^{-4}, i.e., γ_4. Then, it worth remarking that the CC method can lead to the fact that many OFDM symbols are clipped more severely than necessary or are unnecessarily clipped with respect to γ_4 [2]. Fig. 3 shows the zones representing the set of OFDM symbols that are clipped more severely than necessary (AREA2) or unnecessarily clipped (AREA1), when we use a CC method featuring $\rho = 3.5$ dB so as to reduce their PAPR, with respect to Ideal Clipping (see vertical blue line of Fig. 3), and for the same upper bounded PAPR obtained at a CCDF value equal to 10^{-4} (γ_4). The vertical blue line represents the ideal clipping CCDF where $\text{PAPR}_0 = \gamma_4$, which corresponds to the deterministic desired upper bounded PAPR. It is obvious that the output upper bounded PAPR of such an ideal clipping is constant at any value of the CCDF. The main goal of the proposed AC approach is to come close to the ideal clipping. Therefore, AC will degrade less the signal after clipping than the CC method, and features similar performances than that of the latter in terms of PAPR reduction. Hence, once the desired upper bounded PAPR_0 and an OFDM symbol $\mathbf{x}$ featuring an initial PAPR value greater than PAPR_0 have been obtained, the proposed AC approach consists of two stages:

1) Computation of the adaptive threshold $\rho^{(\mathbf{x})}$.

[1] In this paper, we use the zero-inserting scheme to calculate $\mathbf{x}$, i.e., the IDFT operation is applied to the extended vector $\check{\mathbf{X}} = [X_0, \ldots, X_{\frac{M}{2}-1}, \underbrace{0, \ldots, 0}_{(L-1)M \text{ Zeros}}, X_{\frac{M}{2}}, \ldots, X_{M-1}]$.

Having determined the PAPR_0 value, the signal $\mathbf{x}$ needs to be clipped. On that basis, the AC method consists firstly in finding $\rho^{(\mathbf{x})}$ so that the PAPR value of $\mathbf{y}$, obtained after clipping $\mathbf{x}$ featuring $\rho^{(\mathbf{x})}$, is equal to PAPR_0. In other words, the AC method consists of adapting the clipping threshold $\rho^{(\mathbf{x})}$ of each OFDM symbol $\mathbf{x}$ that we want to clip with according to the desired upper bounded PAPR value, i.e., PAPR_0. Therefore, this stage consists of solving the following equation

$$\mathrm{PAPR}_0 = \gamma_4 = \left(10^{\frac{\rho^{(\mathbf{x})}}{10}}\right)\left(\frac{P_{\mathbf{x}}}{P_{\mathbf{y}}}\right), \tag{8}$$

where $P_{\mathbf{y}}$ is the mean power of the clipped signal $\mathbf{y}$.

2) Clip the OFDM symbol $\mathbf{x}$ as in (5) thanks to its adaptive threshold $\rho_{(\mathbf{x})}$.

From (7), it can be noticed that $P_{\mathbf{y}}$ depends on the unknown parameter ρ_n. Therefore, solving (8) is not a trivial problem. Thus, the main challenge of the AC is the computation of the adaptive threshold for each OFDM symbol. In [1], [2], two approaches based on a exhaustive search are proposed, so as to approximate the adaptive threshold. These approaches will be reviewed more clearly hereafter in Section III-A and Section III-B, respectively.

The following section presents a complete analysis of the AC and IAC method [1] and the description of the new proposed approaches for the computation of the adaptive threshold $\rho^{(\mathbf{x})}$.

III. The adaptive threshold computation

After having described the AC method principle in the previous section, we propose, here, a complete analysis of three different methods for computing the adaptive threshold.

A. *Comprehensive search carried out by means of a constant step: AC method*

As it is noticed in Section II, the computation of the adaptive threshold for each OFDM symbol is not a trivial problem since the mean power $\mathrm{P}_{\mathbf{y}}$ of the clipped symbol depends on the unknown $\rho^{(\mathbf{x})}$, see (8). In order to bypass this difficulty, the authors propose in [2] to find an approximation of $\rho^{(\mathbf{x})}$ thanks to a exhaustive search within the interval $[0, \mathrm{PAPR}_0]$. In fact, from (7) it can be noticed that, for each value of PAPR_0 and $\mathbf{x}$ that we want to clip, their adaptive threshold is less than PAPR_0. Besides, if ρ_1 and ρ_2 are two clipping thresholds such that $(\rho_1 - \rho_2)$ is close to zero, the PAPR values of the clipped OFDM symbols using these thresholds are approximately equal. Thus, where $\epsilon > 0$ and $\delta > 0$, the authors propose to check successively $\rho_0 = \gamma_4$, $\rho_1 = \rho_0 - \epsilon$, ... , $\rho_m = \rho_{m-1} - \epsilon$,..., to reach $\rho^{(\mathbf{x})}$, which satisfies

$$(\gamma_4 - \mathrm{PAPR}_{\mathbf{y}}) \leq \delta \tag{9}$$

Fig. 4 describes the chart of the adaptive threshold computation. In the rest of the paper, this approach will be named the AC method. From Fig. 4, it can be noted that, for each OFDM symbol $\mathbf{x}$ featuring with a PAPR value greater that PAPR_0, the PAPR value $\mathrm{PAPR}_{[\mathbf{y}]}$ obtained after a clipping process featuring its adaptive threshold $\rho^{\mathbf{x}}$ is less than $\mathrm{PAPR}_0+\delta$. Thus, if δ is sufficiently small, then $\mathrm{PAPR}_{\mathbf{y}} \simeq \mathrm{PAPR}_0$. Therefore, the CCDF curve of the AC method will approach the CCDF curve of the ideal clipping depicted in Fig. 1. Fig. 5 depicts the PAPR values ($\mathrm{PAPR}_{[\mathbf{y}]}$) obtained after a PAPR reduction carried out by means of the AC and CC method, versus their PAPR values before clipping (the initial PAPR values).

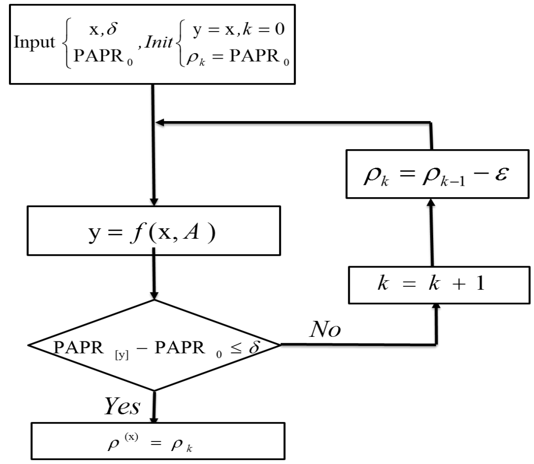

Figure 4. Flow chart of the AC method.

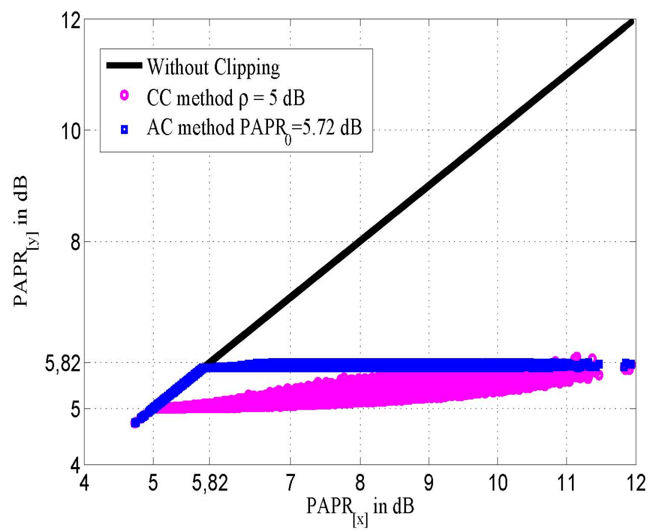

Figure 5. PAPR values after a PAPR reduction obtained by means of the AC and CC methods for OFDM symbols, versus the associated PAPR value before PAPR reduction. $M = 64$, $L = 4$, $\rho = 5$ dB and $\mathrm{PAPR}_0 = \gamma_4(\rho) = 5.82$ dB.

The simulation results depicted in Fig. 5 confirm that in the AC method the output PAPR value of each clipped symbol, i.e., $\mathrm{PAPR}_{[\mathbf{y}]}$ is approximately equal to the desired output PAPR value, i.e., PAPR_0. Therefore, where $\mathrm{PAPR}_0 = \gamma_4(\rho)$, the AC method allows us to prevent from clipping the symbol unnecessarily or more severely than necessary as it is the case where the CC method is used. In fact, from Fig. 5, we remark that the symbols, which feature an initial PAPR value that is included in $[5, 5.82]$ (in dB), are unnecessarily clipped. We can also note that those featuring an initial PAPR value greater than 5.82 dB are severely clipped with respect to the obtained useful PAPR value that is $\gamma_4(\rho) = 5.82$ dB, i.e., the

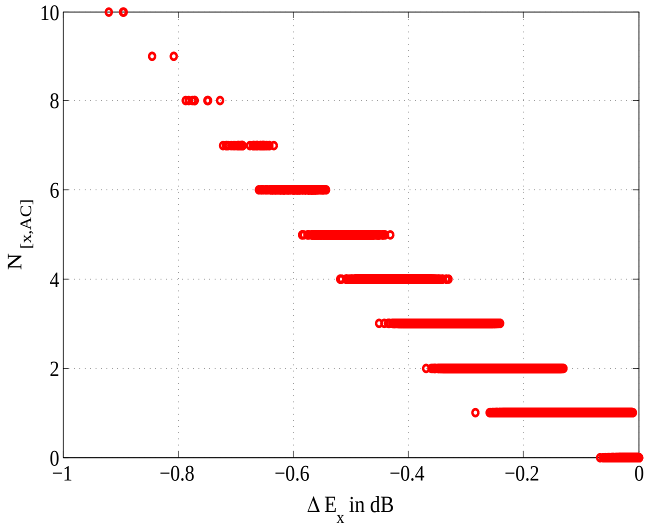

Figure 6. Number of iterations performed by the AC method so as to approach $\rho^{\mathbf{x}}$, versus $\Delta E_{\mathbf{x}}$. $M = 64$, $L = 4$, $\rho = 3.5$ dB and $\text{PAPR}_0 = \gamma_4(\rho) = 4.62$ dB.

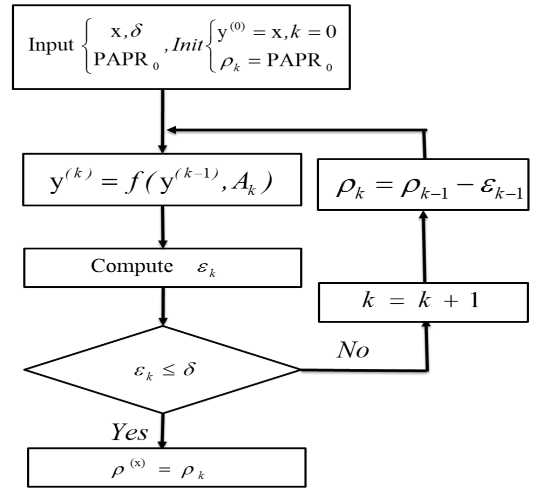

Figure 7. Flow chart of the IAC approach.

upper bounded PAPR at a CCDF clip that is close to zero (here 10^{-4}). Therefore, the AC will degrade less the OFDM symbol obtained by means of clipping than the CC method and and offers similar performances in terms of PAPR reduction. Further simulation pertaining to this aspect will be presented in Section IV.

Since the adaptive threshold has to be computed the AC method is very complex compared to the CC method. From Fig. 4, it can be noticed that, at each iteration, a CC operation is required. Thus, in what follows, the convergence spread, i.e., the mean of number of iterations required so as to find the adaptive threshold of each OFDM symbol, will be discussed. To this end, let us consider $N_{[\mathbf{x},\text{AC}]}$ the number of performed iterations that is needed in order to find the adaptive threshold of the OFDM symbol $\mathbf{x}$. For each $\mathbf{x}$, let us consider $\Delta E_{\mathbf{x}}$ as the mean power variation after PAPR reduction using the CC method or proposed clipping method, and that is defined as follows

$$\Delta E_{\mathbf{x}} = 10\log_{10}\left(\frac{P_{\mathbf{y}}}{P_{\mathbf{x}}}\right) \tag{10}$$

Fig. 6 depicts the number of iterations performed by the AC method, so as to come close $\rho^{\mathbf{x}}$ versus the value of $\Delta E_{\mathbf{x}}$.

The results depicted in Fig. 6 show that the number of iterations performed so as to find the adaptive of each OFDM symbol depends on its content ($\Delta E_{\mathbf{x}}$). In fact, for each OFDM symbol $\mathbf{x}$, it can be noted that when $\Delta E_{\mathbf{x}}$ increases, the number of iterations that are required to find $\rho^{\mathbf{x}}$ increases significantly and the contrary is also true.

B. *Comprehensive search carried out by mean a non constant step (IAC method)*

In this section, we present the IAC method, and the theoretical analysis of its performances in terms of PAPR reduction and convergence spread (number of required iterations to find the $\rho^{(\rho)}$). The theoretical comparison with AC, as regards the convergence speed, will be also presented.

Since $\delta > 0$ and since there exist an OFDM symbol $\mathbf{x}$ featuring a PAPR value greater than PAPR_0, the IAC approach consists of searching the adaptive threshold $\rho^{(\mathbf{x})}$, which satisfies (9). In order to find this threshold we check successively $\rho_0 = \text{PAPR}_0$, $\rho_1 = \rho_0 - \epsilon_1, \ldots, \rho_m = \rho_{m-1} - \epsilon_m$ where ϵ_m is the step that allows us to go from stage $m-1$ to the stage m. In contrast to the work presented in [2], the step ϵ_m is not constant and depends on the content of each OFDM symbol and its clipped version at the previous iteration. To this end, we denote $\mathbf{y}^{(m)}$where $m = 1, 2, ...$, as being the clipped OFDM symbol featuring the threshold ρ_m, and the step ϵ_m at the m^{th} iteration is expressed as follows

$$\epsilon_m = 10\log_{10}\left(\frac{P_{\mathbf{y}^{(m-2)}}}{P_{\mathbf{y}^{(m-1)}}}\right), \tag{11}$$

with the notation $P_{\mathbf{y}^{(-1)}} = P_{\mathbf{x}}$ at the first iteration. The flow chart of the adaptive threshold $\rho^{(\mathbf{x})}$ search in the IAC approach is depicted in Fig. 7.

The clipping level magnitude A_m at the m^{th} iteration can be expressed, from the normalized threshold ρ_m, as

$$\begin{aligned} A_m &= 10^{\frac{\rho_m}{20}}\sqrt{P_{\mathbf{x}}} \\ &= \left(10^{\frac{\rho_0 - \epsilon_1 - \ldots - \epsilon_m}{20}}\right)\sqrt{P_{\mathbf{x}}} \\ &= \left(10^{\dfrac{\text{PAPR}_0}{20}}\right)\left(10^{\dfrac{-\sum_{l=1}^{m}\epsilon_l}{20}}\right)\sqrt{P_{\mathbf{x}}} \\ &= \left(10^{\dfrac{\text{PAPR}_0}{20}}\right)\left(\prod_{l=1}^{m} 10^{\dfrac{-\epsilon_l}{20}}\right)\sqrt{P_{\mathbf{x}}}. \end{aligned} \tag{12}$$

Then, from (11) we obtain after some derivation the following expression of the clipping magnitude at the m-*th* iteration A_m

$$\begin{aligned} A_m &= \left(10^{\frac{\mathrm{PAPR}_0}{20}}\right)\left(\prod_{l=1}^{m}\sqrt{\frac{P_{\mathbf{y}^{(l-1)}}}{P_{\mathbf{y}^{(l-2)}}}}\right)\sqrt{P_{\mathbf{y}}} \\ &= \left(10^{\frac{\mathrm{PAPR}_0}{20}}\right)\sqrt{P_{\mathbf{y}^{(m-1)}}}. \end{aligned} \tag{13}$$

Therefore, by substituting (13) in (6) the PAPR of the clipped signal at the m^{th} iteration satisfies the following expression

$$\mathrm{PAPR}_{[\mathbf{y}^{(m)}]} - \mathrm{PAPR}_0 = \epsilon_{m+1}. \tag{14}$$

If we define $\epsilon_{m+1} \leq \delta$ as the criteria for stopping the IAC method at the m^{th} iteration, then, for each OFDM symbol, the PAPR value of the signal, after PAPR reduction by IAC, is less than $\mathrm{PAPR}_0 + \delta$. Thus, the CCDF curve of the IAC will approach the ideal clipping CCDF. Therefore, the IAC method allows us to get the desired deterministic upper bounded PAPR.

From (13) we remark that, as it is noticed in the introduction, the IAC method is equivalent to clipping the signal iteratively, via an adaptation of the clipping magnitude according to $PAPR_0$ and the content of the clipped signal at the previous iteration. The following Algorithm 1 describes the proposed IAC technique.

Algorithm 1 the IAC algorithm

Require: $\mathbf{x}$ input OFDM signal , $\delta > 0$ and PAPR_0
Ensure: $\mathbf{y_n}$ output signal
  $m \leftarrow 0$
  $\epsilon_m \leftarrow 1$
  $\mathbf{y}^{(-1)} \leftarrow \mathbf{x}$
  while $\left(\mathrm{PAPR}_{\mathbf{y}^{(m)}} - \mathrm{PAPR}_0\right) = \epsilon_m \geq \delta$ **do**
    $m \leftarrow m + 1$
    Compute A_m from equation (13)
    $\mathbf{y}^{(m)} \leftarrow f(\mathbf{y}^{(m-1)}, A_m)$
  end while

Fig. 8 depicts $\mathrm{PAPR}_{[\mathbf{y}]}$ versus $\mathrm{PAPR}_{[\mathbf{x}]}$, where $\mathrm{PAPR}_{[\mathbf{y}]}$ is the PAPR value after a PAPR reduction performed by means of the IAC and the CC methods.

On the basis of the simulation results depicted in Figs .8, we remark that, similarly to the AC method, the output PAPR value of each clipped symbol, i.e., $\mathrm{PAPR}_{[\mathbf{y}]}$, in the IAC method, is approximately equal to the desired output PAPR value PAPR_0, as it is the case in the AC method. Therefore, when $\mathrm{PAPR}_0 = \gamma_4(\rho)$, the IAC method prevents us from clipping the symbol unnecessarily or more severely than necessary as it is the case in CC method. Therefore, the IAC method will degrade less the OFDM symbol after clipping than the CC method, and offers same performances in terms of PAPR reduction. More simulation pertaining to this aspect will be presented in Section IV.

In Fig. 9 the number of iterations performed by the IAC method, so as to reach $\rho^{\mathbf{x}}$, versus $\Delta_{\mathbf{x}}$ is presented. The simulation results show that the IAC method allows a quicker convergence $\rho^{\mathbf{x}}$, in comparison with the AC method (see Fig. 9). Besides, in the IAC method, it can be noticed that the number of required iterations does not increases significantly when $\Delta E_{\mathbf{x}}$ increases.

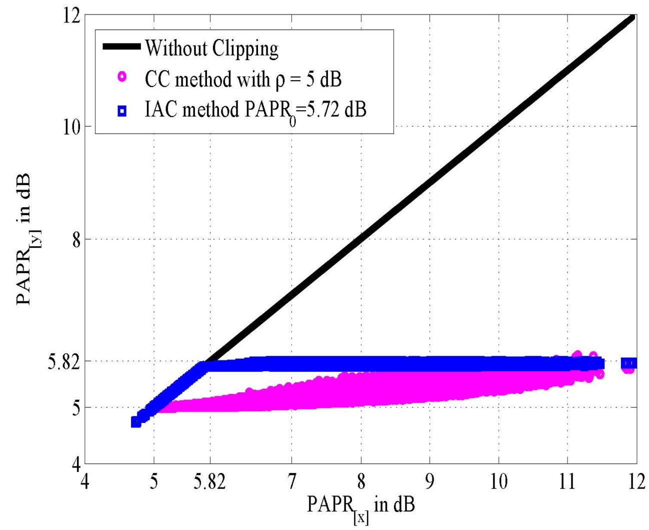

Figure 8. PAPR value obtained after the PAPR reduction of an OFDMs symbols, and performed thanks to the IAC and CC methods, versus the associated PAPR value before PAPR reduction. $M = 64$, $L = 4$, $\rho = 5$ dB and $\mathrm{PAPR}_0 = \gamma_4(\rho) = 5.82$ dB

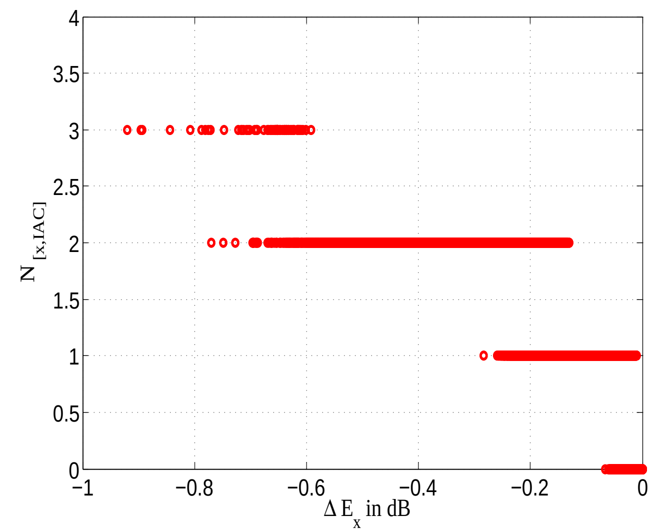

Figure 9. Number of performed iterations by the IAC method so as to find $\mathbf{x}$, versus $\Delta E_{\mathbf{x}}$. $M = 64$, $L = 4$, $\rho = 3.5$ dB and $\mathrm{PAPR}_0 = \gamma_4(\rho) = 4.62$ dB

In what follows, a comparison of the theoretical convergence speeds obtained for the AC and IAC methods will be done thanks to the mean numbers of iterations that are performed by these algorithms for each OFDM symbol.

For each OFDM symbol $\mathbf{x}$ and $\epsilon > 0$, let us consider $N_{[\mathbf{x},\mathrm{AC}]}, N_{[\mathbf{x},\mathrm{IAC}]}$ the number of iterations performed by AC and IAC, respectively, so as to reach $\rho^{(\mathbf{x})}$, subject to the condition in (9). Therefore, $\rho^{(\mathbf{x})}$ is approximated by $\rho_{N_{[\mathbf{x},\mathrm{AC}]}} = \mathrm{PAPR}_0 - N_{[\mathbf{x},\mathrm{AC}]}\epsilon$ and $\rho_{N_{[\mathbf{x}_n,\mathrm{IAC}]}} = \mathrm{PAPR}_0 - \sum_{l=1}^{N_{[\mathbf{x},\mathrm{IAC}]}-1} \epsilon_l$ for AC and IAC, respectively. Let us define the average step used in IAC to find $\rho_{[N_{x_n},\text{IAC}]}$ as

$$\epsilon_{\mathbf{x}} = \frac{1}{N_{[\mathbf{x},\text{IAC}]}-1}\sum_{l=1}^{N_{\mathbf{x},2}-1}\epsilon_l. \tag{15}$$

Proposition 3.1: For each OFDM symbol $\mathbf{x}$, we need to find the value of $\rho_{N_{[\mathbf{x}_n,\text{IAC}]}}$ that depends on the number of iterations performed by AC, and the used step $\epsilon_{\mathbf{x}}$.

To prove this statement (see the proof details in Appendix(A)), it is sufficient to show that the AC method performs $N_{[\mathbf{x}_n,\text{IAC}]}$ iterations, so as to find the $\rho_{N_{\mathbf{x},\text{AC}}}$, where the used step is $\epsilon = \epsilon_{\mathbf{x}}$. In other words, it is sufficient to show that $N_{[\mathbf{x}_n,\text{IAC}]} = N_{[\mathbf{x}_n,\text{AC}]}$ when $\epsilon = \epsilon_{\mathbf{x}}$.

Thus, for each OFDM symbol, the comparison between $N_{[\mathbf{x},\text{AC}]}$ and $N_{[\mathbf{x},\text{IAC}]}$ can be made by means of a comparison between $\epsilon_{\mathbf{x}}$ and ϵ. Since $\mathbf{x}$ is random, we will compare IAC and AC, in terms of convergence spread, on the basis of the average number of iterations required by each algorithms, which is equivalent to compare $\mathbb{E}[\epsilon_{\mathbf{x}}]$, as defined in (16), with ϵ (the constant step in AC).

$$\mathbb{E}[\epsilon_{\mathbf{x}}] \simeq \frac{1}{P_{\mathbf{x}}}\sum_{m=0}^{N_2}\int_0^{+\infty} f(r,A_m)p(r)\mathrm{d}r \tag{16}$$

where $p(r)$ is the probability density function of the OFDM signal's amplitudes. Please note that N_{IAC} (respectively N_{AC}) represents the average number of iterations performed by the IAC (respectively AC) method, which is estimated thanks to the Monte Carlo trial. After some computations [17], we obtain

$$\mathbb{E}[\epsilon_{\mathbf{x}}] = \frac{1}{P_{\mathbf{x}}}\sum_{m=0}^{N_2}\left(1-e^{\frac{-A_m^2}{P_{\mathbf{x}}}}\right) \tag{17}$$

Since, in the IAC method, the stopping criterion is $\epsilon_m < \epsilon$ (see Algorithm 1), the comparison between $\mathbb{E}[\epsilon_{\mathbf{x}}]$ and ϵ can be achieved by comparing $\mathbb{E}[\epsilon_1]$ (the first step in IAC method) with ϵ. Therefore, we can deduce that, for each PAPR_0 $N_{\text{AC}} \geq N_{\text{IAC}}$ if and only if

$$\epsilon_1 = 10\log_{10}\left(\frac{1}{1-e^{-\text{PAPR}_0}}\right) \geq \epsilon.$$

After some derivations, we can conclude that:

$$N_{\text{AC}} \geq N_{\text{IAC}} \text{ If and only if } \text{PAPR}_0 \leq \ln\left(\frac{10^{\frac{\epsilon}{10}}}{10^{\frac{\epsilon}{10}}-1}\right) \tag{18}$$

Figs. 10 and 11, compare respectively N_{IAC} with N_{AC} and $\mathbb{E}[\epsilon_{\mathbf{x}}]$ with $\epsilon = 0.1$ dB.

The simulation results depicted in Figs. 10 and 11 confirm that comparing N_{IAC} with N_{AC} is equivalent to compare $\mathbb{E}[\epsilon_{\mathbf{x}}]$ with ϵ. In fact, we note that where $N_{\text{AC}} \geq N_{\text{IAC}} \Leftrightarrow \mathbb{E}[\epsilon_{\mathbf{x}}] \geq \epsilon$. Besides, it is worth noticing that the IAC method converges more rapidly than the AC method when $\text{PAPR}_0 \leq 6$, which is consistent with equation (18) (with $\epsilon = 0.1$ dB $\Rightarrow$ $10\log_{10}\left[\log\left(\frac{10^{\frac{0.1}{10}}}{10^{\frac{0.1}{10}}-1}\right)\right] = 5.77$ dB $\simeq 6$ dB).

The AC and IAC methods give both an approximation of the adaptive threshold $\rho^{\mathbf{x}}$, via respectively an exhaustive search featuring a constant and a non constant step. In this paper, we propose a new approach that allows to find the exact solution of (8). This approach is based on an approximation of the mean power $P_{\mathbf{y}}$ and is presented in the Section III-C.

C. Mean power approximation based approach: the PAC method

In this section, a new approach to compute the adaptive threshold $\rho^{\mathbf{x}}$ is presented. This approach consists of using an approximation of the mean power $P_{\mathbf{y}}$, see (8), to compute $\rho^{\mathbf{x}}$. Thus, it will be named PAC (Power approximation based approach for AC).

Let us consider $A^{(\mathbf{x})}$ the clipping magnitude obtained from the normalized adaptive threshold $\rho^{(\mathbf{x})}$, i.e.,

$$A^{(\mathbf{x})} = \sqrt{P_{\mathbf{x}}}\left(10^{\frac{\rho^{\mathbf{x}}}{20}}\right) \tag{19}$$

From (8), it can be easily remarked that $0 \leq \rho^{(\mathbf{x})} \leq \text{PAPR}_0$, therefore,

$$A_0 \leq A^{(\mathbf{x})} \leq A_1 \tag{20}$$

where

$$\begin{aligned} A_0 &= \sqrt{P_{\mathbf{x}}} \\ A_1 &= \sqrt{P_{\mathbf{x}}}\left(10^{\frac{\text{PAPR}_0}{20}}\right) \end{aligned} \tag{21}$$

For each clipping magnitude A, let us consider $\mathcal{I}(A)$ the set comprising the index of the $\mathbf{x}$ values, which exceed the clipping magnitude A, i.e.,

$$\mathcal{I}(A) = \{l, \text{ such that } |x_l| > A\} \tag{22}$$

Therefore, from (20) we have

$$\mathcal{I}(A_1) \subset \mathcal{I}(A^{(\mathbf{x})}) \subset \mathcal{I}(A_0), \tag{23}$$

Since the value of $\mathcal{I}(A^{(\mathbf{x})})$ is known, (8) can be expressed as a function of the unknown value $A^{(\mathbf{x})}$ as follows

$$\text{PAPR}_0 \underbrace{\left(\sum_{l \notin \mathcal{I}(A^{(\mathbf{x})})}\frac{|x_l|^2}{LM} + \frac{\text{Cardinal}(\mathcal{I}(A^{(\mathbf{x})}))}{LM}(A^{(\mathbf{x})})^2\right)}_{P_{\mathbf{y}}} = (A^{(\mathbf{x})})^2 \tag{24}$$

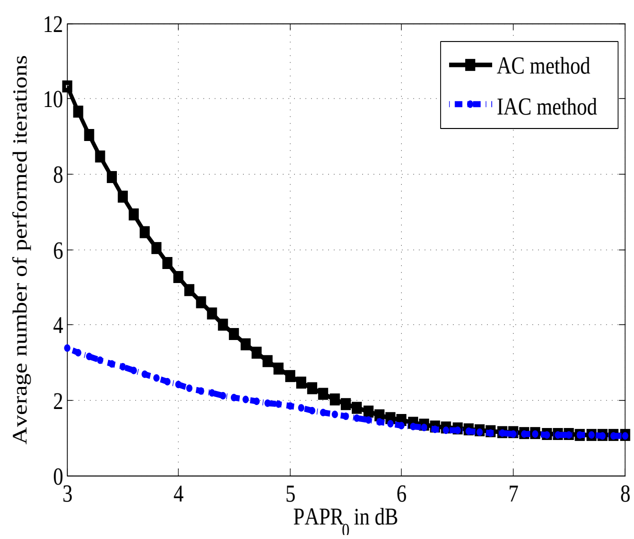

Figure 10. Mean of number of iterations performed by IAC and AC for each OFDM symbol, versus PAPR_0

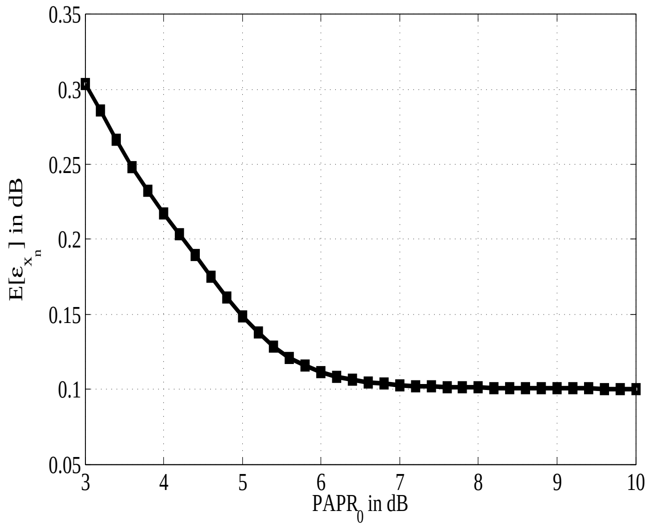

Figure 11. Comparison between $\mathbb{E}[\epsilon_{\mathbf{x}}]$ and $\epsilon = 0.1$ dB, versus PAPR_0

It can be noted from (24), that if $\mathcal{I}(A^{(\mathbf{x})})$ is known, then the mean power of the clipped symbol that uses, i.e., $P_{\mathbf{y}}$, can be expressed explicitly as a function of $\mathbf{x}$ and $A^{(\mathbf{x})}$. Therefore, the suitable clipping magnitude can be computed thanks to (24). Thus, using (23), the PAC method comprises two stages:

Stage 1: Computation of $\mathcal{I}(A^{(\mathbf{x})})$.

Since A_0 and A_1 have been determined, the goal consists of finding A_{min} and A_{max} via a dichotomy search method so that:

$$\begin{cases} A_{min} \leq A \leq A_{max} \\ \underbrace{\mathcal{I}(A_{min}) = \mathcal{I}(A^{(\mathbf{x})}) = \mathcal{I}(A_{max})}_{\#\mathcal{I}(A_{min}) = \#\mathcal{I}(A^{(\mathbf{x})}) = \#\mathcal{I}(A_{max})} \end{cases} \tag{25}$$

Graphically, as A_0 and A_1 are known, the dichotomy search allows us to move from the configuration shown in Fig. 12 to the configuration shown in Fig. 13.

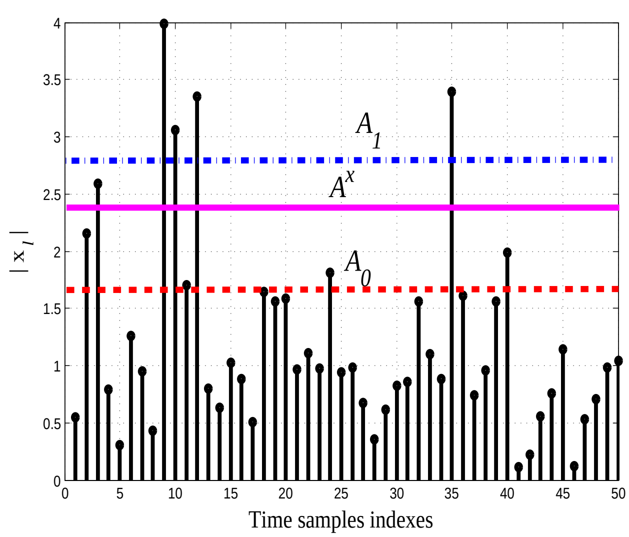

Figure 12. Initial configuration stage.

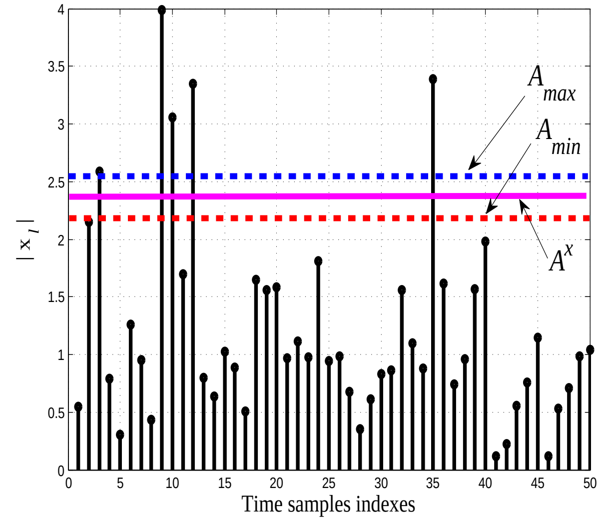

Figure 13. Final configuration stage.

From Fig. 13, we remark that $\mathcal{I}(A^{(\mathbf{x})})$ can be found without knowing $A^{(\mathbf{x})}$.

Stage 2: Computation of $A^{(\rho)}$ by solving (24).

Since $\mathcal{I}(A^{(\mathbf{x})})$ has been determined, this stage consists of solving (24). After some derivations, it can be shown that

$$A^{(\mathbf{x})} = \sqrt{\frac{\frac{\text{PAPR}_0}{LM} \sum\limits_{l \notin \mathcal{I}(A^{(\mathbf{x})})} |x_l|^2}{1 - \text{PAPR}_0 \frac{\text{Cardinal}(\mathcal{I}(A^{(\mathbf{x})}))}{LM}}} \tag{26}$$

After a dichotomy search method, the computation of the set $\mathcal{I}(A^{(\mathbf{x})})$ is described in detail in Fig. 14.

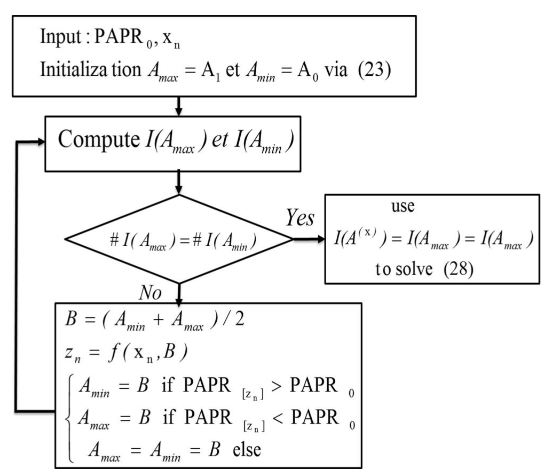

Figure 14. Flow chart of the PAC approach used to find $\mathcal{I}(A^{(\mathbf{x})})$.

Fig. 15 depicts $\text{PAPR}_{[\mathbf{y}]}$ versus $\text{PAPR}_{[\mathbf{x}]}$, where $\text{PAPR}_{[\mathbf{y}]}$ denotes the output PAPR value obtained after a clipping process that uses the PAC and CC methods.

As in previous analysis related to the AC and IAC methods, it can be noticed that the PAC method allows us to prevent the unnecessarily clipping or the more severely than necessary

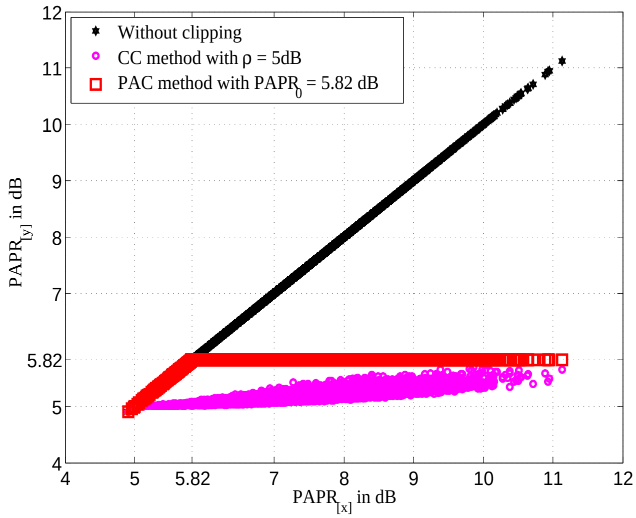

Figure 15. PAPR value after a PAPR reduction of an OFDMs symbols, by means of the PAC and CC methods, versus its PAPR value before PAPR reduction. $M = 64$, $L = 4$, $\rho = 5$ dB and $\text{PAPR}_0 = \gamma_4(\rho) = 5.82$ dB.

clipping, for each OFDM symbol where $\text{PAPR}_0 = \gamma_4(\rho)$ (see Fig 15). In fact, we note that, when the CC method is used, the symbols whose initial PAPR value is included in $[5, 5.82]$ (in dB) are unnecessarily clipped and those with an initial PAPR value is greater than 5.82 dB are severely clipped with respect to the obtained useful PAPR value $\gamma_4(\rho) = 5.82$ dB (i.e., the upper bounded PAPR of the CCDF at a clip rate that is close to zero (here 10^{-4})). Therefore, the PAC method will degrade less the OFDM symbols after the clipping than the CC method for the same PAPR reduction performances. Besides, unlike the AC and IAC methods, the PAPR value of each clipped symbol, in the PAC method, is exactly equal to $\text{PAPR}_0 = \gamma_4(\rho)$. More simulation results will be presented in Section IV. As in the previous section, we will evaluate, in what follows, the number of required iteration so as to find $A^{(\mathbf{x})}$. To this end, for each OFDM symbol $\mathbf{x}$, let us consider $N_{[\mathbf{x},\text{PAC}]}$ the number of performed iterations so as to find $A^{(\mathbf{x})}$.

Fig. 16 depicts the number of iterations performed by the AC method in order to approach $\rho^{\mathbf{x}}$ versus $\Delta_{E_\mathbf{x}}$. Obtained results depicted in Fig. 16 show that the number of iterations performed, so as to find the adaptive threshold of each OFDM symbol does not depend on $\Delta E_\mathbf{x}$. In fact, the curve depicting the number of iterations in function of $\Delta E_\mathbf{x}$ is not monotonic. On the basis of the simulation results depicted in Figs. 6 and 9, it appears that this approach requires generally more iterations than the AC and IAC approaches. In Section IV, the previous approaches and the PAC method will be compared as regards the average number of required iterations versus PAPR_0. In other words, we will compare $\mathbb{E}[N_{[x,\text{AC}]}]$, $\mathbb{E}[N_{[x,\text{IAC}]}]$ and $\mathbb{E}[N_{[x,\text{PAC}]}]$ where $\mathbb{E}[.]$ is the expectation. The Monte Carlo method will be used for this estimation.

IV. Simulation Results

As in the previous section, the performance of the proposed AC, IAC and PAC methods versus that of the CC method are analysed within the framework of a specific PAPR reduction. In other words, if the CC method is performed by means of the predefined normalized threshold ρ, the AC, IAC and PAC will be performed using and $\text{PAPR}_0 = \gamma_4(\rho)$ and $\text{PAPR}_0 = \gamma_4(\rho) - \epsilon$, where $\gamma_4(\rho)$ denotes the PAPR achieved via the CC method. Note that, $\gamma_4(\rho)$ is the upper bounded PAPR for a 10^{γ_4} value. The simulations are performed for an OFDM signal featuring 16-QAM modulation, $M = 64$ and an oversampling factor $L = 4$.

Figs. 17 and 18, with $\rho = 3.5$ dB and 5 dB, respectively, confirm that the CCDF curves of the AC, IAC and PAC approximate the ideal clipping CCDF curve. Besides, the proposed AC methods and the CC method achieve the same upper bounded PAPR value for a CCDF curves clip rate equal to 10^{-4}. It is worth noting that given the depicted results the AC, IAC and PAC methods reach a deterministic upper bounded PAPR. In the following, the AC, IAC and PAC methods are compared with the CC method as regards the BER degradation.

The results depicted in Figs. 19 and 20 show that the AC, IAC, and PAC methods outperform the CC method in terms of BER degradation. The gain obtained for a BER of 10^{-4}, is approximately equal to 0.5 dB in Fig. 20 and 3 dB in Fig. 19. These results confirm the theoretical analysis undertaken in Sections III-A, III-B and III-C, where the authors have shown that, where the CC method is used many OFDM symbols are clipped more severely (see AREA 2 in Fig. 3) than necessary or clipped unnecessarily (see AREA 1 in Fig. 3) with respect to γ_4.

The mean power degradation and the adjacent channels pollution, which is due to the effect of the OOB components, are depicted in Fig. 21 and Fig. 22.

From the simulation results depicted in Fig. 21, it can be noticed that the AC, IAC, and PAC methods decrease less the mean power of the OFDM symbol after a PAPR reduction than the CC method featuring the same achieved PAPR for a CCDF's clip rate equal to 10^{-4}. For example, where $\gamma_4 = 4.72$ dB, $\Delta E = -0.47$ dB in CC method and

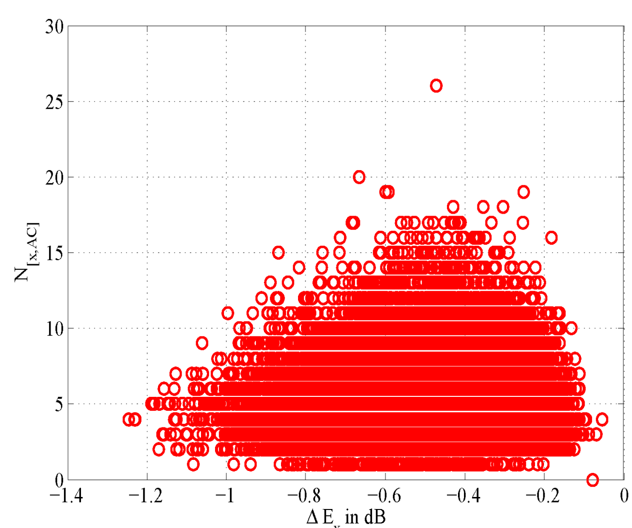

Figure 16. Number of iterations performed by the PAC method so as to find $A^{(\mathbf{x})}$ versus $\Delta E_\mathbf{x}$. $M = 64$, $L = 4$, $\rho = 3.5$ dB and $\text{PAPR}_0 = \gamma_4(\rho) = 4.62$ dB.

$\Delta E = -0.25$ dB in the proposed AC methods.

Fig. 22 depicts the Power Spectrum Density (PSD) of the OFDM signal before and after PAPR reduction where using the proposed AC method and the CC method.

Besides the achieved BER performance in Figs. 19, 20, and attained mean power variation in Fig. 21, it can be noted from Fig. 22 that the proposed AC methods are less polluting than the CC method when $\text{PAPR}_0 = \gamma_4 - \epsilon$, i.e., for the same PAPR value. As a general conclusion, the obtained results pertaining to mean power degradation, Out-Of-Band emission and BER degradations confirm that where $\text{PAPR}_0 = \gamma_4 - \epsilon$, i.e., within the framework of a similar achieved output PAPR, the AC, IAC and PAC methods induce less degradation than the CC method (see Figs. 19, 20, 21, 22 and 23, which is a zoom of Fig. 22).

In the following, we compare the required average number of iterations performed by the proposed AC method so as to find the adaptive threshold of each OFDM symbol that we want to clip. Fig. 24 shows that IAC method converges more quickly than AC and PAC methods. The simulation results show also that the PAC method required more iterations than the AC and IAC methods so as to find the suitable bound of the adaptive magnitude clipping $A^{(\mathbf{x})}$. However, it is important to note that the PAC method gives an exact solution of (8) whereas the AC and IAC give an approximation of the solution. Besides, it can be remarked that $N_1 \geq N_2$ when $\gamma_4 \leq 6$, which is coherent with equation (18) (with $\epsilon = 0.1$ dB $\Rightarrow$ $10\log_{10}\left[\log\left(\frac{10^{\frac{0.1}{10}}}{10^{\frac{0.1}{10}}-1}\right)\right] = 5.77$ dB $\simeq 6$ dB).

V. Conclusion

In this paper, a new clipping method is presented. In the latter, each OFDM symbol is clipped with respect to the desired output upper bounded PAPR_0, which is obtained thanks to an adaptive threshold. Three methods (AC, IAC, and PAC) are proposed for the computation of the adaptive threshold. The theoretical analysis and simulation results achieved in this paper show that the adaptive threshold can be efficiently computed by means of the IAC method. This approach converges more quickly than the one based on exhaustive research featuring a constant step and than the PAC method. However, the PAC method allows finding the exact adaptive threshold for each OFDM symbol. Thanks to these approaches, the proposed AC method achieves the best performances whatever the method to compute the threshold is, and offers similar performances in terms of PAPR reduction. Furthermore, the proposed AC method gives a deterministic desired upper bounded PAPR, which is very important for the IBO definition in the case of high power amplifiers (HPA). Our future work will focus on the extension of the proposed work to other clipping functions such as deep clipping and smooth clipping, combined with Out Of Band noise suppression approaches.

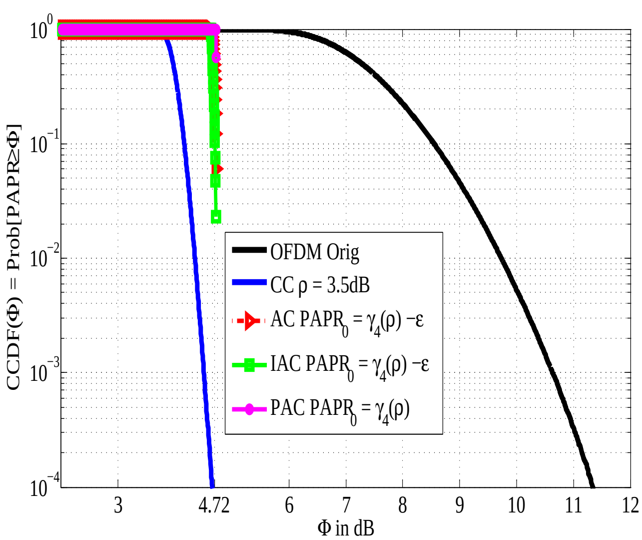

Figure 17. CCDF curves before and after PAPR reduction where using the CC, AC, IAC and PAC methods, and with $\rho = 3.5$ dB.

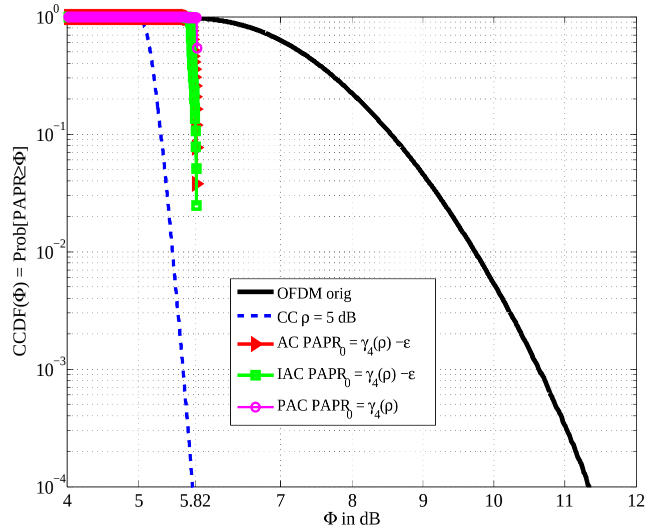

Figure 18. the CCDF curves before and after PAPR reduction where using the CC, AC, IAC and PAC methods, and with $\rho = 5$ dB.

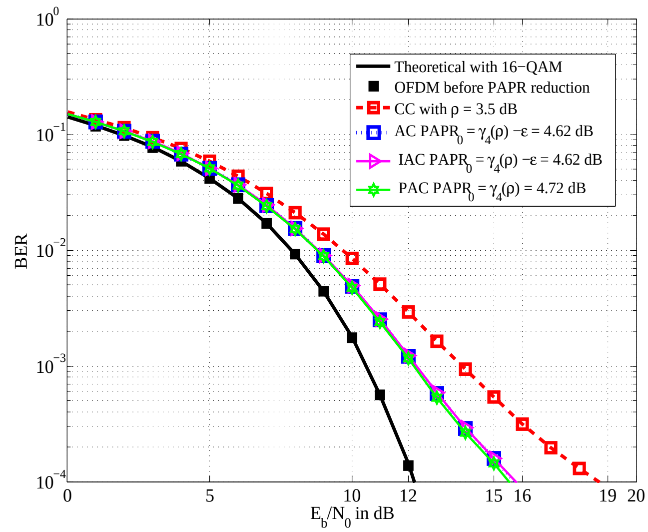

Figure 19. Comparison between the CC method and proposed AC methods, in terms of BER degradation, and with $\rho = 3.5$ dB.

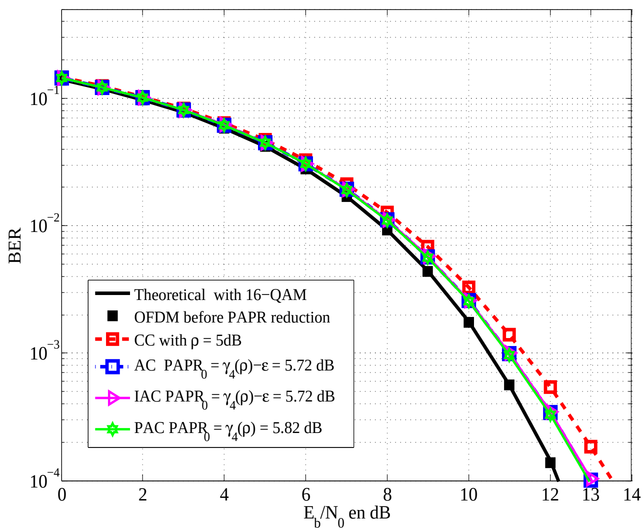

Figure 20. Comparison between the CC method and the proposed AC methods, in terms of BER degradation, and with $\rho = 5$ dB.

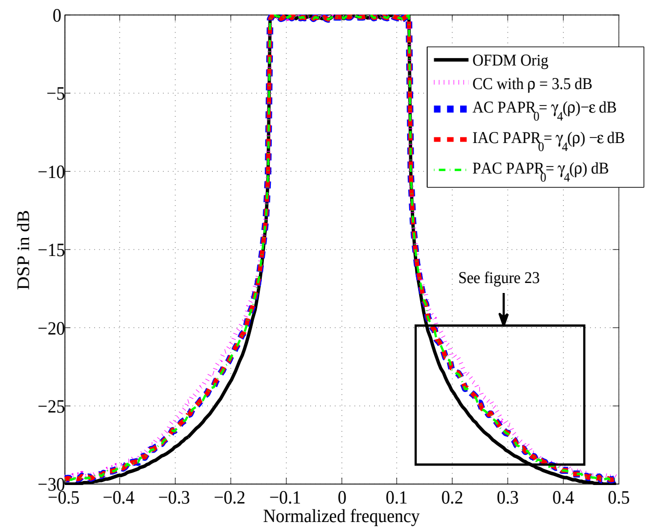

Figure 22. Comparison between PSDs before and after PAPR reduction carried out by means of the proposed methods and the CC method, with $\rho = 3.5$ dB.

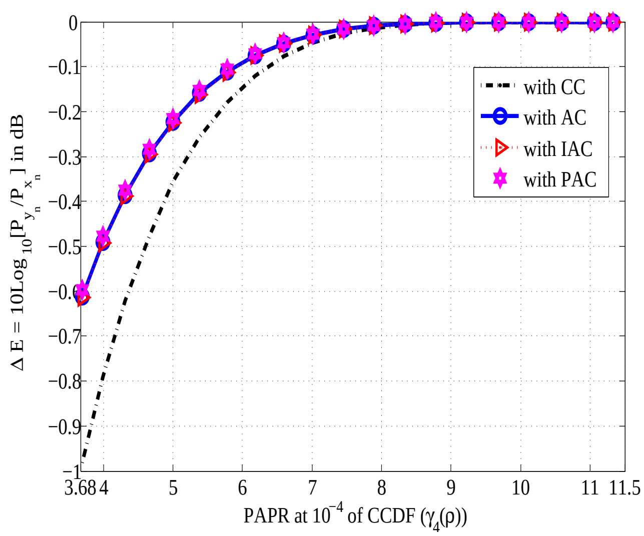

Figure 21. Comparison between the CC method and the proposed AC methods, in terms of Mean Power degradation versus the achieved PAPR for a rate 10^{-4}.

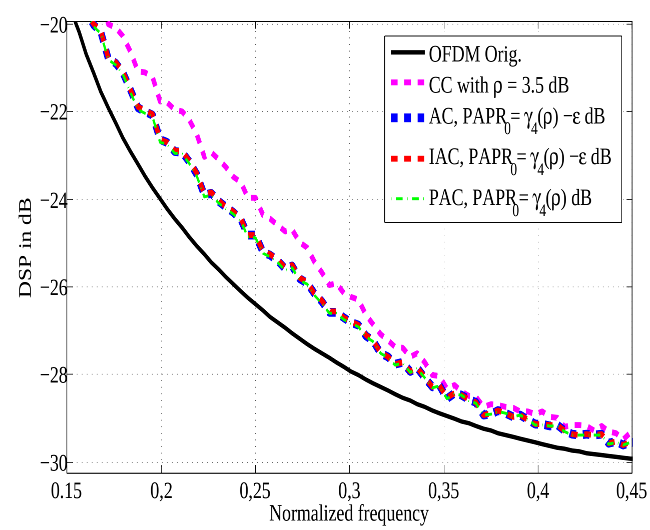

Figure 23. Zoom of Fig. 22. Comparison between the PSD obtained before and after PAPR reduction carried out by means proposed methods and the CC method, with $\rho = 3.5$ dB.

APPENDIX A
PAPR VALUE OF THE CLIPPED OFDM SYMBOL USING THE THRESHOLD ρ_m IN THE AC METHOD

Proof: In this section, we detail the proof related to (15). Therefore, let ρ_m, $m = 1, 2, \ldots$, be the checked step at the m-*th* iteration (see Fig. 4). Then, from (7), the PAPR value of the clipped OFDM symbol using the threshold ρ_m in the AC method can be expressed as follows

$$\begin{aligned} \mathrm{PAPR}_{[\mathbf{y}^{(m)}]} &= \mathrm{PAPR}_0 - (m-1)\epsilon + 10\log_{10}\left(\frac{P_{\mathbf{x}}}{P_{\mathbf{y}^m}}\right) \\ &= \mathrm{PAPR}_0 - (m-1)\epsilon_{\mathbf{x}} + 10\log_{10}\left(\frac{P_{\mathbf{x}}}{P_{\mathbf{y}^m}}\right). \end{aligned}$$

.

After a few derivations and thanks to (15), we obtain:

$$\mathrm{PAPR}_{[\mathbf{y}^{(m)}]} = \mathrm{PAPR}_0 + 10\log_{10}\left(\left[\frac{P_{\mathbf{y}^{(\mathrm{N}_{[\mathbf{x},\mathrm{IAC}]})}}}{P_{\mathbf{x}}}\right]^{\frac{m-1}{\mathrm{N}_{[\mathbf{x},\mathrm{IAC}]}-1}} \frac{P_{\mathbf{x}}}{P_{\mathbf{y}^{(m)}}}\right)$$

Since the number of iterations performed by IAC, so as to compute the normalized threshold for the OFDM symbol $\mathbf{x}$, is $N_{\mathbf{x},2}$, we note that

$$\begin{cases} \left(\mathrm{PAPR}_{[\mathbf{y}^{(m)}]} - \mathrm{PAPR}_0\right) & \geq \quad 10\log_{10}\left[\frac{P_{\mathbf{y}^{(m-1)}}}{P_{\mathbf{y}^{(m)}}}\right] \\ & \geq \quad \epsilon_m > \epsilon \text{ If } m < N_{[\mathbf{x},\mathrm{IAC}]} \\ \left(\mathrm{PAPR}_{[\mathbf{y}^{(m)}]} - \mathrm{PAPR}_0\right) & = \quad \log_{10}\left[\frac{P_{\mathbf{y}^{(N_{[\mathbf{x},\mathrm{IAC}]}-1)}}}{P_{\mathbf{y}^{(N_{[\mathbf{x},\mathrm{IAC}]})}}}\right] \\ & = \quad \epsilon_{N_{\mathbf{x},2}+1} < \epsilon \text{ If } m = N_{[\mathbf{x},\mathrm{IAC}]} \end{cases}.$$

■

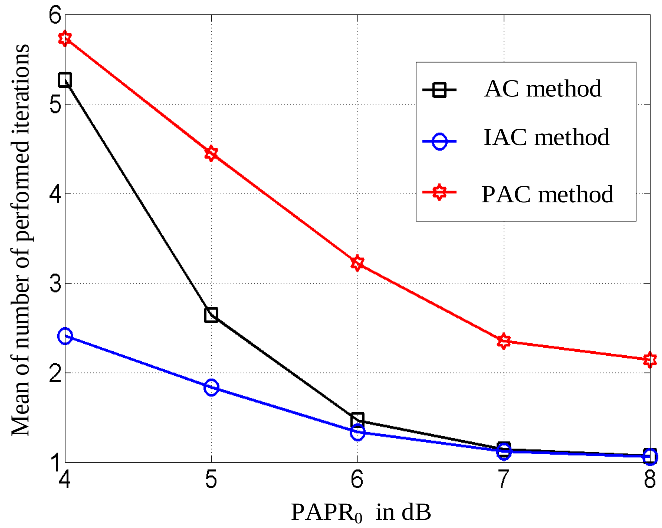

Figure 24. Mean number of iterations performed by IAC and AC for each OFDM symbol in function of PAPR$_0$.

The latter proves that, for each $\mathbf{x}$, the number of iterations performed by IAC is equal to the number of iterations performed by AC, where the step is equal to $\epsilon_{\mathbf{x}}$.

ACKNOWLEDGMENT

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