Symbol Error Rate Analysis of Adaptive Threshold Based Relay Selection with 2-Bits Feedback for Type-2 Relays

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***Abstract*—In this paper, we address the performance analysis of the extended adaptive threshold based relay selection (ATRS) scheme for multiple type-2 relays. More specifically, the symbol error rate (SER) analysis of 2-bits feedback based ATRS scheme is presented. For validation of our analytical results, derived analytical results are cross-verified with results obtained via Monte-Carlo simulations. Some selected results show that with more refined feedback information, the potential performance degradation caused by the minimum (i.e., 1-bit) feedback can be compensated, especially with a 1-bit increase in terms of feedback data rate and additional single comparison process in terms of complexity.**

Keywords–Symbol-Error Rate (SER); closed-form solutions; limited feedback; relay selection; type-2 relays.

I. Introduction

Recently, in [1], the adaptive threshold based relay selection (ATRS) scheme with minimum (i.e., '1-bit') feedback information was proposed by complying with the specifications for type-2 (or user equipment (UE)) relay to meet backward compatibility with the LTE-Advanced standard [2] [4]. Type-2 relay must be transparent to the end user (D) and the retransmitted signal from a selected relay is seen at D, like from the source (S) [1] [2]. If S only has the information of average channel gain of R-D link, like type-2 relay relaying, then the end-to-end error performance is degraded severely due to the insufficient information about R-D link. Here, to meet such backward compatibility required for type-2 relay, one of the possible solutions for selecting relay is that the relay selection process is performed at the transmitter (i.e., the source). In [3], it is done at end user and it turns out that [3] increases the complexity due to selection processing at end user, which is undesired for the mobile UE with limited complexity. In this transmitter-oriented scheme unlike the conventional receiver-oriented method, the relays (Rs) can use in the limited feedback information to the source about channel status information and then the source can use them for selecting the best relay. Based on these observations, in [1], the author proposed the threshold-based relay selection method that requires the minimum feedback load (i.e., '1-bit feedback information').

With [1], there is no need to feedback the full channel information to S that may cause a system overhead. This overhead caused by the channel status information (CSI) exchange is the bottleneck of practical implementations so that the reduction of overhead due to the required CSI exchanges is crucial. Instead of transmitting the full channel information, each relay simply reports to S about its R-D link status (e.g., "unacceptable" or "acceptable"). Then, S selects the relay with the highest S-R link gain among acceptable relays which have "acceptable" R-D link status. Therefore, with ATRS proposed in [1], there is no need to exchange the control (scheduling) message between Rs and D and as a result, it lead to satisfy the backward compatibility with type-2 relay. Further, [1] provides bandwidth efficiency and reduced complexity while it still provides an acceptable performance.

However, with the ATRS scheme [1], there may exist a potential performance degradation caused by applying the minimum (i.e., '1-bit') feedback information instead of the full CSI. More specifically, it is likely to mistakenly discard some links with better R-D channel links. For example, there may exist a relay that does not provide better S-R link compared to the scheduled relay among selected candidates only with two levels R-D link status information. Here, this potential performance degradation can be compensated with more refined feedback information. More specifically, instead of selecting acceptable or unacceptable relays with one threshold, by adopting one more threshold as shown in Figure 1, the status of each relay according to its R-D link gain is classified as three levels (i.e., "unacceptable", "good", or "better"). Then, after selecting each best relay from the good candidate group and the better candidate group, respectively, S selects the best one among these two relays. Here, since each relay needs to notify one of R-D link status among three status levels, 2-bit-based feedback is required. Therefore, with this 2-bits feedback based ATRS scheme, the potential performance degradation can be compensated with a 1-bit increase in terms of feedback data rate and additional single comparison process in terms of complexity.

Based on these observations, in [5], one more threshold was adopted. According to [5], by adopting one additional threshold to '1-bit feedback' based ATRS scheme [1], system can provide the symbol error rate (SER) performance very close to that of the perfect feedback case. Although the possibility of missing better channel links can be eliminated, the authors of [5] provided the SER performance results only from the computer simulations without any analytical derivations. Since the SER results from simulation show the performance only for the tested parameter range, we need the versatile analytical performance results which provide important insight on general range of parameter values [6]–[8]. However, we still need an accurate and efficient performance analysis, since, as far as we are aware, the general performance analysis results are not currently available.

Main Contributions: In this work, we statistically analyze the SER performance of '2-bits feedback' based ATRS scheme for multiple type-2 relays. More specifically, the closed-form

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result of SER is derived and validated with results obtained via Monte-Carlo simulations. Note that the derived closed-form results of SER can be easily evaluated numerically in the standard mathematical packages, such as MATHEMATICA, MATLAB and MAPLE.

The rest of the paper is organized as follows. In Section II, we present the system and channel models including the mode of operation. Then, we provide in Section III the SER performance analysis for M-ary phase-shift keying (PSK) signaling based on the statistical analysis. Finally, some selected results are provided followed by concluding remarks, in Sections IV and V, respectively.

II. SYSTEM AND CHANNEL MODELS

Similar to [1], we assume that all channel links are quasi-static (or block flat fading) and mutually independent, which means that the channels are constant within one transmission duration, but vary independently over different transmission durations. In addition, the fading conditions follow Rayleigh fading model. In the performance analysis, we assume that all relays are statistically identical.

We also consider a network with multiple type-2 relays and decode-and-forward (DF) protocols [9] [10], where a source node, S, communicates with a destination node, D, with the help of multiple type-2 relays, R_i, (there exist N possible relays). The channel gains of S-D, S-R$_i$, and R$_i$-D links are denoted by $h_{s,d}$, h_{s,r_i}, and $h_{r_i,d}$ which are assumed zero-mean complex Gaussian random variables with variances $\delta^2_{s,d}$, δ^2_{s,r_i}, and $\delta^2_{r_i,d}$, respectively.

Each node has only one omni-directional antenna. Therefore, all the nodes operate on half-duplex mode. Furthermore, it is assumed that each relay knows the CSI of its S-R$_i$ link and CSI of its R$_i$-D link based on ACK/NACK from end user (D) [1] [2]. More specifically, based on the system model of the type II relay shown in [1] [2], each relay can overhear the reference signal including ACK/NACK signal periodically sent from D to S. Such overheard signals can be used for estimating each R-D link quality by comparing with pre-determined thresholds. Therefore, it is possible to partially feedback the R-D link channel conditions to S [1]. Given some form of network block synchronization, carrier and symbol synchronization for the network can build equally between the individual transmitters and receivers. Exactly how this synchronization is achieved, and the effects of small synchronization errors on performance, is beyond the scope of this paper.

In the relay selection scheme, similar to [1], we assume TDD mode where $h_{s,d}$ and h_{s,r_i} are known at S, and $h_{r_i,d}$ are known at R, but $h_{r_i,d}$ must be fedback to S by R that causes system overhead. Here, we extend the proposed ATRS scheme with '1-bit feedback information' in [1] as '2-bits feedback information' based scheme by adopting two thresholds to improve the SER performance while slightly increasing system complexity. Note that [5] shows that more specific information assures better R-D channel condition for the best relay, resulting in an improved SER performance [5].

The relay selection strategy is similar to the ATRS scheme with 1-bit feedback [1]. In [1] [5], the relay selection protocol proposed in [11] is adopted to compensate the drawbacks of the

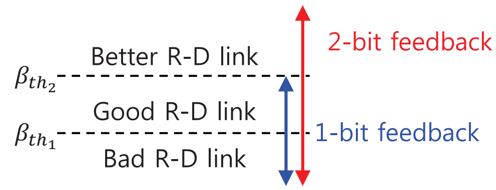

Figure 1. ATRS with 2-bit feedback.

performance degradation due to the limited feedback. In [11], they proposed the relay selection protocol using a modified harmonic mean, which is an appropriate metric to represent the relay's ability on how much help a relay can offer. According to [11], the optimal relay is the one which has the maximum value of the relay's metric, which is a modified version of the harmonic mean function of its source-relay and relay-destination instantaneous channel gains. Therefore, if multiple relays are available and we need to choose one relay only, then the relay with maximum value in terms of the modified harmonic mean is selected. Similarly, in [5], the optimal relay is the one which has the maximum value of the modified version of the harmonic mean among candidate relays. Specifically, the scheme with limited feedback information selects the relay, which maximizes the modified harmonic mean of S-R and R-D links: i) S transmits both to all relays and to D; ii) D transmits an ACK/NACK message, which is overheard at multiple relays to acquire information about the R-D links; iii) eligible relays must communicate their eligibility to S; iv) S selects the best relay in terms of the modified harmonic mean. Here, the main difference of the 2-bits feedback based scheme compared to [1] is to quantize the R-D link with three levels, i.e., "unacceptable", "good" or "better" as shown in Figure 1. The quantized R-D channel gains value can be written as $\beta_{r_i,d} \in \{0, \beta_{th_1}, \beta_{th_2}\}$ (for $i = 1, 2, \ldots, k$ and $k \leq N$). More specifically, in the first stage, each relay notifies S of being "better" or "good" if

$$\begin{aligned} &\beta_{r_i,d} \geq \beta_{th_2} \longrightarrow \text{available with a better link condition,} \\ &\text{or} \\ &\beta_{th_2} > \beta_{r_i,d} \geq \beta_{th_1} \longrightarrow \text{available with a good link condition,} \\ &\text{for } i = 1, 2, \ldots, k_2, \end{aligned} \tag{1}$$

where $k_1 + k_2 = k$, $\beta_{x,y} = |h_{x,y}|^2$, and β_{th_j} (for $j = 1, 2$) is the threshold ($\beta_{th_1} < \beta_{th_2}$). Similar to [1], since such relays are considered as candidates for the best relay and their SNR values are one of the two threshold values (i.e., β_{th_1} or β_{th_2}), choosing the relay who can maximize the modified harmonic mean function of S-R and R-D channel gains based on the S-R link condition will select the best relay. Specifically, the source selects one candidate from the 'better' candidate group whose R-D channel gain exceeds the threshold β_{th_2} is $\beta_b = \max\{\beta_{s,r_1}, \beta_{s,r_2}, \ldots, \beta_{s,r_{k_2}}\}$ and the other candidate from the 'good' candidate group whose R-D channel gain below the threshold β_{th_2} but exceeds the threshold β_{th_1} as $\beta_a = \max\{\beta_{s,r_1}, \beta_{s,r_2}, \ldots, \beta_{s,r_{k_1}}\}$. Then, the source chooses the best S-R link from these two candidates to maximize the modified harmonic mean of S-R and R-D channel gains. Consequently, the optimum relay will have a metric, which is equal to $\max\left\{\beta_{k_1^*}, \beta_{k_2^*}\right\}$, where $\beta_{k_1^*} = \frac{2q_1q_2\beta_{th_1}\beta_a}{q_1\beta_{th_1}+q_2\beta_a}$, $\beta_{k_2^*} = \frac{2q_1q_2\beta_{th_2}\beta_b}{q_1\beta_{th_2}+q_2\beta_b}$, $q_1 = \left(\frac{M-1}{M} + \frac{\sin\left(2\frac{\pi}{M}\right)}{2\pi}\right)^2$ and

$q_2 = \left(\frac{3(M-1)}{2M} + \frac{\sin\left(2\frac{\pi}{M}\right)}{\pi} - \frac{\sin\left(4\frac{\pi}{M}\right)}{8\pi} \right)$, especially for $P_1 = P_2$. If there exists no candidate ($k = 0$), the source randomly chooses one relay among N relays similar to [1].

For cooperative transmission (upon reception of NACK) in the second stage, the best relay forwards data to the end user if decoding is performed correctly and otherwise, the relay remains idle. In addition, we also assume that the system has the total power constraint of $P = P_1 + P_2$, where P is the total maximum transmit power available, P_1 and P_2 are the transmit powers at the source and at the selected relay, respectively. Further, we also assume maximal-ratio combining (MRC) for the signals from source and selected relay to destination, for which the destination estimates the CSI for coherent detection. Then, the instantaneous SNR of MRC output can be evaluated as $\gamma_{s,r_i,d} = \frac{P_1 \beta_{s,d} + P_2 \beta_{r_i,d}}{\sigma_n^2}$ given in [1].

III. Performance Analysis

In this section, by adopting the performance analysis framework applied in [1], the SER performance of the ATRS scheme with 2-bits feedback is analyzed for M-PSK signaling. With the help of [1], the average SER conditioned on the number of candidates k_1 and k_2 can be formulated as

$$\overline{\text{SER}}_{\text{total}} = \sum_{k_1} \sum_{k_2} \overline{\text{SER}}\left(k_1, k_2\right) P\left(K_1 = k_1, K_2 = k_2\right), \tag{2}$$

where $\overline{\text{SER}}\left(k_1, k_2\right)$ is the SER at the destination when there are k_1 candidate relays with 'good' R-D links and k_2 candidate relays with 'better' R-D links and $P\left(K_1 = k_1, K_2 = k_2\right)$ is the probability of having candidate relay subsets of size k_1 and k_2. In deriving $P\left(K_1 = k_1, K_2 = k_2\right)$, the problem can be simplified as "How many candidate relays in each group ('good' and 'better') exist?" because we assume that all the relays are statistically identical. As a result, the probability of having k_1 and k_2 candidates follows the multinomial distribution [12]. Therefore, we can obtain the probability of having k_1 and k_2 candidates in each group as

$$\begin{aligned} P(K_1 = k_1, K_2 = k_2) &= \frac{N!}{k_1! k_2! (N - k_1 - k_2)!} \\ &\times \left(e^{-\frac{\beta_{th_1}}{\delta_{r,d}^2}} - e^{-\frac{\beta_{th_2}}{\delta_{r,d}^2}} \right)^{k_1} \left(e^{-\frac{\beta_{th_2}}{\delta_{r,d}^2}} \right)^{k_2} \left(1 - e^{-\frac{\beta_{th_1}}{\delta_{r,d}^2}} \right)^{N - k_1 - k_2}. \end{aligned} \tag{3}$$

In deriving $\overline{\text{SER}}\left(k_1, k_2\right)$, if there is no relay for co-operation mode, the direct transmission is performed. Otherwise, the relay cooperation mode is performed. As a result, we can formulate $\overline{\text{SER}}\left(k_1, k_2\right)$ with two SER terms of the direct transmission mode and the relay cooperation mode as

$$\begin{aligned} \overline{\text{SER}}\left(k_1, k_2\right) = &P_e(s, r_i | k_1, k_2) P_e(s, d) \\ &+ \left[1 - P_e(s, r_i | k_1, k_2)\right] P_e(s, r_i, d | k_1, k_2), \end{aligned} \tag{4}$$

where $P_e\left(s, r_i | k_1, k_2\right)$, $P_e\left(s, d\right)$, and $P_e\left(s, r_i, d | k_1, k_2\right)$ represent the conditional decoding error at the best relay, the SER for direct transmission, and the conditional SER for cooperative transmission, respectively. Then, we need to evaluate three conditional decoding error terms at the best relay in (4).

A. For the conditional decoding error at the best relay, $P_e\left(s, r_i | k_1, k_2\right)$

In this case, we overall need to consider two cases, separately. More specifically, if the $\beta_{k_2^*} > \beta_{k_1^*}$, the relay for cooperation is selected from k_2 candidates and otherwise, the relay for cooperation is selected from k_1 candidates as

$$\begin{aligned} P_e(s, r_i | k_1, k_2) = &\int_0^\infty P_e(\beta) p_{s,r_{1,i}}(\beta | K_1 = k_1, K_2 = k_2) d\beta \\ &+ \int_0^\infty P_e(\beta) p_{s,r_{2,i}}(\beta | K_1 = k_1, K_2 = k_2) d\beta, \end{aligned} \tag{5}$$

where $P_e(\gamma)$ is the SER formula for M-PSK, $P_e(\gamma) = \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} e^{-\frac{b\gamma}{\sin^2\theta}} d\theta$, given by [1] [8] and $b = \sin^2(\pi/M)$ and M is the modulation order of PSK. For case 1) (i.e., $\beta_{k_2^*} > \beta_{k_1^*}$), we can rewrite as the function of β_a and β_b

$$\begin{aligned} \beta_{k_1^*} > \beta_{k_2^*} \Leftrightarrow &\frac{1}{q_2 * \beta_b} + \frac{1}{q_1 * \beta_{th2}} > \frac{1}{q_2 * \beta_a} + \frac{1}{q_1 * \beta_{th1}} \\ \Leftrightarrow &\frac{1}{\beta_a} - \frac{1}{\beta_b} < \frac{q_2}{q_1} \left(\frac{1}{\beta_{th2}} - \frac{1}{\beta_{th1}} \right). \end{aligned} \tag{6}$$

Then, if we let $X = \frac{1}{\beta_a}$ and $Y = \frac{1}{\beta_b}$, then the valid integral regions of β_a and β_b become $0 < \beta_a < \infty$ and $0 < \beta_b < \frac{\beta_a}{1 - A\beta_a}$, respectively, where $A = \frac{q_2}{q_1} \left(\frac{1}{\beta_{th_2}} - \frac{1}{\beta_{th_1}} \right)$ $(A < 0)$.

Similarly, for case 2) (i.e., $\beta_{k_2^*} < \beta_{k_1^*}$), we can rewrite as the function of β_a and β_b

$$\begin{aligned} \beta_{k_1^*} < \beta_{k_2^*} \Leftrightarrow &\frac{1}{q_2 * \beta_b} + \frac{1}{q_1 * \beta_{th2}} < \frac{1}{q_2 * \beta_a} + \frac{1}{q_1 * \beta_{th1}} \\ \Leftrightarrow &\frac{1}{\beta_a} - \frac{1}{\beta_b} > \frac{q_2}{q_1} \left(\frac{1}{\beta_{th2}} - \frac{1}{\beta_{th1}} \right). \end{aligned} \tag{7}$$

In this case, we consider two cases separately for mathematical convenience. More specifically, for case 2)-i) (i.e., $Y > -A$), the valid integral regions of β_a and β_b become $0 < \beta_a < \frac{\beta_b}{1 + A\beta_b}$ and $0 < \beta_b < \frac{1}{-A}$, respectively. Otherwise (i.e., for case 2)-ii)), the valid integral regions of β_a and β_b become $0 < \beta_a < \infty$ and $\frac{1}{-A} < \beta_b < \infty$, respectively. As results, (5) can be rewritten as the following three integral terms

$$\begin{aligned} &\int_0^\infty P_e(\beta) p_{s,r_{1,i}}\left(\beta | K_1 = k_1, K_2 = k_2\right) d\beta \\ &= \int_0^\infty \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} e^{\left(\frac{-b\beta_a}{\sin^2\theta}\right)} \int_0^{\frac{\beta_a}{1 - A\beta_a}} \frac{k_2}{\delta_{s,r_i}^2} e^{-\frac{\beta_b}{\delta_{s,r_i}^2}} \left(1 - e^{-\frac{\beta_b}{\delta_{s,r_i}^2}} \right)^{k_2 - 1} \\ &\frac{k_1}{\delta_{s,r_i}^2} e^{-\frac{\beta_a}{\delta_{s,r_i}^2}} \left(1 - e^{-\frac{\beta_a}{\delta_{s,r_i}^2}} \right)^{k_1 - 1} d\beta_b d\theta d\beta_a, \end{aligned} \tag{8}$$

and

$$\begin{aligned} &\int_0^\infty P_e(\beta) p_{s,r_{2,i}}\left(\beta | K_1 = k_1, K_2 = k_2\right) d\beta \\ &= \int_0^{-\frac{1}{A}} \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} e^{\left(\frac{-b\beta_b}{\sin^2\theta}\right)} \int_0^{\frac{\beta_b}{1 + A\beta_b}} \frac{k_1}{\delta_{s,r_i}^2} e^{-\frac{\beta_a}{\delta_{s,r_i}^2}} \left(1 - e^{-\frac{\beta_a}{\delta_{s,r_i}^2}} \right)^{k_1 - 1} \\ &\frac{k_2}{\delta_{s,r_i}^2} e^{-\frac{\beta_b}{\delta_{s,r_i}^2}} \left(1 - e^{-\frac{\beta_b}{\delta_{s,r_i}^2}} \right)^{k_2 - 1} d\beta_a d\theta d\beta_b \\ &+ \int_{-\frac{1}{A}}^\infty \frac{1}{\pi} \int_0^{\frac{(M-1)\pi}{M}} e^{\left(\frac{-b\beta_b}{\sin^2\theta}\right)} \frac{k_2}{\delta_{s,r_i}^2} e^{-\frac{\beta_b}{\delta_{s,r_i}^2}} \left(1 - e^{-\frac{\beta_b}{\delta_{s,r_i}^2}} \right)^{k_2 - 1} d\theta d\beta_b. \end{aligned} \tag{9}$$

In (8), after applying the binomial theorem [13], (8) and then with the help of the integral identity [14, Eq. (07.33.07.0001.01)] and Taylor series expansions of exponential functions [13], the closed-form expression of (8) can be obtained as

$$\sum_{j=0}^{k_2-1}\sum_{l=0}^{k_1-1}\binom{k_2-1}{j}\binom{k_1-1}{l}k_2k_1\frac{(-1)^{j+l+1}}{1+j} \times\left[\sum_{n=0}^{\infty}F_1\left(\frac{\left(1+l+\frac{b\delta_{s,r_i}^2}{\sin^2\theta}\right)\left(\frac{A\delta_{s,r_i}^2}{1+j}\right)^n}{U\left(n,0,-\frac{1}{A}\left(\frac{b}{\sin^2\theta}+\frac{1+l}{\delta_{s,r_i}^2}\right)\right)}\right)-F_1\left(1+l+\frac{b\delta_{s,r_i}^2}{\sin^2\theta}\right)\right], \tag{10}$$

where $U(a,b,z)$ is Tricomi's confluent hypergeometric function [14, Eq. (07.33.02.0001.01)] and $F_1\left(x\left(\theta\right)\right)$ is given in [1, Eq. (22)] as $F_1\left(x\left(\theta\right)\right)=\frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}}\frac{1}{x(\theta)}\,d\theta$. Here, $U(a,b,z)$ and $F_1\left(x\left(\theta\right)\right)$ can be evaluated in the standard mathematical packages. Note that the expression in (10) involves an infinite summation for the term of n. However, it is found that the summand in (10) decay exponentially (or slightly faster) with the increase of n, because Stirling's approximation specifies that $n!$ grows as $\exp\left(n\ln n\right)$ [13]. As results, due to the factorial term in Tricomi's confluent hypergeometric function, $U(\cdot,\cdot,\cdot)$, as the function of n, a truncated summation with a finite number of terms can reliably achieve a required accuracy.

For the first integral term in (9), after applying binomial theorem and then expanding the exponential function as a Taylor series similar to previous case, with the help of [15, (3.381.3)], the closed-form expression of the first integral term in (9) as the function of $F_1\left(\cdot\right)$

$$\sum_{j=0}^{k_2-1}\sum_{l=0}^{k_1-1}\binom{k_1-1}{l}\binom{k_2-1}{j}k_1k_2\frac{(-1)^{j+l}}{1+l}\left[\sum_{n=0}^{\infty}\frac{1}{n!}\Gamma\left(1-n,-\frac{1+l}{A\delta_{s,r_i}^2}\right)\left(\frac{A^2\left(\delta_{s,r_i}^2\right)^2}{1+l}\right)^{-n}e^{\frac{j-l}{A\delta_{s,r_i}^2}}F_1\left(e^{-\frac{b}{A\sin^2\theta}}\left(1+j+\frac{b\delta_{s,r_i}^2}{\sin^2\theta}\right)^{1-n}\right) -e^{\frac{1+j}{A\delta_{s,r_i}^2}}F_1\left(e^{-\frac{b}{A\sin^2\theta}}\left(1+j+\frac{b\delta_{s,r_i}^2}{\sin^2\theta}\right)\right)\right]. \tag{11}$$

For the second integral term in (9), similarly, after applying the binomial theorem and then simply integrating over β_b, the closed-form expression can be obtained as

$$\sum_{j=0}^{k_2-1}\binom{k_2-1}{j}k_2(-1)^j e^{\frac{1+j}{A\delta_{s,r_i}^2}}F_1\left(e^{-\frac{b}{A\sin^2\theta}}\left(1+j+\frac{b\delta_{s,r_i}^2}{\sin^2\theta}\right)\right). \tag{12}$$

B. For the SER for direct transmission, $P_e(s,d)$

In this case, the PDF of S-D link is independent of the number of candidates and its channel gain follows the exponential distribution. Therefore, the SER of directly transmission from source to destination can be evaluated as

$$P_e(s,d)=\int_0^{\infty}P_e(\beta)p_{s,d}(\beta)d\beta=\frac{1}{\pi}\int_0^{\frac{(M-1)\pi}{M}}\frac{\sin^2\theta}{\sin^2\theta+b\delta_{s,d}^2}d\theta. \tag{13}$$

Then, the closed-form expression of (13) can be simply obtained as

$$P_e(s,d)=F_1\left(1+\frac{b\delta_{s,d}^2}{\sin^2\theta}\right). \tag{14}$$

C. For the conditional SER for cooperative transmission, $P_e\left(s,r_i,d|k_1,k_2\right)$

In this case, the conditional SER for cooperative transmission at D when each candidates are k_1 and k_2 can be formulated by considering two cases (s.t. 'good' or 'better')

$$\begin{aligned} &P_e(s,r_i,d|k_1,k_2)\\ &=P\left[\beta_{k_1^*}>\beta_{k_2^*}|K_1=k_1,K_2=k_2\right]\int_0^{\infty}P_e(\beta)p_{s,r_{1,i},d}(\beta)d\beta\\ &\quad+P\left[\beta_{k_1^*}<\beta_{k_2^*}|K_1=k_1,K_2=k_2\right]\int_0^{\infty}P_e(\beta)p_{s,r_{2,i},d}(\beta)d\beta. \end{aligned} \tag{15}$$

In (15), we need to evaluate two integral terms. For the first integral term, it can be evaluated by performing the integration of the exponential function over β. Therefore, we can obtain the closed-form expression of (15) as

$$\int_0^{\infty}P_e(\beta)p_{s,r_{1,i},d}(\beta)d\beta =F_1\left(\left(\frac{e^{\frac{\beta_{th_1}}{\delta_{r,d}^2}}-e^{\frac{\beta_{th_2}}{\delta_{r,d}^2}}}{e^{-\frac{b\beta_{th_2}}{\sin^2\theta}+\frac{\beta_{th_1}}{\delta_{r,d}^2}}-e^{-\frac{b\beta_{th_1}}{\sin^2\theta}+\frac{\beta_{th_2}}{\delta_{r,d}^2}}}\right)\left(1+\frac{b\delta_{r,d}^2}{\sin^2\theta}\right)\left(1+\frac{b\delta_{s,d}^2}{\sin^2\theta}\right)\right). \tag{16}$$

For the second integral term, similar to (16), we can obtain the closed-form expression as

$$\int_0^{\infty}P_e(\beta)p_{s,r_{2,i},d}(\beta)d\beta=F_1\left(e^{\frac{b\beta_{th_2}}{\sin^2\theta}}\left(1+\frac{b\delta_{r,d}^2}{\sin^2\theta}\right)\left(1+\frac{b\delta_{s,d}^2}{\sin^2\theta}\right)\right). \tag{17}$$

In (15), the probability where the R-D link of the selected relay is 'good' can be evaluated as

$$P\left[\beta_{k_1^*}>\beta_{k_2^*}|K_1=k_1,K_2=k_2\right]=\int_0^{\infty}\frac{k_1}{\delta_{s,r_i}^2x^2}e^{-\frac{1}{\delta_{s,r_i}^2x}}\left(1-e^{-\frac{1}{\delta_{s,r_i}^2x}}\right)^{k_1-1} \left(\int_{x-A}^{\infty}\frac{k_2}{\delta_{s,r_i}^2y^2}e^{-\frac{1}{\delta_{s,r_i}^2y}}\left(1-e^{-\frac{1}{\delta_{s,r_i}^2y}}\right)^{k_2-1}dy\right)dx, \tag{18}$$

where $A=\frac{q_2}{q_1}\left(\frac{1}{\beta_{th_2}}-\frac{1}{\beta_{th_1}}\right)$. Here, by applying the binomial theorem to the inner integral term and then with the help of Taylor series expansions of exponential functions [13], with the help of the integral identity [14, Eq. (07.33.07.0001.01)], we can finally obtain the closed-form expression of (18) as

$$P\left[\beta_{k_1^*}>\beta_{k_2^*}|K_1=k_1,K_2=k_2\right]=\sum_{l=0}^{k_2-1}\binom{k_2-1}{l}\frac{(-1)^lk_2}{1+l} \times\left[1+\sum_{j=0}^{k_1-1}\sum_{n=0}^{\infty}\binom{k_1-1}{j}\frac{k_1(-1)^{j+1}}{1+j}A^{-n}U\left(n,0,\frac{-1-j}{A\delta_{s,r_i}^2}\right)\left(\frac{1+l}{\delta_{s,r_i}^2}\right)^n\right]. \tag{19}$$

Similarly, with (19), the probability where the R-D link of the selected relay is 'better' in (15) can be evaluated as

$$P\left[\beta_{k_1^*}<\beta_{k_2^*}|K_1=k_1,K_2=k_2\right]=1-P\left[\beta_{k_1^*}>\beta_{k_2^*}|K_1=k_1,K_2=k_2\right]. \tag{20}$$

IV. NUMERICAL RESULTS

In this section, as a validation of our analytical results for the SER performance, we compare in Figure 2 the analytical results with the simulation results obtained via Monte-Carlo simulation over i.i.d. Rayleigh fading channels. Here, for the fair comparison of the SER performance between 1-bit and 2-bits feedback based schemes, we consider the equal power allocation and the fixed threshold, especially to show the effect of more refined feedback information on the SER performance. Note that the derived analytical results match the simulation results and we believe that it is available to accurately predict the performance with them.

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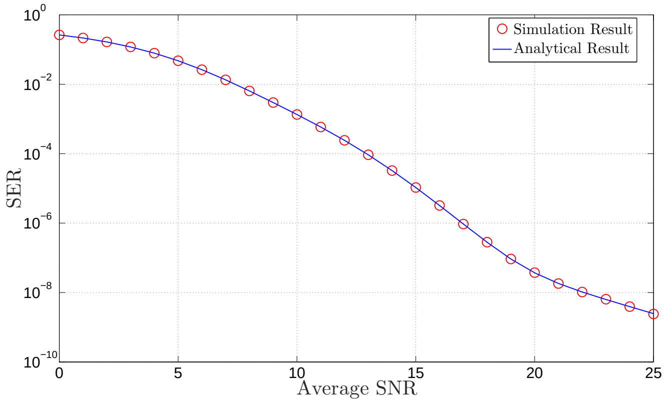

Figure 2. Performance comparison between the simulation and analytical results with $N = 6$, $\beta_{th_1} = 10$dB, $\beta_{th_2} = 12$dB, and $\overline{\gamma}_{SR} = \overline{\gamma}_{RD} = \overline{\gamma}_{SD} = \overline{\gamma}$.

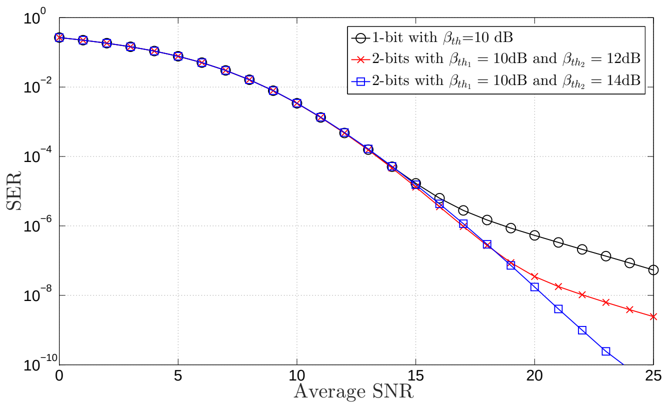

Figure 3. SER performance with varying threshold values and $\overline{\gamma}_{SR} = \overline{\gamma}_{RD} = \overline{\gamma}_{SD} = \overline{\gamma}$ for $N = 6$.

Figure 3 shows that as our original expectation, the proposed ATRS scheme with 2-bits feedback information achieves better performance than 1-bit feedback based scheme. More specifically, the possibility that the 2-bits feedback based scheme can provide the better performance compared to the 1-bit feedback based scheme is high because through one additional bit in terms of a feedback information and one additional comparison operation in terms of the complexity, the 2-bits feedback based scheme has the higher ability to compensate the potential performance degradation. Further, if we consider that the S-R link condition of each candidates selected from both 'better' and 'good' candidate groups is similar, the performance improvements of the 2-bits feedback based scheme is getting increasing compared to the 1-bit feedback based scheme as the threshold, β_{th_2}, increases.

V. Conclusions

In this paper, we analyzed the SER performance of the extended ATRS scheme based on '2-bits feedback' information as closed-form expressions. For validation of analytical results, derived analytical results were cross-verified with results obtained via Monte-Carlo simulations. Based on some selected results, we confirmed that with more refined feedback information, the potential performance degradation caused by the 1-bit feedback can be compensated, especially with a 1-bit increase in terms of feedback data rate and additional single comparison process in terms of complexity.

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