# Efficient Adaptive Equalizer Combined with LDPC Code for Vehicular Communications

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***Abstract*—Low density parity check (LDPC) code is very powerful for the error correction in the communication systems. So, LDPC code has been adopted for the standard of IEEE 802.11p (vehicular ad-hoc networks) and digital video broadcasting-satellite2 (DVB-S2). Features of LDPC code are iterative decoding and sparse parity check matrix. However, when multipath fading channel is considered, LDPC code needs long parity check matrix and pretty big iteration number of decoding. We propose a fast and efficient adaptive equalizer with LDPC code for compensating the carrier frequency offset (CFO) and phase noise in the vehicular environments. In addition, this proposed system has less iterations than the conventional system without combining structure. So, the proposed system has lower complexity. In this paper, we investigate the system model and evaluate BER performance for the vehicular environments by the simulation results, considering the CFO and phase noise in the OFDM system.**

***Keywords-LDPC; adapitve equalizer; CFO; phase noise; OFDM; Sparse Parity Check Matrix; iteration number.***

## I. INTRODUCTION

There are some impairments degrading the OFDM communication system. One is phase noise caused by the oscillator in transceiver; another is CFO (carrier frequency offset). These components produce the ICI (inter-subcarrier interference). Until now, in fact, there are many methods to compensate CFO [1]-[2] or phase noise [3]-[5] in OFDM system. Therefore, we can overcome ICI effect by using forward error correction code and equalization in OFDM system [6]-[12].

Especially, LDPC code is well known as the good performance channel coding method. LDPC code was firstly proposed by Robert G. Gallager in 1962 [13] and re-considered by Mackay and Neal in 1997 [14] [15]. LDPC is the error correction code which shows the closest performance to Shannon's limit [14]. Recently, LDPC code has been commonly used in such as Mobile WiMax and DVB-S2. LDPC code has highly error correction performance at low Signal to Noise Ratio (SNR) environment and is able to encode fast by using sparse parity check matrix. However, when multipath fading channel is considered, LDPC code needs long sparse parity check matrix and iteration number of decoding.

In this paper, we propose an adaptive equalizer combined with LDPC code. The proposed system requires a smaller iteration number of LDPC decoding than the system without combining. In addition, this combined adaptive equalizer can compensate ICI effect which degrades the BER performance, such as CFO and expand ICI in OFDM system[1]-[5]. So, we consider about CFO and phase noise effects in OFDM system. At first, we analyze the basic principle of LDPC code and frequency domain equalizer [6]-[12]. After that, we evaluate BER performance of the combined adaptive equalizer for vehicular environments.

## II. SYSTEM MODEL

Fig. 1 shows the system model of the combined adaptive equalizer with LDPC code in OFDM system [6]. We use LDPC code and channel estimator to estimate channel characteristic in OFDM communication system. It can eliminate ISI (inter-symbol interference) effects by using CP (cyclic prefix). Then, it can estimate the channel characteristic through the long preamble and equalize the channel fading. Especially, we design and propose a feedback loop composed up of LDPC decoder to adaptive equalizer. That is, adaptive equalizer is combined with LDPC decoder iteratively. The combined equalizer calculates the control factor in feedback loop iteratively. So, we can call adaptive equalizer combined with LDPC code.

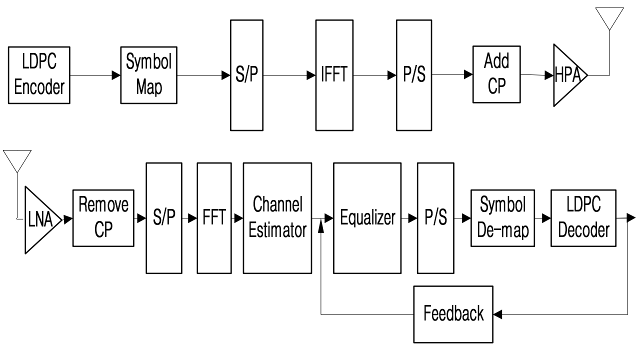


Figure 1. System model of the adaptive equalizer.

## III. LOW DENSITY PARITY CHECK CODE

LDPC code is one of the block coding methods. It can be expressed as $(N, K)$. It has a sparse parity check matrix (H matrix) of $(N-K)\times N$ size. H matrix is non-systematic sparse parity check matrix.

### A. LDPC Encoding

The received vector is corrupted by an error vector $\mathbf{e}$ as follows [16].

$$\mathbf{r}=\mathbf{c}\oplus\mathbf{e}=\begin{bmatrix}\mathbf{p} & \mathbf{m}\end{bmatrix}\oplus\mathbf{e}, \quad (1)$$

where $\mathbf{p}$ is parity vector and $\mathbf{m}$ is message vector. The parity vector and message vector are assumed to be located at the former or latter part of the code vector, respectively. The decoder of the receiver is supposed to apply for the received signal vector to find the syndrome vector as [16]:

$$\begin{aligned}\mathbf{s}=\mathbf{r}H^T &= \left(\begin{bmatrix}\mathbf{p} & \mathbf{m}\end{bmatrix}\oplus\mathbf{e}\right)\begin{bmatrix}H_1^T\\ H_2^T\end{bmatrix}. \\ &=\mathbf{p}H_1^T\oplus\mathbf{m}H_2^T\oplus\mathbf{e}H^T\end{aligned} \quad (2)$$

Noting that this syndrome vector should be zero for the non-error case, that is, $\mathbf{e}=0$.

$$\mathbf{s}=\mathbf{p}H_1^T\oplus\mathbf{m}H_2^T=\mathbf{0}. \quad (3)$$

We can write the parity vector $\mathbf{p}$ in terms of the message vector $\mathbf{m}$ as [16]

$$\mathbf{p}=\mathbf{m}H_2^T H_1^{-T}. \quad (4)$$

This amounts to the generator matrix

$$G=\begin{bmatrix}H_2^T H_1^{-T} & I\end{bmatrix}, \quad (5)$$

so the code vector can be generated by post-multiplying the message vector $\mathbf{m}$ with the generator matrix G as [16]

$$\mathbf{c}=\mathbf{m}G=\begin{bmatrix}\mathbf{m}H_2^T H_1^{-T} & \mathbf{m}\end{bmatrix}=\begin{bmatrix}\mathbf{p} & \mathbf{m}\end{bmatrix}. \quad (6)$$

### B. LDPC Decoding

The LDPC decoder calculates the probability in variable nodes and check nodes, respectively [17] [18]. Decoding algorithms can be divided into sum-product (SP) algorithm and min-sum (MS) algorithm. SP algorithm has better decoding performance, but higher complexity. So, we choose SP algorithm to decode because of decoding performance. The LDPC decoder operates through 4 steps. First step is the initializing step. Second step is the updating check nodes step. Third step is the updating variable nodes step. Final step is decision step.

## IV. Channel Estimation and Equalization

A frequency-domain training sequence is

$$X(k)=X_R(k)+jX_I(k) \quad (7)$$

with the corresponding channel output.

$$Y(k)=Y_R(k)+jY_I(k). \quad (8)$$

Channel estimation with $X(k)$ and $Y(k)$ is

$$H_p=\frac{Y(k)}{X(k)} \quad (9)$$

The long preamble used for channel estimation is

$$X(k)=1 \quad \text{or} \quad -1(\text{with} \quad X_I(k)=0) \quad (10)$$

The channel estimation signal can be simplified as

$$H_p=\frac{Y(k)}{X(k)}=\frac{Y_R(k)+jY_I(k)}{X_R(k)} \quad (11)$$

Since the long preamble contains two repeated training sequences, the average of the FFT ($Y_1(k)$ and $Y_2(k)$) of the channel outputs can be taken for the better channel estimation.

$$H_p=\frac{1}{2}\frac{Y_1(k)+Y_2(k)}{X(k)} \quad (12)$$

So, the estimated channel can equalize the output to compensate the channel effect.

$$\hat{X}(k)=\frac{Y'}{\hat{H}_p} \quad (13)$$

## V. Proposed Adaptive Equalizer Combined with LDPC Code

Fig. 2 shows the feedback system of the combined adaptive equalizer in the receiver side. Fig. 2 shows subsystem after FFT operation in OFDM receiver side. Original equalization type is the zero forcing, but the combined equalizer with LDPC decoder is almost same to the decision feedback equalizer because it performs feedback loop between equalizer and LDPC decoder iteratively. So, we call as adaptive equalizer combined with LDPC code.

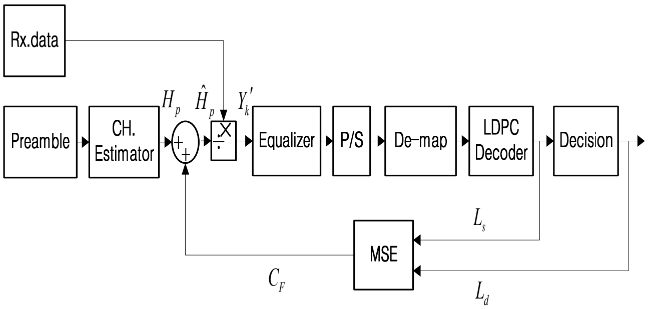


Figure 2. Block diagram of combined adaptive equalizer in receiver.

As shown in Fig. 2, after through LDPC decoder, the proposed system feeds $C_F$ back to adaptive equalizer from LDPC decoder to compensate channel estimation with $L_s$ and $L_d$ . $L_s$ and $L_d$ are decoder output and decision bit, respectively.

$$\tilde{L}_s = norm(L_s) \tag{14}$$

$$\hat{L}_k = \sum_{k \in s_d} \tilde{L}_s^{*} L_d \tag{15}$$

$$C_F = \frac{\hat{L}_k}{\sum_{k \in s_d} \left|\tilde{L}_s\right|^2} \tag{16}$$

$\hat{H}_p$ is the channel characteristic after estimation by the long preamble adding $C_F$ .

$$\hat{H}_p = H_p + C_F \tag{17}$$

We can improve performance by the following equation with compensated channel $\hat{H}_p$ .

$$Y_k' = \frac{Y_k}{\hat{H}_p} \tag{18}$$

Therefore, the combined adaptive equalizer can achieve better performance like the decision feedback equalizer. Basically, equalization type is based on the zero forcing, but the combined adaptive equalizer with LDPC decoder is almost same to the decision feedback equalizer because of feedback loop between equalizer and LDPC decoder.

## VI. SIMULATION RESULTS

Table 1 shows simulation parameters for the combined adaptive equalizer with LDPC code. In this paper, we consider IEEE 802.11p format. Code rate is 3/4 and parity check matrix size is 720. At first, we generate 78 bits per data block. We change block size to 540 bits through the buffer and encode as 720 bits. Then, we change again to 104 bits in order to modulate through the buffer. Finally, we can output as 52 data symbols. We consider the AWGN and multipath channel. The multipath channel considers the ITU vehicular channel model. Furthermore, we adapt the normalized CFO and phase noise power as 0.03 and -12dBc, respectively.

TABLE I. SIMULATION PARAMETERS.

| Parameters | Values |
|---|---|
| # of data subcarriers | 52 |
| # of pilot subcarriers | 4 |
| # of padded zeros | 7 |
| FFT size | 64 |
| # of samples in a GI | 16 |
| Modulation level | 4QAM |
| Multipath channel | ITU vehicular channel A |

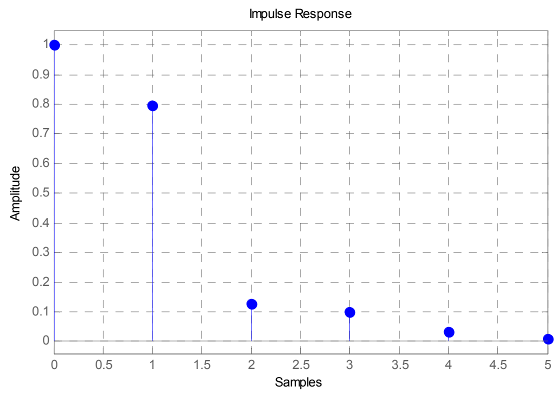


Figure 3. Impulse response of considered multipath channel.

Fig. 3 shows the impulse response of the considered multipath channel. This multipath channel is called as ITU-Vehicular channel A. The multipath channel can be simplified in the digital FIR filter model. Overall 5 delay paths may exist.

Fig. 4 shows BER curve according to iteration number of the LDPC decoding in AWGN channel. The iterations is higher, BER performance can be improved.

Fig. 5 shows BER performance of the combined adaptive equalizer and without channel estimation and equalization. The combined adaptive equalizer has much better performance than without channel estimation and equalization over considered CFO and phase noise effects. When iteration number is 5, the SNR is less than 5dB at $10^{-5}$ error probability. So, the combined adaptive equalizer is able to compensate the CFO and phase noise.

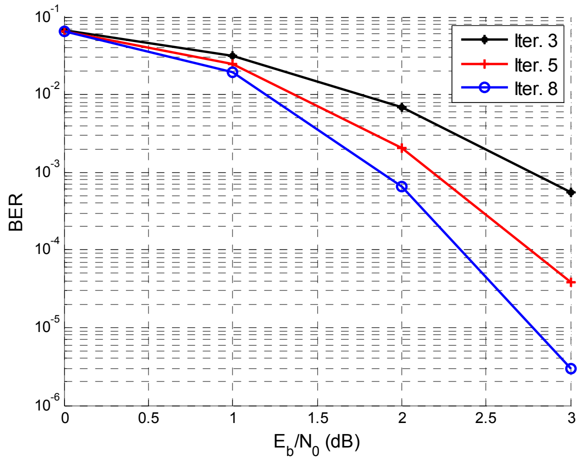


Figure 4. BER performance according to iteration number in AWGN.

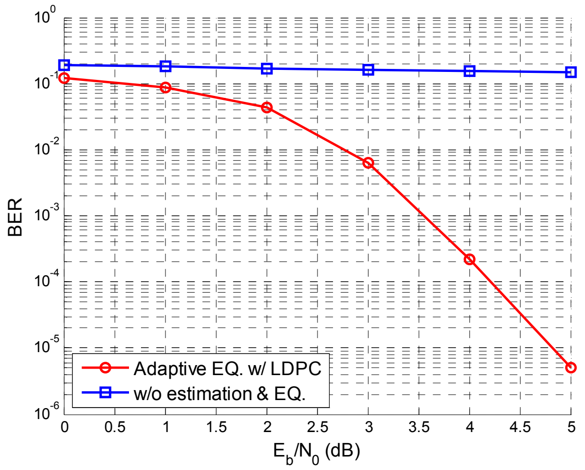


Figure 5. Comparison BER performance of the combined adaptive equalizer and BER without channel estimation and equalization.

Fig. 6 shows BER performance of the combined adaptive equalizer and without combining system under same iteration number situation. Performance of the combined adaptive equalizer is better than without combining system. The combined adaptive equalizer is better performance about 0.5dB at $10^{-4}$ when iteration number is 5.

Fig. 7 shows closely similar BER performance between the combined adaptive equalizer and without combining system at same iterations. When SNR is 3.5dB, BER of the combined adaptive equalizer and without combining system is exactly same. Where, iteration number of the combined adaptive equalizer is 3, but without combining system is 5. Because complexity of system is closely related to iteration number, the combined adaptive equalizer has lower complexity than without combining system.

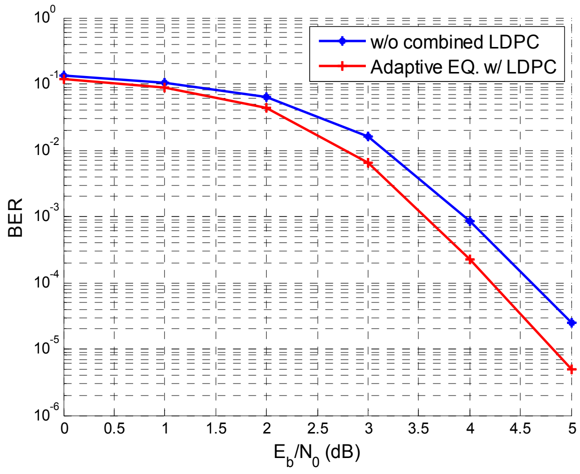


Figure 6. BER performance of the combined adaptive equalizer and BER without combining.

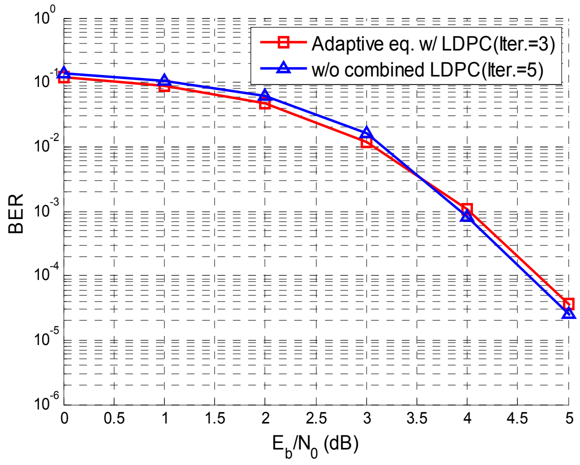


Figure 7. Comparison BER performance of adaptive equalizer combined with LDPC and BER without combining.

## VII. CONCLUSION

We propose an efficient adaptive equalizer combined with LDPC code to compensate the CFO and phase noise in vehicular environments. To estimate the channel, combined adaptive equalizer uses the long preamble. After passing through the LDPC decoder, the proposed system calculates $C_F$ with $L_s$ and $L_d$. Then, it feeds $C_F$ back into the equalizer from the LDPC decoder. So, BER performance of the combined adaptive equalizer can be improved. On the other hands, the combined adaptive equalizer can achieve lower complexity when it has same performance. The combined adaptive equalizer has good BER performance with CFO and phase noise. As a result of Fig. 5, the combined adaptive equalizer shows better BER performance than without combining. Also, as a result of Fig. 6, iteration number of combined adaptive equalizer is lower than the system without combining when both of performances are closely similar. That is, complexity is lower than the system without combining because complexity is related to iterations of the LDPC decoder. So, the combined adaptive equalizer can compensate CFO and phase noise through combining in OFDM system. Therefore, it can improve BER performance and reduce complexity of the LDPC decoder. Additionally, we can expect implementation of the combined adaptive equalizer for vehicular communications.

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