

# Event-triggered Robust Output Feedback Controller for a Networked Roll Control System

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**Abstract**—Development of control systems that help to reduce the danger in risky driving situations while also improving stability and comfort are one of the main research focus nowadays. The use of low-cost sensors integrated in these systems is important for their commercialisation. Also, more complex architectures require a larger number of sensors and connections. The delays generated by these sensors, in addition to those of the network, must be addressed to provide robust control. The novelty of this paper is a robust  $H_\infty$  Output Feedback Controller based on an active suspension system for roll control that takes into account the effect of the delays in the network. An event-triggering condition is included in order to not saturate the network.

**Keywords**— $H_\infty$  control; event-triggering; networked control systems; roll stability; active suspension

## I. INTRODUCTION

The development of advanced vehicle safety systems is a major research focus in the road transport sector today. These systems can be divided into active and passive systems. Passive systems attempt to minimise the damage that vehicle occupants may suffer in an accident from a mitigating perspective. Active systems act to prevent a potential accident by taking control of vehicle dynamics. The latter requires sensors to monitor the dynamic state of the vehicle, as well as controllers that provide a fast and reliable response.

One of the main causes of fatal accidents in road transport is rollover. Danger of rollover becomes even greater in vehicles with a higher centre of gravity, such as SUVs, vans or trucks. For this reason, previous work has focused on designing rollover controllers to achieve a certain vehicle behaviour. Vehicle rollover stability can be improved through different control strategies, such as four wheel steering, differential braking, active suspension, or active stabilizer. In [1], a robust rollover risk suppression controller is proposed by reducing lateral acceleration through a steer-by-wire system. In [2], an integrated controller based on active steering and braking is proposed, taking the expected path of the driver to achieve better anti-rollover effect into account. A differential braking rollover mitigation control strategy through a polytopic description of the vehicle is presented in [3]. In [4], an  $H_\infty$  controller of roll stability is proposed, taking into account the effect of delays and noise of low cost sensors. Overall, the

forementioned studies were good, however a control signal was generated even when not necessary, as no rules were defined in order not to update it when the system does not change substantially.

Accurate knowledge of the variables involved in rollover dynamics is key to designing reliable controllers that help improve vehicle stability and save lives. Some of them, such as roll angle and sideslip angle, cannot be measured directly by sensors installed in series production vehicles due to their high cost. Previous work analysed the use of low-cost sensors to estimate roll angle using different methods [5] [6] which, while demonstrating good performance, could present relevant noise and delays related to the low-cost nature of the architecture.

In real control systems, the plant, controller, sensors and actuators are usually located far from each other. Therefore, signals must be sent through a communication network. Networked Control Systems (NCSs) bear with delays generated in transmission process which affect the system. In autonomous or intelligent vehicles, the architecture network can increase its complexity significantly and higher bandwidth is required in order to not saturate the network. This leads to an increment of the delay between sensors and controllers which could lead to an unstable system behaviour. Therefore, an event-triggering condition can be included in the controller design so that the amount of data transmitted over time is reduced. This condition, which is evaluated at every sampled time step, determines if the state of the system has changed significantly enough. Only in that case, the observed variables from the sensors are transmitted through the network. The event-triggering method has proven to be an useful way to avoid saturating the network by not sending data packets through it unless is strictly necessary [7] [8].

In order to compensate the delays in the network and the noise in sensor signals, a robust control design is required to improve the performance of the system. Robust LMI-based  $H_\infty$  control is a good option for this kind of problems, as proven in previous works [9] [10].

The novelty of this paper is the development of a robust  $H_\infty$  Output Feedback Controller for a NCSs that takes into account the effect of the delays over the network. An event-triggering

condition mechanism is included in order to not saturate the network. Lyapunov-Krasovskii functional approach and LMI restrictions are used to guarantee system stability and  $H_\infty$  criteria. The controller is based on an active suspension system, taking the anti-roll moment as control input and the roll rate as measured data.

The document is structured as follows. Section II presents the mathematical formulation of the problem, introducing the vehicle model used for the design of the controller and the control algorithm. Section III includes the simulation results where the proposed system is compared with the vehicle without control. Section IV states the conclusion of this study.

## II. PROBLEM FORMULATION

This section describes the problem of event-triggered  $H_\infty$  Output Feedback Controller for roll stability. The diagram of the network communication and control sequence is depicted in Figure 1. Every component is described in the following subsections.

### A. Vehicle model

The vehicle model used in the design of the  $H_\infty$  Output Feedback Controller describes the roll vehicle motion as seen in Figure 2, where  $\phi$  is the roll angle and  $a_y$  is the lateral acceleration.

$$\dot{x}(t) = Ax(t) + B_u \tilde{u}(t) + B_{a_y} a_y(t) + B_{\phi_r} \phi_r(t) + B_d d_s(t) \quad (1)$$

$$y(t) = C_1 x(t) \quad (2)$$

$$z(t) = C_2 x(t) \quad (3)$$

with  $x = [\phi, \dot{\phi}]^T$  as the state vector defined by the roll angle  $\phi$  and roll rate  $\dot{\phi}$ ,  $y(t) = \phi$  is the observed measurement from the system,  $z(t)$  is the control output to be minimized though the event-triggered control input  $u = M_x$ , where  $M_x$  is the anti-roll moment provided by actuators located at every wheel suspension.  $a_y$  is the lateral acceleration of the vehicle,  $\phi_r$  is the road bank angle and  $d_s$  is the unknown vector disturbance.  $a_y$ ,  $\phi_r$  and  $d_s$  are an input disturbance to the system, which can be represented as  $\omega(t) = [a_y(t) \ \phi_r(t) \ d_s(t)]$ , with an input

disturbance matrix  $B_\omega = [B_{a_y} \ B_{\phi_r} \ B_d]$ , leading the system presented in (1) to

$$\dot{x}(t) = Ax(t) + B_u \tilde{u}(t) + B_\omega \omega(t) \quad (4)$$

$$y(t) = C_1 x(t) \quad (5)$$

$$z(t) = C_2 x(t) \quad (6)$$

### B. $H_\infty$ Output Feedback Controller

The control input is defined as follows

$$u(t) = Ky(t) \quad (7)$$

where  $K$  is the control gain matrix to be determined so that, the closed-loop  $H_\infty$  performance of the closed-loop system (1) is guaranteed, which is satisfied if the following inequality remains true

$$\|z^T(t)z(t)\|_2 < \gamma^2 \|\omega^T(t)\omega(t)\|_2 \quad (8)$$

### C. Event-triggering mechanism

In order to reduce the Transmission Rate (TR), the following event-triggered rule is defined so that the control output is sent to the actuators only when necessary

$$e^T(t)K^T\Omega Ke(t) \leq \varepsilon^2 \tilde{u}^T(t)\Omega \tilde{u}(t) \quad (9)$$

where  $e(t)$  is the difference between the current vehicle measurement and the measurement from the last triggering instant,  $\tilde{u}(t)$  is the last transmitted control input,  $\varepsilon$  is a user defined threshold and  $\Omega > 0$  is a positive definite matrix to be designed

### D. Network communication

The observed measurement  $y(t)$  from the vehicle is sampled every  $h$  milliseconds, then the event-triggering mechanism decides whether to neglect this information or to update the control input that the actuators must supply. Every time the event-triggering mechanism sends new information through the communication network, this presents a maximum delay of  $\tau_M$  milliseconds. When the system state varies significantly, actuators will react to stabilize it after a maximum period of  $\tau = h + \tau_M$  milliseconds.

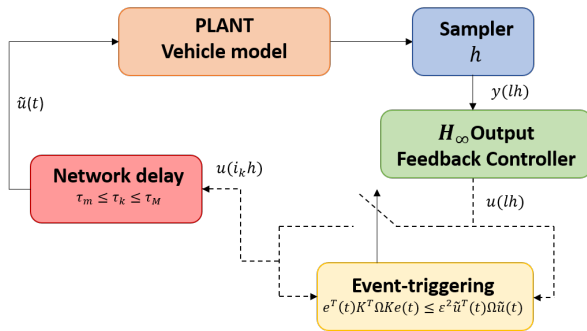


Fig. 1. Diagram of the networked control system.

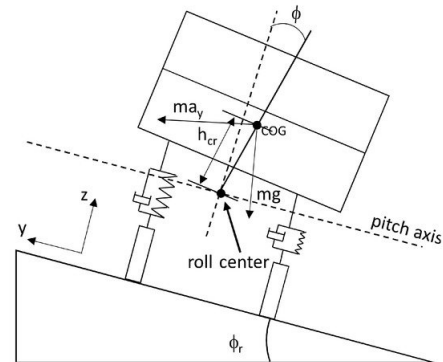


Fig. 2. Vehicle model used in the controller.

### E. Lyapunov stability

In order to guarantee the system stability, the following Lyapunov-Krasovskii functional is chosen

$$V(t) = V_1(t) + V_2(t) + V_3(t) \quad (10)$$

where

$$\begin{aligned} V_1(t) &= x^T(t)Px(t) \\ V_2(t) &= \int_{t-\tau}^t \dot{x}^T(s)C_1^T K^T SKC_1 x(s)ds \\ V_3(t) &= \int_{t-\tau}^t (s - (t - \tau))\dot{x}^T(s)C_1^T K^T SRC_1 \dot{x}(s)ds \end{aligned} \quad (11)$$

$$(12)$$

with symmetrical positive matrices  $P > 0$ ,  $R > 0$ ,  $S > 0$  to be determined such that

$$\dot{V}(t) < 0 \quad (13)$$

### III. SIMULATION RESULTS AND DISCUSSION

The control gain  $K$  is obtained through the minimization problem

$$\min \gamma^2 \quad (14)$$

$$\text{subject to } P > 0, R > 0, S > 0, \Omega > 0 \quad (15)$$

A feasible solution was found using the MATLAB LMI Toolbox, with the matrices

$$\begin{aligned} \gamma^2 &= 16.96 \\ P &= \begin{bmatrix} 14.306391 & 0.607416 \\ 0.607416 & 0.464377 \end{bmatrix} \\ R &= 3.4 * 10^{-9} \\ S &= 1.25 * 10^{-10} \\ K &= -13552.53 \\ \Omega &= 1 \\ \epsilon^2 &= 0.1 \end{aligned}$$

The sampling period defined for the observer in this study is  $h = 20$  ms and the maximum and minimum delays over the network are assumed as  $\tau_M = 20$  ms and  $\tau_m = 10$  ms respectively, which are common times for data sampling and communication delays [4] [7]. It is not expensive to use data logging devices with higher sampling rates, however, a sampling frequency of 50 Hz is enough for this study.

For the validation of the proposed controller, the CarSim software is used together with Simulink, where the following experiments were set

- 1) A double line change at a speed of 100 km/h
- 2) A double line change at a speed of 120 km/h, in order to study the performance in a more severe test

The results for the first test are depicted through Figures 3-10. Figures 3 and 5 show the roll rate and angle obtained from this simulation, where it can be seen that the proposed controller achieves a better performance, as the roll rate and angle are lowered through time. These results are summarized

in Table I. Figures 4 and 6 compare the Power Spectral Density (PSD) of the roll rate and angle respectively, where the proposed controller enhances the lateral behaviour, as the PSD is lowered for all frequencies. The anti-roll moment provided by the actuators is depicted in Figure 7. The Event Triggering instants are depicted in Figure 10; the event-triggering mechanism achieves a TR of 29.14% for this experiment. The results for the second test are depicted through Figures 11-18 and summarized in Table II. Once again, the controlled system achieves a better behaviour than the system without control. The TR is 25.33% for this case, which proves that the network usage is reduced.

Normalized load transfer (NLT) was calculated in addition to the above mentioned results. This variable is an accurate measure for evaluating roll stability control systems. It guarantees that the vehicle will not roll over while its value for both axles is within the  $\pm 1$  range. The normalized load transfer can be calculated as:

$$NLT_f = \frac{\Delta F_{zf}}{F_{zf}}, NLT_r = \frac{\Delta F_{zr}}{F_{zr}} \quad (16)$$

where  $F_{zf}$  and  $F_{zr}$  are the total load on the front and rear axle, respectively. Values of NLT for experiments 1 and 2 are depicted through Figures 8-9 and Figures 16-17, respectively. NLT remains within the determined range in all experiments and a decrease of the NLT is observed for both experiments at the front axle, which demonstrates the effectiveness of the controller.

Overall, the vehicle's dynamic response shows a smoother behaviour, ensuring safety and comfort inside the vehicle. Roll angle and rate have been significantly decreased compared to [4], and the TR has been reduced as we have implemented an event triggering mechanism which is more flexible than the one presented in [7] and facilitates the design of the controller.

### IV. CONCLUSIONS

An output-feedback event-triggered  $H_\infty$  controller that enhances the lateral behaviour of the vehicle has been introduced in this paper. A Lyapunov-Krasovskii functional is chosen to assure the system stability despite the affection of communication delays. An event-triggering rule is designed in order to reduce the communication resources. The controller is designed under a guaranteed  $H_\infty$  performance index  $\gamma$ , which represents the impact of the disturbances on the roll rate and angle. To analyze the performance of the proposed controller, the RMS and MAX values of the roll rate and angle are compared, leading to a reduction of up to 50% of the roll rate and angle in the worst cases. The Event-Triggered mechanism reduces the use of network communication resources usage by up to 70%.

TABLE I  
ROLL BEHAVIOUR OF THE VEHICLE UNDER THE EXPERIMENT 1

	RMS Roll Rate (°/s)	MAX Roll Rate (°/s)	RMS Roll Angle (°)	MAX Roll Angle (°)
Passive system	3.57	4.15	1.97	3.31
Active system (proposed)	1.70	3.31	1.14	2.22

TABLE II  
ROLL BEHAVIOUR OF THE VEHICLE UNDER THE EXPERIMENT 2

	RMS Roll Rate (°/s)	MAX Roll Rate (°/s)	RMS Roll Angle (°)	MAX Roll Angle (°)
Passive system	4.13	11.05	2.18	3.27
Active system (proposed)	2.21	5.38	1.30	2.26

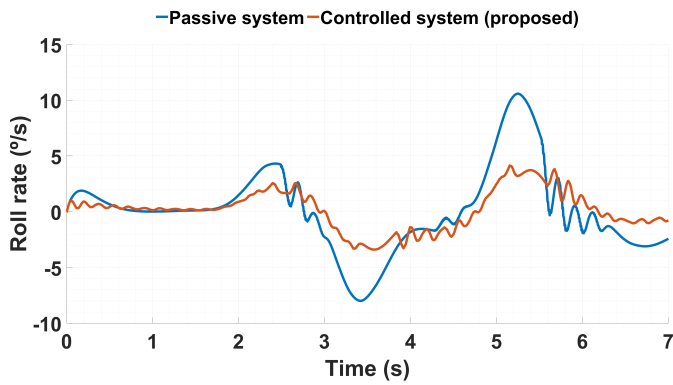


Fig. 3. Roll rate under the experiment 1

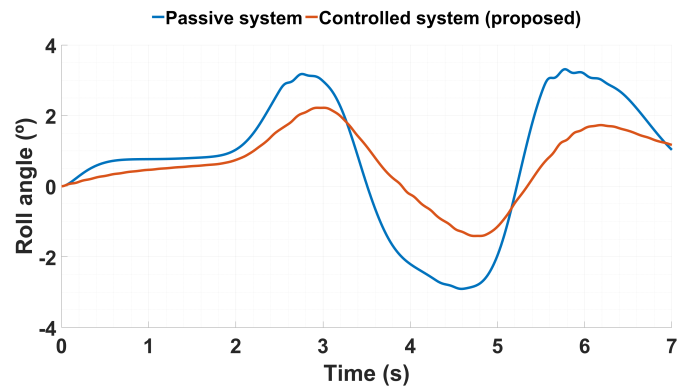


Fig. 5. Roll angle under the experiment 1

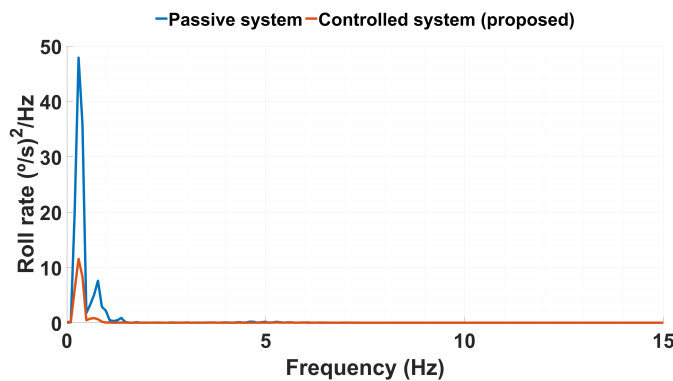


Fig. 4. Roll rate PSD under the experiment 1

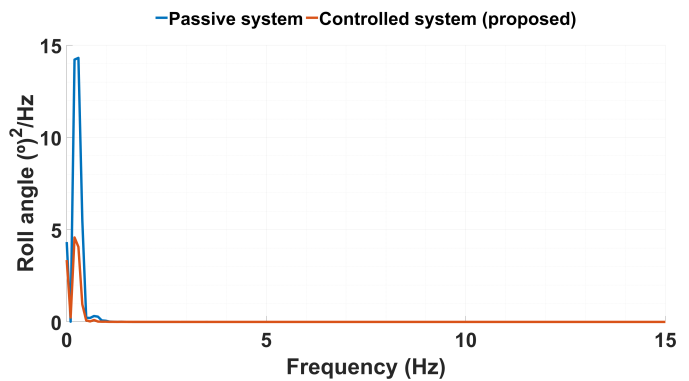


Fig. 6. Roll angle PSD under the experiment 1

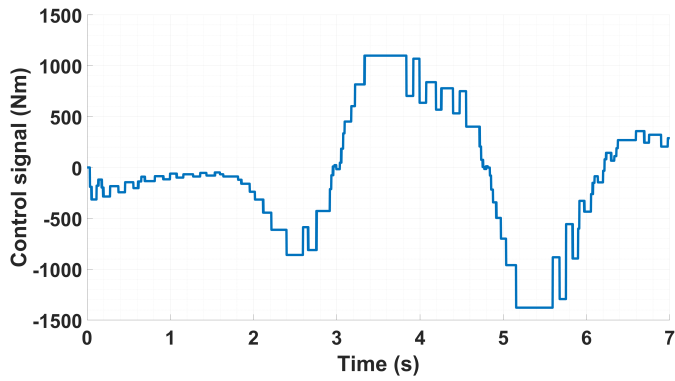


Fig. 7. Control signal under the experiment 1

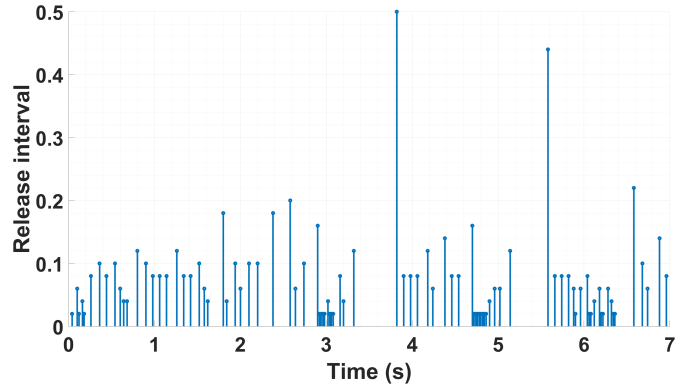


Fig. 10. Event Triggering instants under the experiment 1

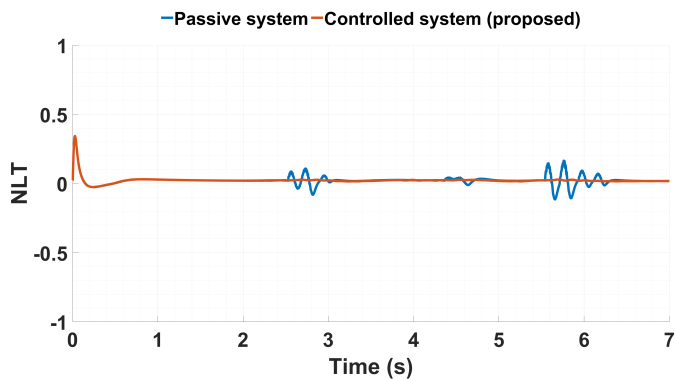


Fig. 8. NLT of front axle under the experiment 1

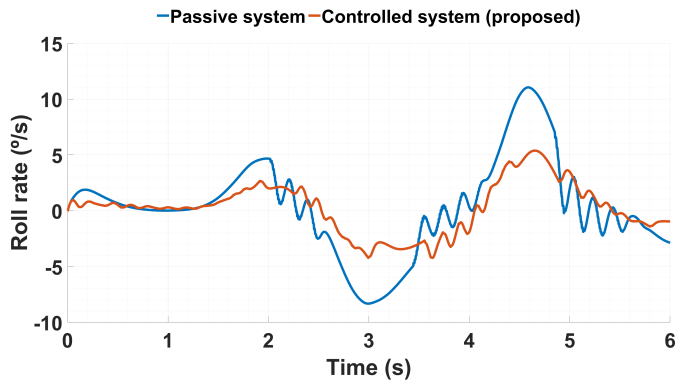


Fig. 11. Roll rate under the experiment 2

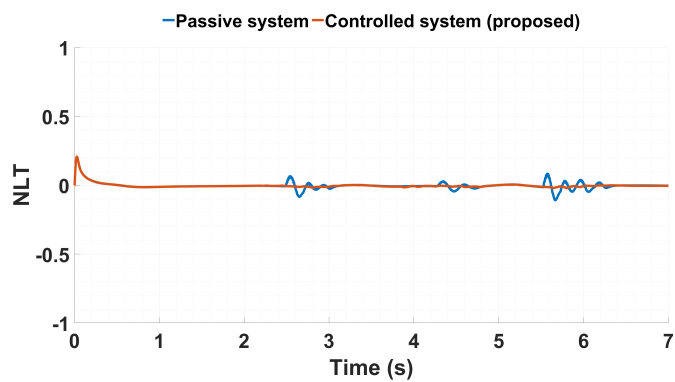


Fig. 9. NLT of rear axle under the experiment 1

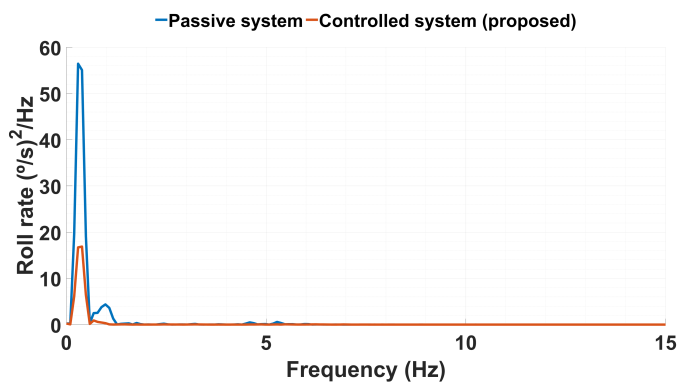


Fig. 12. Roll rate PSD under the experiment 2

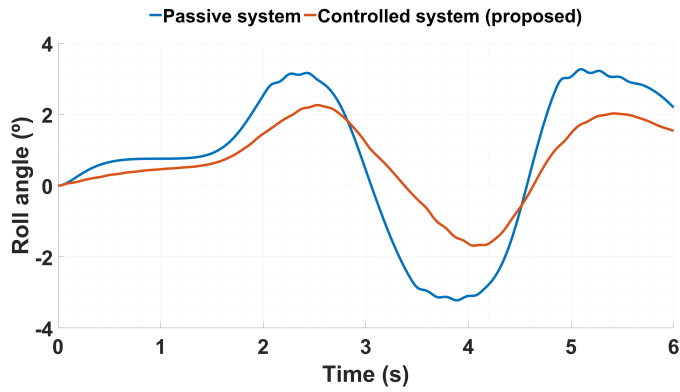


Fig. 13. Roll angle under the experiment 2

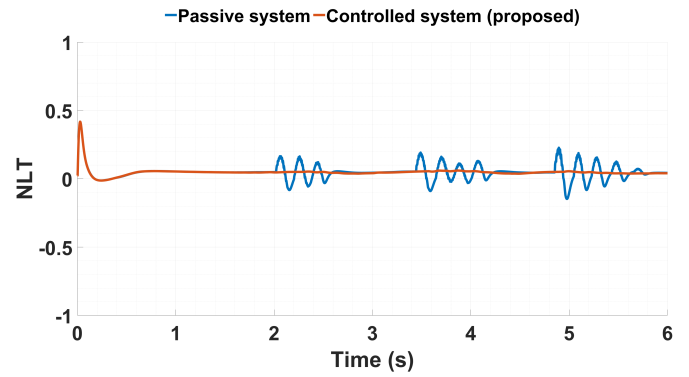


Fig. 16. NLT of front axle under the experiment 2

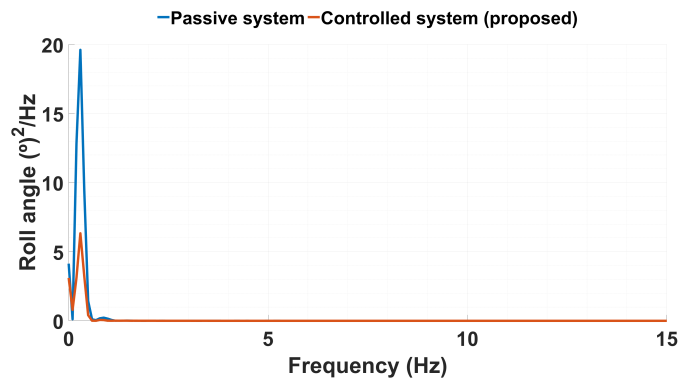


Fig. 14. Roll angle PSD under the experiment 2

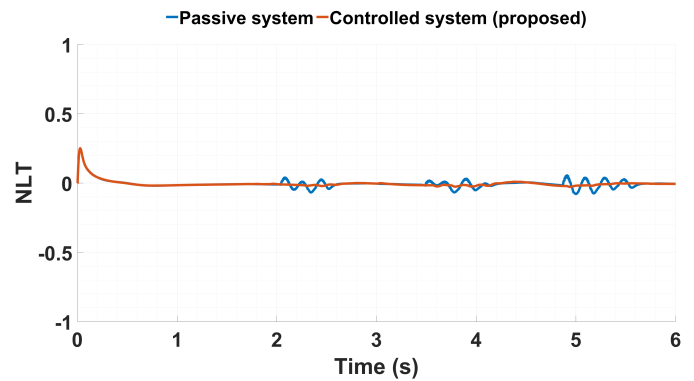


Fig. 17. NLT of rear axle under the experiment 2

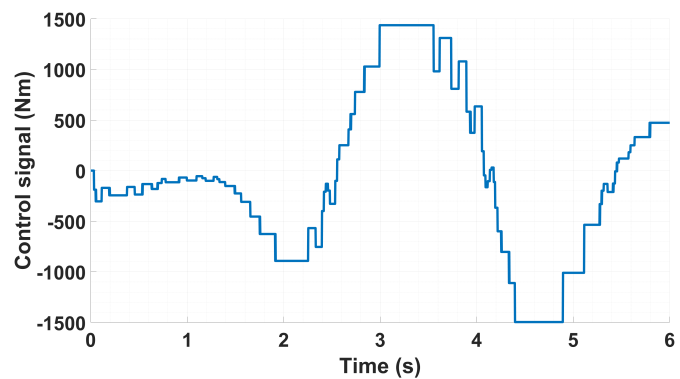


Fig. 15. Control signal under the experiment 2

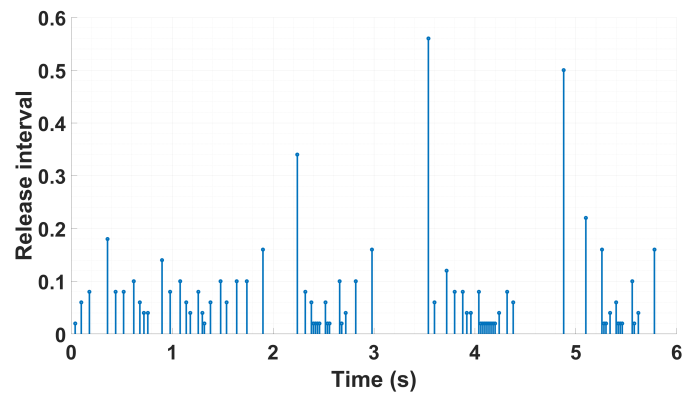


Fig. 18. Event Triggering instants under the experiment 2

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