Economic Aspects of Intelligent Network Selection: A Game-Theoretic Approach

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***Abstract*—The Digital Marketplace is a market-based framework where network operators offer communications services with competition at the call level. It strives to address a tussle between the actors involved in a heterogeneous wireless access network. However, as with any market-like institution, it is vital to analyse the Digital Marketplace from the strategic perspective to ensure that all shortcomings are removed prior to implementation. This paper presents some preliminary results of such an analysis.**

***Keywords*-network selection; economics; Digital Marketplace**

I. Introduction

With the advent of 4th Generation wireless systems, such as WiMAX and 3GPP Long Term Evolution (LTE), the world of wireless and mobile communications is becoming increasingly diverse in terms of different wireless access technologies available [1], [2]; each of these technologies has their own distinct characteristics. Mirroring this diversity, multimode terminals (GSM/UMTS/Wi-Fi) currently dominate the market permitting the possibility of selecting the most appropriate access network to match the Quality of Service (QoS) requirements of a particular session/call. A number of approaches have examined this issue utilising techniques as disparate as neural networks [3] and multiple attribute decision making [4]. The applicability of these techniques can be extended to fixed networks that employ multihoming where the problem becomes one of path selection [5], [6].

This work complements previous studies of intelligent network selection by considering economic aspects. From this perspective the exclusive one-to-one relationship between network operators and their subscribers no longer holds; subscribers are free to choose which operator and which access technology they would like to utilise at call set-up time. From the users' perspective, different coverage and QoS characteristics of each access network will lead to the ability to seamlessly connect at any time, at any place, and to the technology which offers the most optimal quality available for the best price. This is referred to as the *Always Best Connected* networking paradigm [7]. From the network operators' perspective, on the other hand, the integration of wireless access technologies will allow for more efficient usage of the network resources, and might be the most economic way of providing both universal coverage and broadband access [1].

On the other hand, since many different actors with opposing interests are involved, it may also lead to a 'tussle' [8]. For example, the end-users seek to obtain the best quality for the best price, while the network operators aim at maximising their profit and performing efficient load balancing. The conflict will become even more aggravated should the service provision be separated from the network operators [9]. Hence more sophisticated management techniques may be required to manage such a complex system.

Over the last decade, several different approaches have been proposed as possible solutions to the problem when economic competition is considered. Antoniou *et al.*, and Charilas *et al.* model the problem as a noncooperative game between wireless access networks which aims at obtaining the best possible tradeoff between networks' efficiency and available capacity, while, at the same time, satisfying the users' QoS [10], [11]. Ormond *et al.* propose an algorithm for intelligent cost-oriented and performance-aware network selection which maximises consumer surplus [12], [13]. Niyato *et al.* propose two game-theoretic algorithms for intelligent network selection mechanism which performs intelligent load balancing to avoid network congestion and performance degradation [14]. Khan *et al.* model the problem as a procurement second-price sealed-bid auction where network operators are the bidders and user is the buyer [15], [16]. Lastly, Irvine *et al.* propose a market-based framework called the Digital Marketplace (DMP), where network operators offer communications services with competition at the call level [17]–[19].

Although each proposed solution is technically valid, only the DMP strives to address tussle between the actors involved. Not only does the DMP consider the technical challenges but also the economic issues. However, as with any market-like institution, it is vital to analyse the DMP from the strategic perspective (using game theory, or otherwise) to ensure that all shortcomings are removed prior to implementation. This paper presents some preliminary results of such an analysis.

The rest of this paper is organised as follows. In Section II, an overview of the DMP is given. Section III presents the results of the analysis. Section IV discusses future work, while Section V draws conclusions.

II. The Digital Marketplace

The DMP was developed with the heterogeneous mobile and wireless communications environment in mind, where users have the ability to select a network operator that reflects their preferences best on a per-call basis. In other

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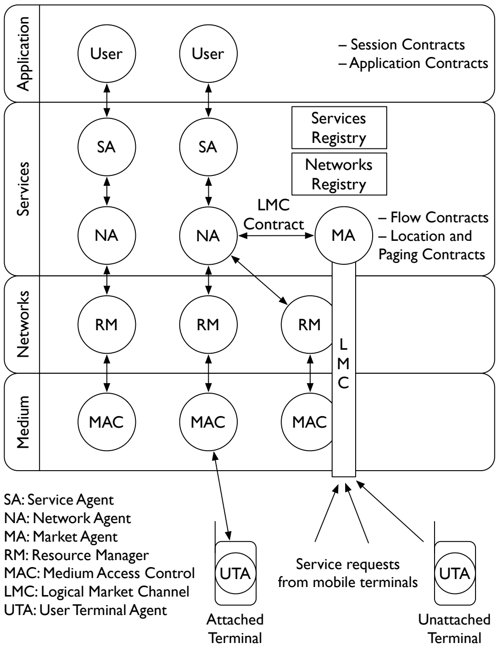

Figure 1. The Digital Marketplace (adapted from [17])

words, the end-users have the freedom of choice, while the network operators manage service requests appropriately.

The conceptual framework of the DMP is shown in Figure 1. The DMP is defined using a four-layer communications stack: *application layer*, *services layer*, *networks layer*, and *medium layer*. The end-users who effectively reside in the application layer are able to negotiate network access on a per call basis. To this end, they have two ways of accomplishing it: they can either go into a business relationship with a service provider (service agent, SA, in Figure 1) who will act on their behalf, or they can personally participate in the negotiation process with a network operator (network agent, NA). In both cases, the process is supervised by a market provider (market agent, MA), and takes place in the services layer. Before the negotiation occurs, the end-user is required to forward her service requirements to either the SA or the NA. This is done using a common communications channel referred to as a logical market channel (LMC). The LMC itself is negotiated between the MA and the registered NAs at the marketplace initialisation stage.

The network selection mechanism in the DMP is based on a procurement first-price sealed-bid (FPA) auction. The network operators represent the sellers (or bidders) who compete for the right to sell their product (transport service) to the buyer; i.e., either the service provider or the end-user. However, unlike in a standard procurement FPA auction, here, bidders do not bid only on prices, but also on reputation; i.e., when selecting the winner, the buyer takes into consideration both the offered price of the product and the bidder's reputation. The reputation is directly proportional to the number of calls that have been decommitted in the past by the respective network operator.

An FPA auction, in an economic terminology, is an example of an allocation mechanism; that is, a system where economic transactions take place and goods are allocated [20]. As briefly mentioned in the Introduction, it is vital to analyse it from the strategic perspective, and establish what the most probable outcome will be; how the bidders will most likely bid; etc. In this way, all the shortcomings and inefficiencies can be addressed prior to implementation.

III. Modelling and Analysis

A. Notation and Preliminaries

The following notation and concepts are assumed throughout the rest of this paper.

1) Probability Theory and Statistics: Let X denote a random variable (r.v.) with the support $[a, b]$, where $a < b$ and $a, b \in \mathbb{R}$. By F_X we mean a cumulative distribution function of the X r.v.; therefore, for any $x \in \mathbb{R}$, $F_X(x) = P\{X \leq x\}$, where $P\{X \leq x\}$ denotes the probability of the event such that $X \leq x$. If F_X admits a density function, it shall be denoted by $f_X \equiv F'_X$.

The expected value of X, denoted by $E[X]$, is defined as $E[X] = \int_{-\infty}^{\infty} x dF_X(x)$. Similarly, if u is a function of X, then the expected value of $u(X)$ is defined as $E[u(X)] = \int_{-\infty}^{\infty} u(x) dF_X(x)$.

Let $X_1, \ldots, X_n$ be independent continuous r.v.s with distribution function F and density function $f \equiv F'$. If we let $X_{i:n}$ denote the ith smallest of these r.v.s, then $X_{1:n}, \ldots, X_{n:n}$ are called the *order statistics* [21], [22]. In the event that the r.v.s are independently and identically distributed (i.i.d.), the distribution of $X_{i:n}$ is

$$F_{X_{i:n}}(x) = \sum_{k=i}^{n} \binom{n}{k} (F(x))^k (1 - F(x))^{n-k}, \tag{1}$$

while the density of $X_{i:n}$ can be obtained by differentiating Eq. (1) with respect to x [23]. Hence,

$$f_{X_{i:n}}(x) = \frac{n!}{(n-i)!(i-1)!} f(x)(F(x))^{i-1}(1 - F(x))^{n-i}.$$

2) Game Theory: Let $\Gamma^B = [N, \{S_i\}, \{u_i(\cdot)\}, \Theta, F(\cdot)]$ be a *Bayesian game with incomplete information*. Formally, in this type of games, each player $i \in N$ has a utility function $u_i(s_i, s_{-i}, \theta_i)$, where $s_i \in S_i$ denotes player i's action, $s_{-i} \in S_{-i} = \times_{j \neq i} S_j$ denotes actions of all other players different from i, and $\theta_i \in \Theta_i$ represents the type of player i. Letting $\Theta = \times_{i \in N} \Theta_i$, the joint probability distribution of the $\theta \in \Theta$ is given by $F(\theta)$, which is assumed to be common knowledge among the players [24]–[26].

In game Γ^B, a *pure strategy* for player i is a function $s_i : \Theta_i \to S_i$, where for each type $\theta_i \in \Theta_i$, $s_i(\theta_i)$ specifies the action from the feasible set S_i that type θ_i would choose. Therefore, player i's pure strategy set $\mathscr{S}_i$ is the set of all such functions.

Player i's *expected utility* given a profile of pure strategies $(s_1(\cdot), \ldots, s_N(\cdot))$ is given by

$$\tilde{u}_i(s_1(\cdot), \ldots, s_N(\cdot)) = E[u_i(s_1(\theta_1), \ldots, s_N(\theta_N), \theta_i)], \tag{2}$$

where the expectation is taken over the realisations of the players' types, $\theta \in \Theta$. Now, in game Γ^B, a strategy

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profile $(s_1^*(\cdot), \ldots, s_N^*(\cdot))$ is a *pure-strategy Bayesian Nash equilibrium* if it constitutes a Nash equilibrium of game $\Gamma^N = [N, \{\mathscr{S}_i\}, \{\tilde{u}_i(\cdot)\}]$; that is, if for each player $i \in N$,

$$\tilde{u}_i(s_i^*(\cdot), s_{-i}^*(\cdot)) \geq \tilde{u}_i(s_i(\cdot), s_{-i}^*(\cdot)) \tag{3}$$

for all $s_i(\cdot) \in \mathscr{S}_i$, where $\tilde{u}_i(s_i(\cdot), s_{-i}(\cdot))$ is defined as in Eq. (2).

B. Problem Definition and Assumptions

The formal description of the network selection mechanism employed in the DMP is as follows. The model is a modified version of procurement FPA auction. Thus, formally, it represents a Bayesian game of incomplete information, Γ^B, as defined in Section III-A2. There are N bidders who bid for the right to sell their product to the buyer.

Formally, each bidder $i \in N$ is characterised by the utility function $u_i(\cdot)$ such that

$$u_i(b, c, r) = \begin{cases} b_i - c_i & \text{if } \beta(b_i, r_i) < \min_{j \neq i} \beta(b_j, r_j), \\ 0 & \text{if } \beta(b_i, r_i) > \min_{j \neq i} \beta(b_j, r_j), \end{cases} \tag{4}$$

where $b = (b_i, b_{-i})$ represents the bid price vector, $c = (c_i, c_{-i})$ the type vector, and $r = (r_i, r_{-i})$ the reputation vector. The type of each bidder is assumed to represent the cost of (or minimum price for) the service under consideration. Let $\beta : \mathbb{R}_+ \times [0,1] \to \mathbb{R}_+$, defined by

$$\beta(b_i, r_i) = w_{price} \cdot b_i + w_{penalty} \cdot r_i \quad \forall i \in N, \tag{5}$$

denote the *compound bid*. The winner of the auction is determined as the bidder whose compound bid is the lowest one; i.e., bidder i is the winner if

$$\beta(b_i, r_i) < \min_{j \neq i} \beta(b_j, r_j).$$

In the event that there is a tie

$$\beta(b_i, r_i) = \min_{j \neq i} \beta(b_j, r_j),$$

the winner is randomly selected with equal probability.

It is, moreover, assumed that the price and reputation weights $(w_{price}, w_{penalty})$ are announced by the buyer to all bidders before the auction. Thus, there is no uncertainty in knowing how much the buyer values the offered price of the service over the reputation of the seller (or vice versa). Furthermore,

$$w_{price} + w_{penalty} = 1, \quad 0 \leq w_{price}, w_{penalty} \leq 1.$$

In order to simplify the notation, it is assumed throughout the rest of this paper that $w = w_{price}$.

The buyer and the bidders are risk neutral.

The costs c_i for each $i \in N$ are private knowledge. Thus, they are particular realisations of the r.v.s C_i for each $i \in N$. Furthermore, it is assumed that each C_i is i.i.d. over the interval $[0,1]$, and admits a continuous distribution function F_C and its associated density function f_C.

Similarly, the reputations r_i for each $i \in N$ are private knowledge. Thus, they are particular realisations of the r.v.s R_i for each $i \in N$. Furthermore, it is assumed that each R_i is i.i.d. over the interval $[0,1]$, and admits a continuous distribution function F_R and its associated density function f_R. It is crucial to observe that the higher the reputation, the lower the value of r_i.

The bidding strategy functions $b_i = b_i(c_i, r_i) : [0,1] \times [0,1] \to \mathbb{R}_+$ are nonnegative in value for all $i \in N$.

In equilibrium, every bidder $i \in N$ uses the same strictly increasing in all of its variables bidding strategy function; i.e., $b_i = b_i(c_i, r_i) = b(c_i, r_i), \forall i \in N$. In this case, the equilibrium profile $(b^*(\cdot), \ldots, b^*(\cdot))$ is called *symmetric*.

The aim is to solve the game for pure-strategy symmetric Bayesian Nash equilibrium(-a) as defined in Eq. (3), Section III-A2.

C. Analysis and Results

First of all, it should be noted that the problem is far more complicated than the one encountered when solving standard FPA auction. Thus, the arguments and the heuristic approach of derivation of the equilibrium bidding strategy, although effective in standard FPA setting (for example, see [27]–[29]), are useless in this case. Not only is the bidding strategy function $b(c_i, r_i)$ dependent on two variables, but also the probability of winning involves finding the minimum of a linear combination of $b(C_j, R_j)$ and R_j r.v.s; that is,

$$P\{i \text{ wins}\} = P\left\{\beta(b_i, r_i) < \min_{j \neq i} \beta(b_j, r_j)\right\}.$$

(For simplicity the possibility of a tie has been neglected.)

Simplification of the problem by letting $b(c_i, r_i) = b(c_i)$ for all $i \in N$ is also insufficient. Going even further and assuming that every bidder knows the reputations of their opponents does not simplify the problem enough for the analytical analysis to be viable. Then the problem becomes

$$\max_{b_i} E\left[b_i - c_i \,\middle|\, wb_i + (1-w)r_i < \min_{j \neq i}(wb(C_j) + (1-w)r_j)\right].$$

Noting that

$$\min_{j \neq i}(wb(C_j) + (1-w)r_j) \geq w \min_{j \neq i} b(C_j) + (1-w) \min_{j \neq i} r_j,$$

and assuming that $w \neq 0$, yields

$$\max_{b_i} E\left[b_i - c_i \,\middle|\, b^{-1}\left(b_i + \frac{1-w}{w}(r_i - \min_{j \neq i} r_j)\right) < \min_{j \neq i} C_j\right] \tag{6}$$

where we have used the fact that $b(\cdot)$ is strictly increasing, and hence, it is invertible and $\min_x b(x) = b(\min_x x)$ for all x.

Let $C_{1:N-1} = \min_{j \neq i} C_j$ be the lowest order statistic of an i.i.d. random sample C_j for all $j \neq i$ with the distribution function $F_{C_{1:N-1}}$. Hence, the identity (6) becomes

$$\max_{b_i} \left(b_i - c_i\right)\left(1 - F_C\left(b^{-1}\left(b_i + \frac{1-w}{w}(r_i - \min_{j \neq i} r_j)\right)\right)\right)^{N-1} \tag{7}$$

where we have used the fact that the distribution function of an i^{th} order statistic of an i.i.d. random sample is defined as in Eq. (1).

Finally, recalling that at a symmetric equilibrium $b_i = b(c_i)$ and letting $k = \frac{(1-w)}{w}(r_i - \min_{j \neq i} r_j)$, the identity (7) becomes

$$b'\left(b^{-1}(b(c_i)+k)\right)\cdot\left[1-F_C(b^{-1}(b(c_i)+k))\right]^{N-1}$$
$$=(N-1)(b(c_i)-c_i)\left[1-F_C(b^{-1}(b(c_i)+k))\right]^{N-2}$$
$$\cdot f_C(b^{-1}(b(c_i)+k)). \quad (8)$$

It is rather difficult (if even possible) to solve the resulting ordinary differential equation in (8). Therefore, it can be concluded that even serious simplification of the problem is not enough to heuristically derive an optimal bidding strategy function for each player $i \in N$.

1) Special Case $w = 0$: However, the problem becomes simpler when $w = 0$. For then, the utility of each bidder i is

$$u_i(b,c,r)=\begin{cases} b_i-c_i & \text{if } r_i<\min_{j\neq i} r_j,\\ 0 & \text{if } r_i>\min_{j\neq i} r_j.\end{cases} \quad (9)$$

Since the probability of winning, i.e., the probability of the event such that $r_i < \min_{j\neq i} R_j$ for all $i \in N$, does not depend on the value of the bid, b_i, it is clear that bidders will have an incentive to bid abnormally high.

Proposition 1. *In the Digital Marketplace, when c_i are i.i.d. over the interval $[0,1]$ for all $i \in N$ and r_i are i.i.d. over the interval $[0,1]$ for all $i \in N$, the bidders will have an incentive to bid abnormally high whenever $w = 0$. That is, $b_i \to \infty$ for all $i \in N$.*

The formal proof of Proposition 1 is given in Appendix A.

2) Special Case $w = 1$: When $w = 1$, on the other hand, the problem becomes that of standard FPA auction. The utility of each bidder i is given by

$$u_i(b,c,r)=\begin{cases} b_i-c_i & \text{if } b_i<\min_{j\neq i} b_j,\\ 0 & \text{if } b_i>\min_{j\neq i} b_j.\end{cases} \quad (10)$$

The bidders will then try to solve, for all $i \in N$

$$\max_{b_i} E\left[b_i-c_i \,\middle|\, b_i<\min_{j\neq i} b(C_j)\right]$$
$$=\max_{b_i} E\left[b_i-c_i \,\middle|\, b^{-1}(b_i)<\min_{j\neq i} C_j\right]$$
$$=\max_{b_i} E\left[b_i-c_i \mid b^{-1}(b_i)<C_{1:N-1}\right]$$
$$=\max_{b_i}\int_{b^{-1}(b_i)}^1 (b_i-c_i)dF_{C_{1:N-1}}(t)$$
$$=\max_{b_i}(b_i-c_i)(1-F_{C_{1:N-1}}(b^{-1}(b_i))), \quad (11)$$

where, as before, $C_{1:N-1}=\min_{j\neq i} C_j$ be the lowest order statistic of an i.i.d. random sample C_j for all $j \neq i$ with the distribution function $F_{C_{1:N-1}}$, and its associated density $f_{C_{1:N-1}}$. The first-order condition yields

$$1-F_{C_{1:N-1}}(b^{-1}(b_i))-(b_i-c_i)\frac{f_{C_{1:N-1}}(b^{-1}(b_i))}{b'(b^{-1}(b_i))}=0. \quad (12)$$

Recalling that at a symmetric equilibrium $b_i = b(c_i)$, the identity (12) becomes

$$b'(c_i)-b(c_i)\frac{f_{C_{1:N-1}}(c_i)}{1-F_{C_{1:N-1}}(c_i)}=-c_i\frac{f_{C_{1:N-1}}(c_i)}{1-F_{C_{1:N-1}}(c_i)},$$

or equivalently,

$$(b(c_i)(1-F_{C_{1:N-1}}(c_i)))'=-c_i f_{C_{1:N-1}}(c_i).$$

Since $b(1) = 1$, we have

$$b(c_i)=\frac{1}{1-F_{C_{1:N-1}}(c_i)}\int_{c_i}^1 t\,dF_{C_{1:N-1}}(t)$$
$$=\frac{N-1}{(1-F_C(c_i))^{N-1}}\int_{c_i}^1 t(1-F_C(t))^{N-2}f_C(t)dt. \quad (13)$$

Thus, the symmetric bidding strategy in Eq. (13) is the most likely candidate for a symmetric pure-strategy Bayesian Nash equilibrium of the standard FPA auction when $w = 1$.

Proposition 2. *In the Digital Marketplace, when c_i are i.i.d. over the interval $[0,1]$ for all $i \in N$ and r_i are i.i.d. over the interval $[0,1]$ for all $i \in N$, the symmetric equilibrium bidding strategy function of the standard procurement first-price sealed-bid auction,*

$$b^*_{FPA}(c_i)=\frac{1}{1-F_{C_{1:N-1}}(c_i)}\int_{c_i}^1 t\,dF_{C_{1:N-1}}(t), \quad (14)$$

constitutes a symmetric pure-strategy Bayesian Nash equilibrium of the Digital Marketplace variant of a procurement first-price sealed-bid auction whenever $w = 1$.

The formal proof of Proposition 2 is given in Appendix A.

The next natural question to ask is whether $b^*_{FPA}(\cdot)$ constitutes an equilibrium for $w \neq 1$. The following conjecture summarises this point,

Conjecture 3. *In the Digital Marketplace, when c_i are i.i.d. over the interval $[0,1]$ for all $i \in N$ and r_i are i.i.d. over the interval $[0,1]$ for all $i \in N$, $w = 1$ whenever the symmetric equilibrium bidding strategy function of the standard procurement first-price sealed-bid auction, $b^*_{FPA}(\cdot)$, constitutes a symmetric pure-strategy Bayesian Nash equilibrium of the Digital Marketplace variant of a procurement first-price sealed-bid auction.*

The conjecture can be rephrased as "If $w \neq 1$, then $b^*_{FPA}(\cdot)$ does not constitute a symmetric pure-strategy Bayesian Nash equilibrium of the Digital Marketplace variant of a procurement first-price sealed-bid auction." The formal proof of this statement is rather difficult. However, the following argument shows why it might hold.

Suppose for the time being that $b^*(c_i) = b^*_{FPA}(c_i)$ for every value of the price weight $w \in [0,1]$. It is possible to estimate numerically how well such a bidding strategy performs for all values of w. To this end, a simple Monte Carlo simulation scenario was constructed where the bidders' costs and reputations were pseudo-randomly generated and drawn from a uniform distribution $\mathcal{U}(0,1)$.

Table I and Figure 2 depict a particular output from the simulation for $N = 3$ bidders. In this particular example, for $w \in (0.65,1]$, bidder 1 who is characterised by the lowest cost of all three bidders, wins the auction; that is, his compound bid is the lowest. At $w = 0.65$, an intersection occurs of bidder 1's and 3's compound bids, and after that, for $w \in [0,0.65)$, bidder 3 becomes the winner. If the simulation was repeated n times, and the intersection would fall within a close neighbourhood of $w = 0.65$ in the vast majority of cases, then $b^*(\cdot)$ is quite

Table I
THE OUTPUT FROM ONE RUN OF THE MONTE CARLO SIMULATION FOR $N = 3$ BIDDERS

	Cost, c_i	Reputation, r_i	Bid, $b^*(c_i)$
Bidder 1	0.2548	0.3889	0.5032
Bidder 2	0.2728	0.5528	0.5152
Bidder 3	0.4084	0.2031	0.6056

likely to be an equilibrium bidding strategy in the interval $w \in (0.65, 1]$. This is predicated on the fact that, as $w \to 1$, the offered price dominates the value of the compound bid; that is, the offered price is weighted more than the reputation (see Eq. (5)).

The methodology is as follows:

1) Generate cost/reputation/bid triplet using the Monte Carlo methods.
2) Find the winner for $w = 1$, bidder i, say (in Figure 2 that would be bidder 1).
3) Decrease the value of w until bidder i no longer wins, and save the value of w for which that happens. Henceforth, such an event shall be denoted by I, and called *the event when an intersection has occurred.*
4) If the intersection did not occur, $I = 0$, increase the counter that counts the frequency of such an event, and then discard that run.
5) Repeat n number of times.

The case when $n = 10,000$ runs, and $N = 3$ bidders is depicted in the three figures: Figure 3 depicts the evolution of the intersections against the length of the simulation; Figure 4 shows the empirical density function of the intersections; and Figure 5 depicts the empirical distribution function of the intersections. The probability of an intersection occurring equals $P\{I = 1\} = 0.67$. It can be concluded from the figures that, on average, the intersections occur at $\bar{w} = 0.6$, which represents the mean of the distribution. However, the peak observed in a close neighbourhood of $\bar{w}$ is not significant enough to conclude that bidding according to $b^*(\cdot)$ is the best strategy one can take for $w \in (\bar{w}, 1]$.

A more formal argument goes as follows. Figure 5 depicts the probability that an intersection has occurred within an interval $(-\infty, w]$ given that an intersection has occurred, $I = 1$; that is, if the former event is denoted by W, then the figure describes $P\{W \in (-\infty, w] \mid I = 1\}$. From this, the probability of winning for bidder i (as defined in the list above) given any w is

$$\begin{aligned} &P\{\text{winning} \mid w\} = \\ &= 1 - P\{W \in [w, \infty) \cap I = 1\} \\ &= 1 - P\{W \in [w, \infty) \mid I = 1\} P\{I = 1\} \\ &= 1 - (1 - P\{W \in (-\infty, w] \mid I = 1\})\, P\{I = 1\}. \end{aligned} \tag{15}$$

In order to verify Eq. (15), set $w \in \{0.25, 0.75\}$ and run a Monte Carlo simulation which counts the number of times when the bidder with the lowest cost is the winner; i.e., the winner of the auction for $w = 1$. When $w = 0.25$, $P\{\text{winning} \mid w = 0.25\} = 1 - (1 - 0.13)0.67 = 0.4171$ according to Eq. (15), while the numerically obtained result $P\{\text{winning} \mid w = 0.25\} = 0.4136$. When $w = 0.75$, $P\{\text{winning} \mid w = 0.75\} = 1 - (1 - 0.68)0.67 = 0.7856$ according to Eq. (15), while the numerically obtained result $P\{\text{winning} \mid w = 0.75\} = 0.7866$.

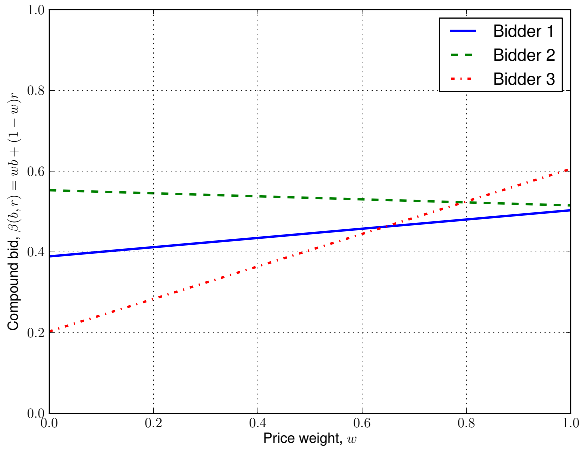

Figure 2. The performance of standard FPA bidding strategy, $b^*(\cdot)$, for the values of type, reputation and bid aggregated in Table I

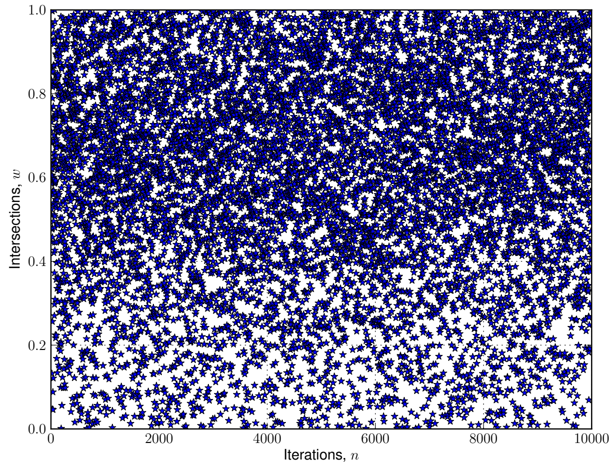

Figure 3. Intersections depicted as time series for $n = 10,000$ runs and $N = 3$ bidders

Clearly, the prediction based on Eq. (15) converges to the numerically obtained result. Moreover, it is worth noting that for $w = 0.25$, bidding according to $b^*(\cdot)$ guarantees the probability of winning for the bidder with the lowest cost of only 0.4171 which is below 50%. Thus, the bidders will definitely deviate from $b^*(\cdot)$ for low values of w. On the other hand, for $w = 0.75$, $b^*(\cdot)$ seems to achieve a relatively high probability of winning for the bidder with the lowest cost; i.e., the probability of 0.7856. However, the argument is incomplete in the sense that it only considers the probability of winning rather than the expected utility.

IV. FUTURE WORK

There are a number of potentially fruitful research directions worthy of further investigation. Firstly, a formal proof or disproof of Conjecture 3 is a necessary step in the analysis of the behaviour of the bidders.

Secondly, since the problem appears complex for N

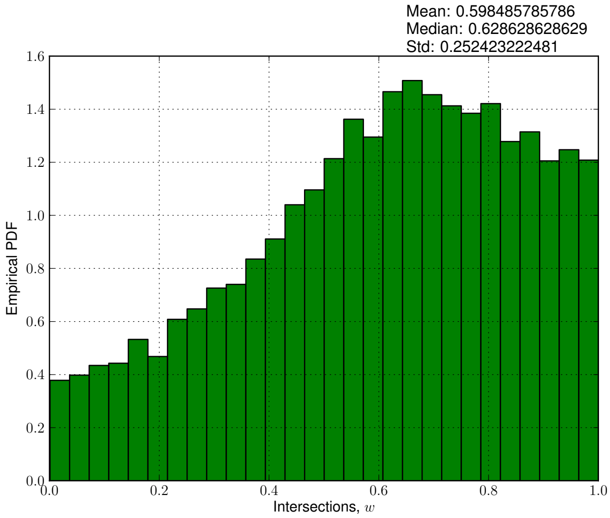

Figure 4. The histogram of the time series shown in Figure 3

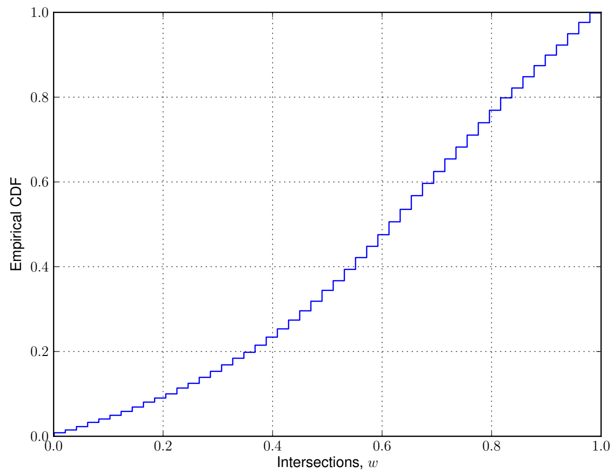

Figure 5. The empirical probability distribution associated with the histogram in Figure 4

bidders, restricting it to $N = 2$ bidders might prove beneficial. If the analysis was successful in this restricted case, perhaps it would be possible to generalise the solution(s) to an arbitrary N.

Lastly, the bidding model presented in this paper assumes that the buyer has no budget constraints. A situation virtually impossible in real life. Therefore, one of the future directions would be to modify the model by allowing the buyer to have a fixed budget.

V. Conclusions

This paper has presented some preliminary results of the game-theoretic analysis of network selection mechanism proposed in the Digital Marketplace. All things considered, it can be concluded that the analysis of the Digital Marketplace variant of procurement first-price sealed-bid auction is rather complex. It is, however, vital to have at least partially accurate predictions of the behaviour of the bidders prior to implementation.

The problem appears to be too complicated for the analytical analysis to be successful in finding a closed-form solution. On the other hand, some light was shed on the problem when $w = 0$ and $w = 1$. In the first case, it was shown that bidders will find it beneficial to submit abnormally high bids, since their bid is independent of the probability of winning the auction. In the latter case, when $w = 1$, it was shown that the problem reduces to a standard procurement first-price sealed-bid auction, and hence, a symmetric equilibrium bidding strategy function was derived and proved to indeed constitute an equilibrium of the game. It was also pointed out (informally, using Monte Carlo simulation) that the same bidding strategy most likely does not constitute an equilibrium for values of $w \neq 1$.

Appendix
Proofs

Proof of Proposition 1: Let $w = 0$. Each bidder i will then try to solve

$$\begin{aligned}
&\max_{b_i} E\left[b_i - c_i \,\middle|\, r_i < \min_{j\neq i} R_j\right] \\
&= \max_{b_i} E\left[b_i - c_i \mid r_i < R_{1:N-1}\right] \\
&= \max_{b_i} \int_{r_i}^{1} (b_i - c_i) dF_{R_{1:N-1}}(t) \\
&= \max_{b_i} (b_i - c_i)(1 - F_R(r_i))^{N-1}.
\end{aligned}$$

Since $1 - F_R(r_i) \geq 0, \forall r_i \in [0, 1]$, and since $b_i \in \mathbb{R}_+$ and $\mathbb{R}_+$ is not bounded from above, this implies that the maximisation problem is unbounded; that is, $b_i \to \infty$, which concludes the proof. ■

Proof of Proposition 2: Let $w = 1$. Suppose that all but bidder 1 follow the symmetric equilibrium bidding strategy, $b^*_{FPA}(\cdot)$. We will argue that it is optimal for bidder 1 to follow $b^*_{FPA}(\cdot)$ as well. First of all, notice that $b^*_{FPA}(\cdot)$ is a strictly increasing and continuous function. Thus, in equilibrium, the bidder with the lowest cost submits the lowest bid and wins the auction. It is not optimal for bidder 1 to bid $b_1 < b^*_{FPA}(0)$. Suppose, therefore, that bidder 1 bids an amount $b_1 \geq b^*_{FPA}(0)$. Denote by $\hat{c}_1 = b^{*-1}_{FPA}(b_1)$ the value for which b_1 is the equilibrium bid. Thus, bidder 1's expected utility from bidding $b^*_{FPA}(\hat{c}_1)$ while her cost is c_1 becomes

$$\begin{aligned}
&U(b^*_{FPA}(\hat{c}_1), c_1) = \\
&= E\left[b^*_{FPA}(\hat{c}_1) - c_1 \,\middle|\, b^*_{FPA}(\hat{c}_1) < \min_{j\neq 1} b^*_{FPA}(C_j)\right] \\
&= E\left[b^*_{FPA}(\hat{c}_1) - c_1 \,\middle|\, \hat{c}_1 < \min_{j\neq 1} C_j\right] \\
&= (b^*_{FPA}(\hat{c}_1) - c_1)(1 - F_{C_{1:N-1}}(\hat{c}_1)) \\
&= \int_{\hat{c}_1}^{1} t f_{C_{1:N-1}}(t)dt - c_1(1 - F_{C_{1:N-1}}(\hat{c}_1)) \\
&= 1 - c_1 + F_{C_{1:N-1}}(\hat{c}_1)(c_1 - \hat{c}_1) - \int_{\hat{c}_1}^{1} F_{C_{1:N-1}}(t)dt.
\end{aligned}$$

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We thus obtain that

$$U(b^*_{FPA}(c_1), c_1) - U(b^*_{FPA}(\hat{c}_1), c_1) =$$
$$= F_{C_{1:N-1}}(\hat{c}_1)(\hat{c}_1 - c_1) - \int_{c_1}^{\hat{c}_1} F_{C_{1:N-1}}(t)dt \geq 0$$

regardless of whether $\hat{c}_1 \geq c_1$ or $\hat{c}_1 \leq c_1$. We have thus argued that if all other bidders follow the strategy $b^*_{FPA}(\cdot)$, bidder 1 with a cost c_1 cannot benefit by bidding anything other than $b^*_{FPA}(c_1)$. Since similar argument can be used to show that it is optimal for any other bidder $i \neq 1$ with cost c_i to follow $b^*_{FPA}(c_i)$, $b^*_{FPA}(\cdot)$ is a symmetric equilibrium bidding strategy of the Digital Marketplace variant of procurement first-price sealed-bid auction whenever $w = 1$, which concludes the proof. ■

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